The Heterotic String:

From Super-Geometry to the LHC

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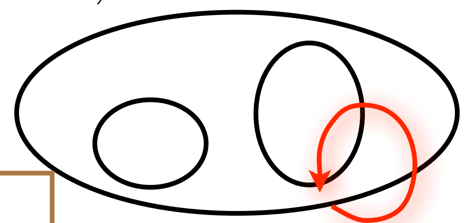
Heterotic Compactifications

$D = 10, \quad g_{MN}, \quad A^a_M, E_8$

$X, D = 6$



V,G

"slope" stable

W,F

$\mathbb{R}^4$

$N = 1\ SUSY$

$H = [E_8, G]$

$\mathcal{H} = [H, F]$

$$H^1(V)^F \Rightarrow matter$$

$$H^1(V^*)^F \Rightarrow conjugate\ matter$$

$$H^1(\wedge^2 V)^F \Rightarrow Higgs$$

$$H^1(V \otimes V^*)^F \Rightarrow Bundle\ Moduli$$

- Heterotic Standard Model: $V\ , G = SU(4)$, $W\ , F = \mathbb{Z}_3 \times \mathbb{Z}_3$

  Braun, He, Ovrut, Pantev 2006

$\mathbb{R}^4$ Theory Gauge Group:

Gauge connection $G = SU(4) \Rightarrow$

$$E_8 \to \quad H = Spin(10)$$

Wilson line $F = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow$

$$Spin(10) \to \quad \mathcal{H} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

rank Spin(10)=5 plus F Abelian ⇒ extra gauged $U(1)_{B-L}$.

Note that

$$\mathbb{Z}_2 \ (R - \text{parity}) \subset U(1)_{B-L}$$

⇒ no rapid proton decay. But must be spontaneously broken above the scale of weak interactions.

$\mathbb{R}^4$ Theory Spectrum:

$$E_8 \xrightarrow{V} Spin(10) \Rightarrow$$

$$248 = (1, 45) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 10) \oplus (15, 1 \ )$$

The Spin(10) spectrum is determined from $n_R = h^1(X, U_R(V))$.
For example,

$$n_{16} = h^1(X, V) = 27$$

$$Spin(10) \xrightarrow{F} SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \Rightarrow$$

The $3 \times 2 \times 1_Y \times 1_{B-L}$ spectrum is determined from $n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3}$. For example, $R = \mathbf{16}$

Tensoring and taking invariant subspace gives 3 families of quarks/leptons each transforming as

$$Q_L = (3,2,1,1), \quad u_R = (\bar{3},1,-4,-1), \quad d_R = (\bar{3},1,2,-1)$$

$$L_L = (1,2,-3,-3), e_R = (1,1,6,3), \quad \nu_R = (1,1,0,3)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.

Similarly we get 1 pair of Higgs-Higgs conjugate fields

$$H = (1,2,3,0), \quad \bar{H} = (1,\bar{2},-3,0)$$

That is, we get exactly the matter spectrum of the MSSM!

In addition, there are $n_1 = h^1(X, V \times V^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli

$$\phi = (1,1,0,0)$$

Supersymmetric Interactions:

The most general superpotential is

$$W=\sum_{i=1}^{3}(\lambda_{u,i}Q_iHu_i+\lambda_{d,i}Q_i\bar{H}d_i+\lambda_{\nu,i}L_iH\nu_i+\lambda_{e,i}L_i\bar{H}e_i)$$

Note B-L symmetry forbids dangerous B and L violating terms

$$LLe, \quad LQd, \quad udd$$

Can we evaluate Yukawa couplings from first principles? Yes!

a) Texture:

$$W=\ldots\lambda LHr+\ldots$$

Braun, He, Ovrut 2006

⇒ a Yukawa coupling is the triple product

$$H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3}\otimes H^1(X,\wedge^2V)^{\mathbb{Z}_3\times\mathbb{Z}_3}\otimes H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3}\longrightarrow\mathbb{C}$$

Internal super-geometry (X elliptically fibered over dP9 base) ⇒ in flavor diagonal basis for each of u,d,ν,e

$$\lambda_1=0, \quad \lambda_2,\lambda_3\neq0$$

That is, naturally light first family and heavy second/third families.

b) Explicit Calculation:

Braun, Brelidze, Douglas, Ovrut 2008
Anderson, Braun, Karp, Ovrut 2009

The triple product ⇒

$$\lambda = \int_X \sqrt{g_{\mu\nu}}\psi_L^a \psi_H^{[b,c]} \psi_r^d \epsilon_{abcd} d^6 x$$

where

$$\nabla^2_{**}\psi^* = \lambda\psi^* \ , \lambda = 0$$

⇒ need to calculate the metric and eigenfunctions of the Laplacian. Unfortunately, a Calabi-Yau manifold does not admit a continuous symmetry. ⇒ the metric, gauge connection and, hence, the Laplacian are unknown! Remarkably, these can be well-approximated by numerical methods.

Ricci-Flat Metrics, Scalar Laplacians and Gauge Connections on Calabi-Yau Threefolds

Let $s_\alpha, \alpha = 0, \ldots, N_k - 1$ be degree-k polynomials on the CY and $h_{\rm bal}^{\alpha\bar\beta}$ a specific matrix. Defining

$$g^{(k)}_{({\rm bal})i\bar j} = \frac{1}{k\pi}\partial_i\partial_{\bar j} \ln \sum_{\alpha,\bar\beta=0}^{N_k-1} h_{\rm bal}^{\alpha\bar\beta} s_\alpha \bar s_{\bar\beta}$$

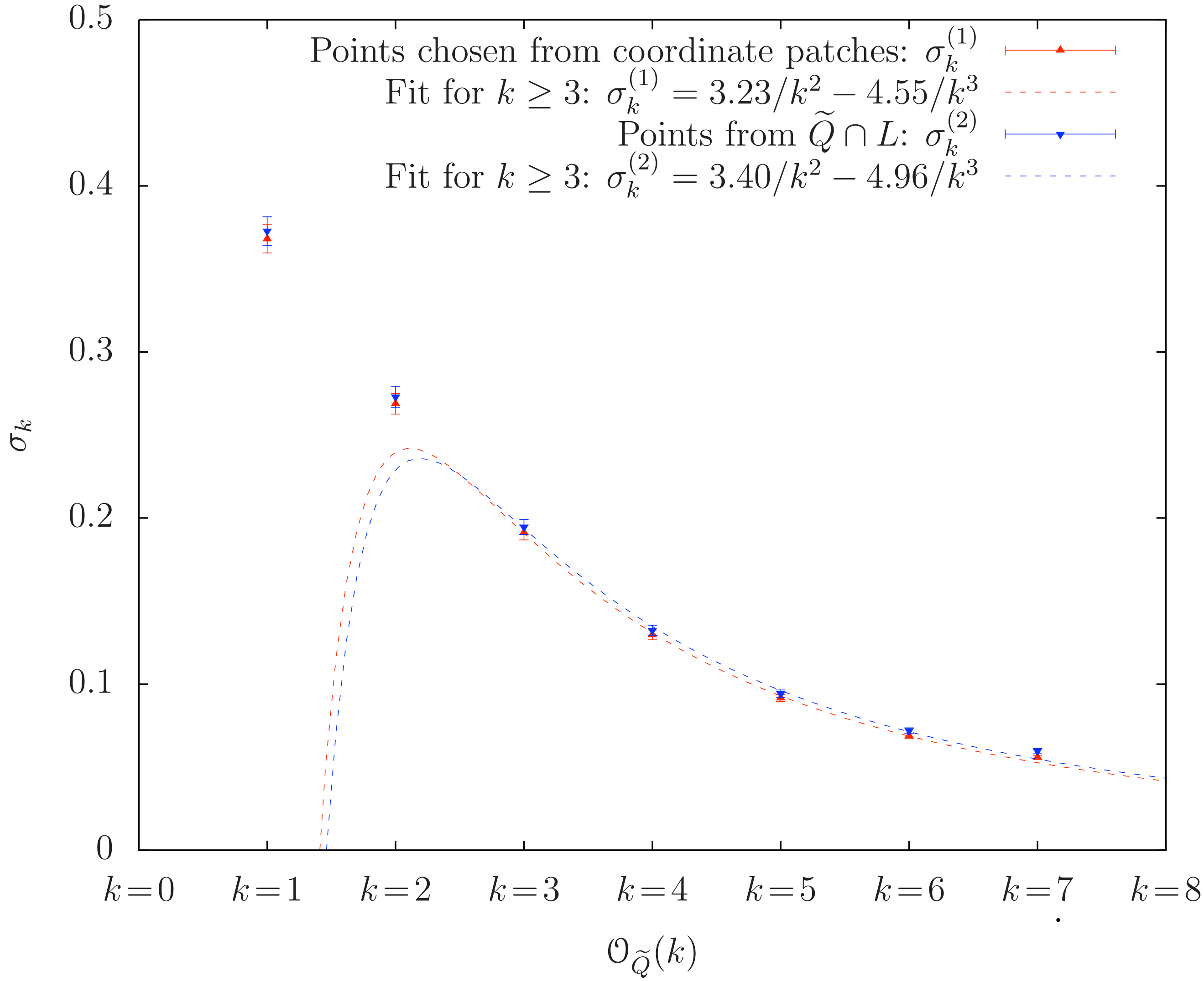
then

$$g^{(k)}_{({\rm bal})i\bar j} \xrightarrow{k\to\infty} g^{CY}_{i\bar j}$$

Expressed this way, $g^{(k)}_{({\rm bal})i\bar j}$ at any finite k is not very enlightening. More interesting is how closely they approach $g^{CY}_{i\bar j}$ for large k. This can be estimated using

$$\sigma_k(\tilde Q) = \frac{1}{Vol_{CY}(\tilde Q)} \int_{\tilde Q} \left| 1 - \frac{\omega_k^3 \Big/ Vol_K(\tilde Q)}{\Omega \wedge \bar\Omega \Big/ Vol_{CY}(\tilde Q)} \right| dVol_{CY}$$

Fermat quintic:



The error measure σ_k for the metric on the Fermat quintic, computed with the two different point generation algorithms

Scalar Laplacians:

Given a metric $g_{\mu\nu} \Rightarrow$

$$\Delta = -\frac{1}{\sqrt{g}}\partial_\mu(g^{\mu\nu}\sqrt{g}\partial_\nu)$$

Solve the eigen-equation

$$\Delta\phi_{m,i} = \lambda_m\phi_{m,i}\ ,\ i = 1,\ldots\mu_m$$

where μ_m is the multiplicity from continuous/finite symmetry.

Choose a basis $\{f_a|a = 1,\ldots,k\} \Rightarrow$ the eigen-equation becomes

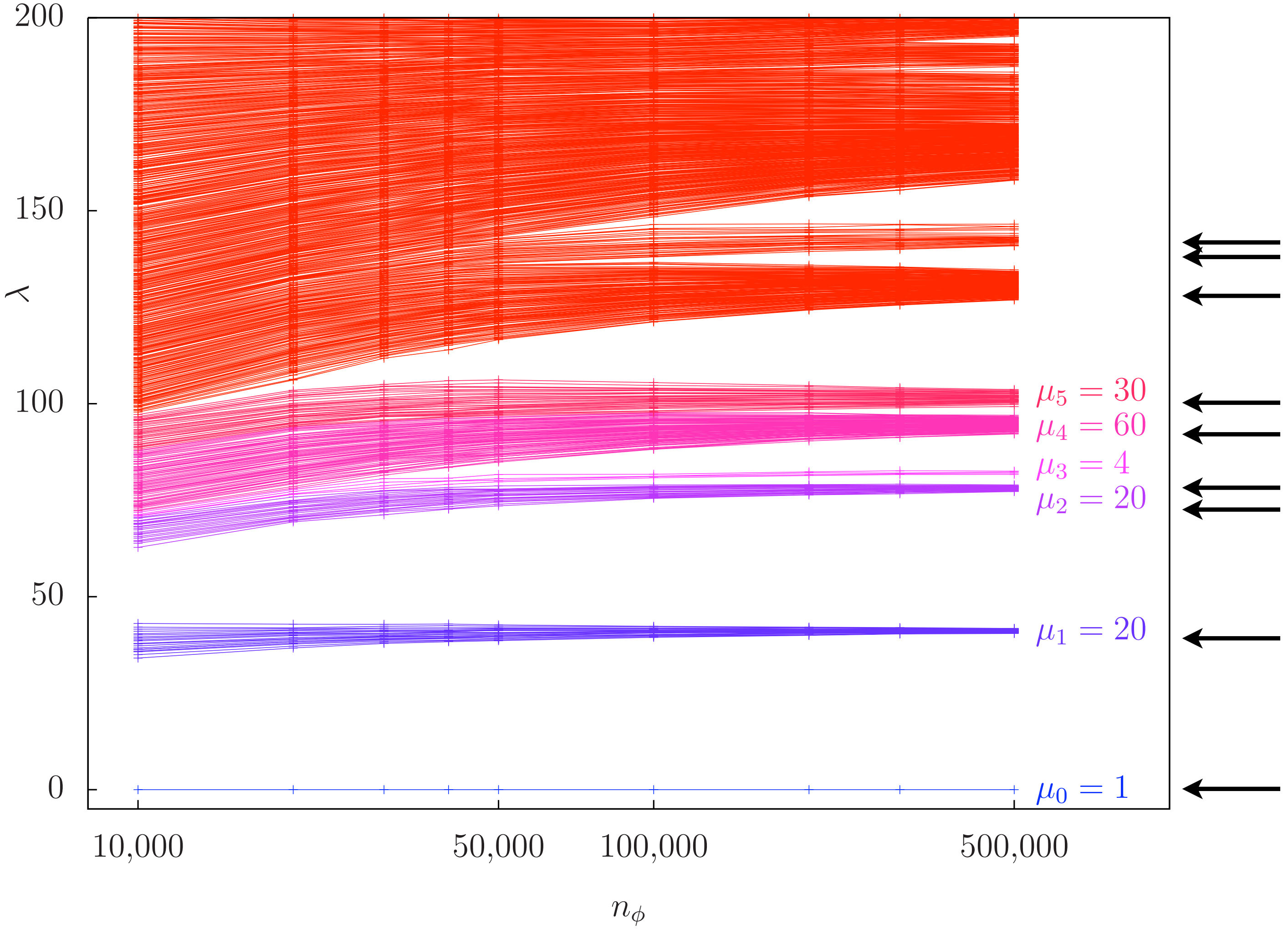
$$\sum_b \langle f_a|\Delta|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle = \sum_b \lambda_m\langle f_a|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle$$

Numerical Solution:

1) Solve numerically for λ_n and ϕ_n

2) For fixed k let $n_\phi \rightarrow \infty$

Fermat quintic:



Eigenvalues of the scalar Laplace operator on the Fermat quintic. The metric is computed at degree $k_h = 8$, using $n_h = 2{,}166{,}000$ points. The Laplace operator is evaluated at degree $k_\phi = 3$ using a varying number n_ϕ of points.

SU(N) Gauge Connections:

Let $z^a_\alpha, \ \alpha = 0, \ldots, N_{k_H} - 1$ be degree-k_H polynomials on the CY carrying the N-representation of U(N) and $H^{\alpha\bar{\beta}}_{\text{bal}}$ a specific matrix. Defining an SU(N) connection

$$A^{(k_H)a\bar{b}}_{\text{(bal)i}} = \partial_i (ln \sum_{\alpha,\bar{\beta}}^{N_{k_H}-1} H^{\alpha\bar{\beta}}_{\text{bal}} z^a_\alpha \bar{z}^{\bar{b}}_{\bar{\beta}} - g^{a\bar{b}} ln \sum_{\alpha,\bar{\beta}}^{N_{k_H}-1} h^{\alpha\bar{\beta}}_{\text{bal}} s_\alpha \bar{s}_{\bar{\beta}})$$

then

$$A^{k_H}_{\text{(bal)i}} \stackrel{k_H \to \infty}{\longrightarrow} A^H_i$$

where A^H_i satisfies the Hermitian Yang-Mills equations. That is

$$\omega^{i\bar{j}} F^{(k_H)}_{\text{(bal)i}\bar{\text{j}}} = \omega^{i\bar{j}} \partial_{\bar{j}} A^{(k_H)}_{\text{(bal)i}} \stackrel{k_H \to \infty}{\longrightarrow} 0$$

Expressed this way $A^{k_H}_{\text{(bal)i}}$ at any finite k_H is not enlightening. More interesting is how closely they approach A^H_i for large k_H. This can be estimated using

$$\tau_{k_H}(A) = \frac{1}{2\pi V_{CY}(\tilde{Q})} \int_{\tilde{Q}} \sum_{a=1}^{N} |\lambda_a| dVol_{CY} \quad \text{where} \quad \omega^{i\bar{j}} F^{(k_H)}_{(\text{bal})\text{i}\bar{\text{j}}} = diag(\lambda_1, \ldots, \lambda_N)$$

Fermat quintic:

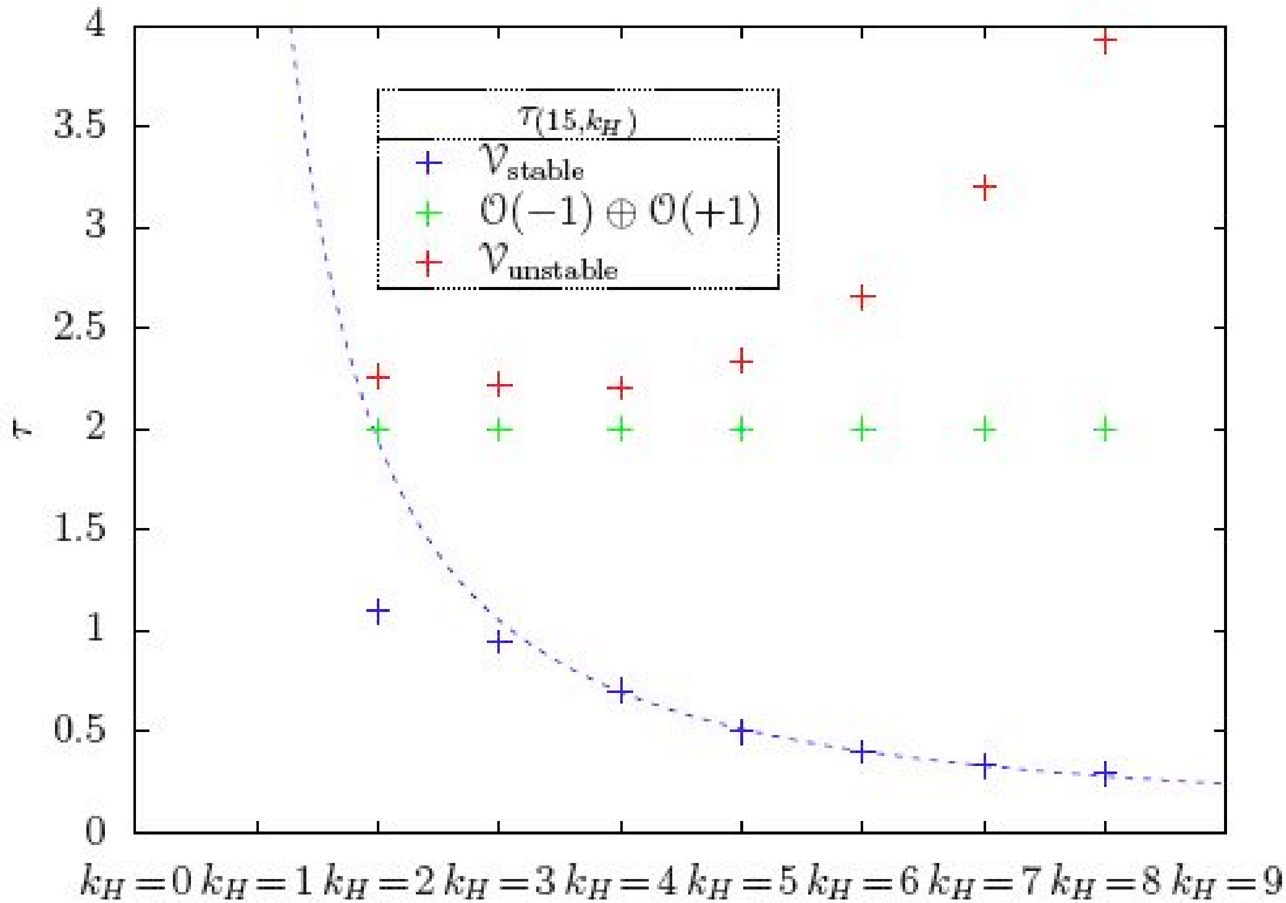


Figure 7: $\tau^{(untw)}_{(15,k_H,15)}$.

Supersymmetry Breaking, the Renormalization Group and the LHC

Ambroso, Ovrut 2009

Soft Supersymmetry Breaking:

N=1 Supersymmetry is spontaneously broken by the moduli during compactification ⇒ soft supersymmetry breaking interactions. The relevant ones are

$$V_{2s} = m_{\nu_3}^2 |\nu_3|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 - (BH\bar{H} + hc) + \dots$$

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \dots$$

At the compactification scale $M_C \simeq 10^{16} GeV$ these parameters are fixed by the vacuum values of the moduli. For example

$$m_{\nu_3}^2 = m_{\nu_3}^2(\langle \phi \rangle)$$

However, at a lower scale μ measured by $t = ln(\frac{\mu}{M_C})$ these parameters change under the renormalization group.

For example,

$$16\pi^2 \frac{dm_{\nu_3}^2}{dt} \simeq \frac{3}{4} {g_4}^2 \sum_{i=1}^{3} (m_{\nu_i}^2 + \dots) \ , \ 8\pi^2 \frac{d\xi_{B-L}}{dt} = \dots + \sqrt{\frac{3}{4}} g_4 Tr(Y_{B-L} m^2)$$

Solving these, at a scale $\mu \simeq 10^4 GeV \Rightarrow t_{B-L} \simeq -25$

$$m_{\nu_3}(t_{B-L})^2 = m_\nu(0)^2 - 1.9 \ m_\nu(0)^2 \ , \quad \xi_{B-L}(t_{B-L}) = -8.57 \ m_\nu(0)^2$$

Including the D-term effect

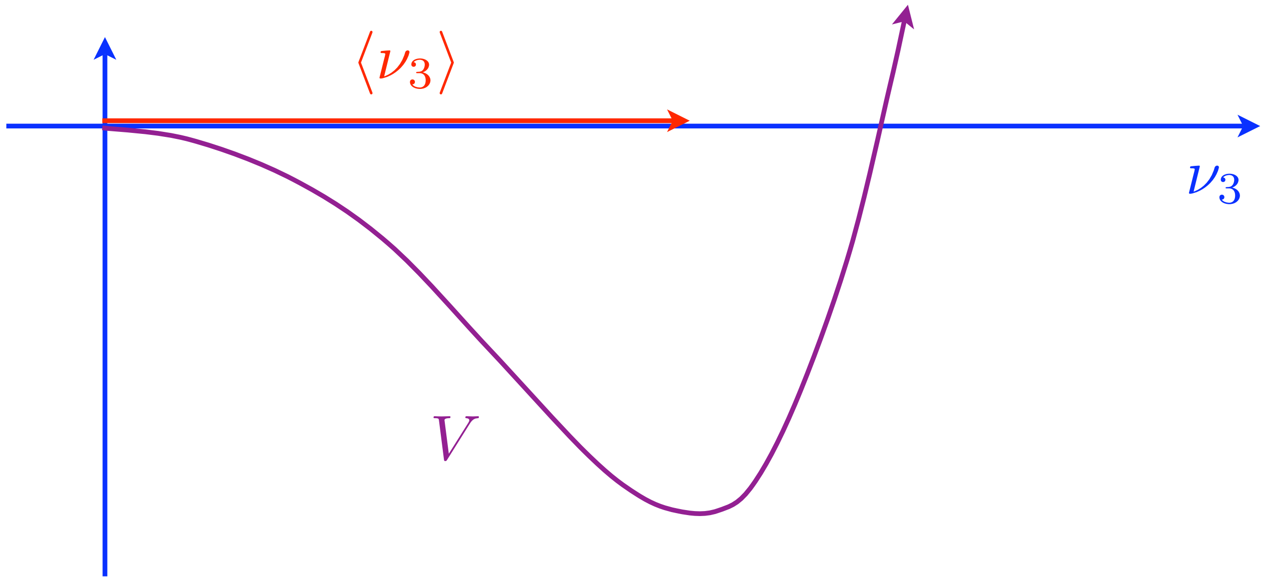
$$m_{\text{eff}\,\nu_3}(t_{B-L})^2 = m_{\nu_3}(t_{B-L})^2 + \sqrt{\frac{3}{4}} g_4 \xi_{B-L}$$

$\Rightarrow$

$$m_{\text{eff}\,\nu_3}(t_{B-L})^2 = -4 m_\nu(0)^2$$

Therefore, we expect the spontaneous breaking of B-L at t_{B-L}.

Result:



The vacuum expectation value at t_{B-L} is

$$\langle \nu_3 \rangle = \frac{2m_\nu(0)}{\sqrt{\frac{3}{4}} g_4}$$

⇒ a B-L vector boson mass of

$$M_{A_{B-L}} = 2\sqrt{2} m_\nu(0)$$

At this scale, no other symmetry is broken.

Similarly, at the electroweak scale $\mu \simeq 10^2 GeV \Rightarrow t_{EW} \simeq -29.6$

$$m_{H'}(t_{EW})^2 \simeq -\frac{\Delta^2}{tan\beta^2} m_H(0)^2 \quad , \quad m_{\bar{H}'}(t_{EW})^2 \simeq m_H(0)^2$$

where $tan\beta = \frac{\langle H \rangle}{\langle \bar{H} \rangle}$ and $0 < \Delta^2 < 1$ is related to $M_3(0)$. $\Rightarrow$ at t_{EW} electroweak symmetry is broken by the expectation value

$$\langle H'^0 \rangle = \frac{2\Delta \ m_H(0)}{tan\beta \sqrt{\frac{3}{5} g_1^2 + g_2^2}}$$

$\Rightarrow$ a Z-boson mass of

$$M_Z = \frac{\sqrt{2}\Delta \ m_H(0)}{tan\beta} \simeq 91 GeV$$

It follows that there is a B-L/EW gauge hierarchy given by

$$\frac{M_{A_{B-L}}}{M_Z} \simeq \frac{tan\beta}{\Delta}$$

Our approximations are valid for the range $6.32 \leq tan\beta \leq 40$.

For $\Delta = \frac{1}{2.5}$, the B-L/EW hierarchy in this range is

$$15.8 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 100$$

We conclude that this vacuum exhibits a natural hierarchy of $\mathcal{O}(10)$ to $\mathcal{O}(100)$ $\Rightarrow$

$$1.42 \times 10^3 GeV \lesssim M_{A_{B-L}} \lesssim 0.91 \times 10^4 GeV$$

All super-partner masses are related through intertwined renormalization group equations. ⇒ Measuring some masses predicts the rest!

The slepton and squark masses to leading order are

$$\langle\langle m^2_{\nu_{1,2}}\rangle\rangle \simeq 36.4\ m_H(0)^2, \quad \langle\langle m^2_{\nu_3}\rangle\rangle \simeq 8.87\ m_H(0)^2,$$
$$\langle\langle m^2_{N_i}\rangle\rangle \simeq \langle\langle m^2_{E_i}\rangle\rangle \simeq\ 6.65\ m_H(0)^2,\ \langle\langle m^2_{e_i}\rangle\rangle \simeq 4.75\ m_H(0)^2$$

and

$$\langle\langle m^2_{U_3}\rangle\rangle \simeq \langle\langle m^2_{D_3}\rangle\rangle \simeq 0.109\ m_H(0)^2,$$
$$\langle\langle m^2_{U_{1,2}}\rangle\rangle \simeq \langle\langle m^2_{D_{1,2}}\rangle\rangle \simeq 0.442\ m_H(0)^2,$$
$$\langle\langle m^2_{u_{1,2}}\rangle\rangle \simeq \langle\langle m^2_{d_i}\rangle\rangle \simeq 1.075\ m_H(0)^2,\ \langle\langle m^2_{u_3}\rangle\rangle \simeq 0.409\ m_H(0)^2$$

where

$$m_H(0) = \frac{tan\beta}{\sqrt{2}\Delta}\ M_Z$$

Note that all mass squares are positive and, hence, the B-L/EW vacuum is a stable local minimum!