Title:

Yukawa Couplings and Right-Handed Neutrinos in F-Theory Compactifications

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Outline

1. Yukawa Couplings and Bundles in Heterotic String
2. Yukawa Couplings and Singularities in F-theory
3. Extending Heterotic - F-theory Duality
4. Right Handed Neutrinos as Complex Structure Moduli
5. Yukawa Couplings for Neutral Fields
• SU(5): up-type quarks Yukawa couplings $10^{ij} \ 10^{kl} \ H(5)^m \epsilon_{ijklm}$

• $\epsilon_{ijklm}$ cannot be realised in brane configurations

• We can this term from a coset $E_6/SU(5)$ and then embed $E_6$ into $E_8$.

• Need the $E_8$ group obtained in Heterotic, M theory and F-theory

• We need to describe the breaking of $E_8$ to $SU(5)_{GUT}$. This can be done in both Heterotic and F-theory

• In heterotic $E_8 \times E_8$ turn on an $SU(5)$ vector bundle within one $E_8$

• The correspondence between the representations $\rho(V)$ and and those of the unbroken $SU(5)_{GUT}$ is:

• $V \leftrightarrow 10$ ($V$ is the 5 of vector bundle SU(5)), $\wedge^2 V \leftrightarrow \bar{5}$

• How to explicitly build the vector bundle?
Consider an elliptic fibered Calabi-Yau 3-fold $Z$, $\pi_Z : Z \rightarrow B_2$ and a rank-$N$ vector bundle $V$ on $Z$.

Need to build a spectral surface of degree $N$ over $B_2$ and a line bundle $\mathcal{N}_V$ over $C_V$.

The chiral multiplets in the low energy theory are identified with $H^1(Z, \rho(V))$.

This is non-zero only along the matter curves

$$\bar{c}_{\rho(V)} = C_{\rho(V)} \cdot \sigma$$

Specific example: Rank-5 Bundles

Spectral surface $C_V$ of rank-5 bundle $V$ is given by

$$s = a_0(u, v) + a_2(u, v)x + a_3(u, v)y + a_4(u, v)x^2 + a_5(u, v)xy = 0;$$

The matter curve of the fundamental representation $\bar{c}_V = C_V \cdot \sigma$ is given by the zero locus of $a_5$. 
The matter curve of $\wedge^2 V$: defining equation of the spectral surface factorises locally as

$$s = (Ax + B)(Py + Qx + R)$$

The factorisation condition is equivalent to

$$P^{(5)} := a_0 a_5^2 - a_2 a_3 a_5 + a_4 a_3^2 = 0$$

The two matter curves $\bar{c}_V$ and $\bar{c}_{\wedge^2 V}$ intersect in $B_2$ with two different types of intersections:

(a) Multiplicity 1: $a_5 = 0$ and $a_4 = 0$, and hence $P^{(5)} = 0$

(d) Multiplicity 2: $a_5 = 0$ and $a_3 = 0$, and hence $P^{(5)} = 0$.

The form of $P^{(5)}$ reveals that $\bar{c}_{\wedge^2 V}$ forms a double point at each type (d) intersection point.

The covering matter curve $\tilde{c}_{\wedge^2 V}$ is obtained by blowing up the double points of the matter curve $\bar{c}_{\wedge^2 V}$, and the map $\tilde{\pi}_D : D \rightarrow \tilde{c}_{\wedge^2 V}$ becomes a degree-2 cover.
Lesson: use the covering matter curves to describe Yukawa couplings

Yukawa couplings in F-theory

• Hints from Heterotic String but valid for models with no Heterotic dual

• $\mathcal{N} = 1$ supersymmetry: F-theory is compactified on an elliptic fibered Calabi–Yau 4-fold $\pi : X \to B_3$

• The discriminant $\Delta$ of this elliptic fibration may have several irreducible components

$$\Delta = \sum_i n_i S_i, \; S_i \text{ are divisors of } B_3, \; n_i \text{ their multiplicities}$$

For $\Delta = nS + D'$, matter multiplets charged under the gauge group on $S$ at $S \cdot D'$

• F-theory phenomenology: Beasley-Heckmann-Vafa and Donagi-Weijnholt

• Many aspects of gauge theory associated with the discriminant locus $S$, only on the geometry of $X$ around $S$. 
• The study of F-theory on $X$ reduces to the study of an 8-dimensional field theory on $S$ times Minkowski.

The matter multiplets “see” only the geometry along the $S \cdot D'$ codimension-2 loci of $B_3$

• The Yukawa couplings –from codimension-3 loci of $B_3$. Thus, one can go a long way in phenomenology by studying only the local geometry of F-theory compactification.

• Suppose a zero mode exists for $\phi_{mn}(u_1, u_2) du_m \wedge du_n$ on $S$ - transverse fluctuation of D7-branes in Type IIB orientifold compactification on a Calabi–Yau 3-fold $X$ ($(u_1, u_2)$ coordinates of $S'$).

• This corresponds to deforming geometry of $X$, $\Delta = n'' S'' + S' + D'$

• Singularity along the irreducible discriminant locus $S$ is reduced from $g$ to the commutant $g''$ of $\phi$ in $g$. 
Example: Generic Rank-2 Deformation of $A_{N+1}$ Singularity

The most generic form of deformation to $A_{N-1}$ is given by two parameters, $s_1$ and $s_2$:

$$Y^2 = X^2 + Z^N(Z^2 + s_1Z + s_2).$$  \hfill (1)

An alternative parametrization of deformation

$$2\alpha\phi_{12}(u_1, u_2) = (0, \cdots, 0^N, \tau_{N+1}, \tau_{N+2}),$$  \hfill (2)

Easy case: $s_1(u_1, u_2) = F_1u_1 + F_2u_2$, \hspace{1cm} $s_2(u_1, u_2) = F_1F_2u_1u_2$ so $2\alpha\phi_{12} = (0, \cdots, 0^N, F_1u_1, F_2u_2),$

The irreducible decomposition of $su(N+2)$ is

$su(N+2)$-$\text{adj.}$ $\rightarrow$ $su(N)$-$\text{adj}$ $+ \left[ N^{(-0)} + N^{(0,-)} + 1^{(+,-)} \right] + \text{h.c.}$
Zero-mode equations give solutions:

• For the $N^{(-,0)}$ and $\bar{N}^{(+,0)}$ components,

\[
\tilde{\chi}_\mp = c_\mp \exp \left[ -F_1 |u_1|^2 \right], \quad \tilde{\psi}_{\bar{1} \mp} = \pm c_\mp \exp \left[ -F_1 |u_1|^2 \right], \quad \tilde{\psi}_{\bar{2}} = 0. \tag{3}
\]

• For the $N^{(0,-)}$ component,

\[
\tilde{\chi} = c(u_1) \exp \left[ -F_2 |u_2|^2 \right], \quad \tilde{\psi}_{\bar{2}} = c(u_1) \exp \left[ -F_2 |u_2|^2 \right], \quad \tilde{\psi}_{\bar{1}} = 0. \tag{4}
\]

• This simple case has a IIB interpretation in terms of open strings between

$N -- N + 1$ D7-branes

$N -- N + 2$ D7 branes respectively
Complicated Case:

\[ s_1 = 2u_1, \quad s_2 = u_2, \]

This second case of the deformation of \( A_{N+1} \) singularity is described by the field theory on a local patch of \( S \) with \( 2\alpha \phi_{12} \) given by

\[
\tau_+ \equiv \tau_{N+1} = -u_1 + \sqrt{u_1^2 - u_2}, \quad \tau_- \equiv \tau_{N+2} = -u_1 - \sqrt{u_1^2 - u_2}.
\]

(5)

The decomposition is

\[ su(N + 2)\text{-adj.} \rightarrow su(N)\text{-adj.} + su(2)\text{-adj.} + (2, N) + (2, \bar{N}) \]

(6)

Resolve \( A_{N+1} \) with \( N + 1 \) cycles, 2 of them being
\[ C_{\pm} : (x, y, z) = (r(z)i \cos \theta, r(z) \sin \theta, z) \quad z \in [0, z_{\pm}] \quad \theta \in [0, 2\pi] \] 

\[ r(z) \equiv \sqrt{z^N(z - z_-)(z - z_+)} \] 

The vev of \(2\alpha\phi_{12},\)

\[ \left( +\sqrt{u_1^2 - u_2}, -\sqrt{u_1^2 - u_2} \right) \] 

becomes \(\times (-1)\) of its own around the branch locus \(u_1^2 - u_2 = 0.\)

Overall, we need to introduce a branch cut extending out from the branch locus \(u_1^2 - u_2 = 0\)

\(su(N+2)\text{-adj.}\) fields are glued to themselves after twisting by Weyl group of \(su(2) \subset su(N + 2)\) algebra—across the branch cut (cf. Katz-Morrison identification).

This is not a simple theory of fields in the \(su(N + 2)\text{-adj.}\) representation.
Introduce a new surface: $C(u, x), \ (u, x) = (u_1, \sqrt{u_1^2 - u_2})$ as the space of all possible values for $\phi$. This is a covering space similar to the Heterotic covering matter curve.

Apply this observation to $E_6, D_6 \rightarrow A_4$:

$A_4 : y^2 = x^3 + a_5xy + a_4zx^2 + a_3z^2y + ..., \ and \ consider \ case \ when$

$a_4 \rightarrow 0, \ a_5 \rightarrow 0 \ i.e. \ E_6 \rightarrow A_4: \ up$-type Yukawa

$a_3 \rightarrow 0, \ a_5 \rightarrow 0 \ i.e. \ D_6 \rightarrow A_4: \ down$-type Yukawa

The zero-mode wavefunctions of SU(5) - 10 representation are determined as $\text{diag}(-a_4/2 \pm \sqrt{a_4/2}^2 - a_5)$ and at small $a_4$ is goes like $e^{-|a_5|^{3/2}}$

The zero-mode wavefunctions of SU(5) - 5 representation are determined as $-a_4$ and has a Gaussian normal to the matter curve $e^{-|a_4|^2}$

For the $D_6 \rightarrow A_4$ there is no branch-cut and it can be represented by flat D7-branes intersecting at angles.
These are local models with wavefunctions defined on the surface S. The 0-modes should also be defined as line bundles of globally defined curves.

Main observation: the charged matter multiplets in F-theory are sheaves on spectral covers and not on matter curves. The spectral surfaces are key notion to generalize objects like D-branes and gauge bundles on them.

Supersymmetric compactification of F-theory is described by 8-dim. field theory with a Higgs bundle \((F, \phi)\) as background.

\[
\bar{\partial}_m \phi = 0, \partial_m \bar{\phi} = 0, F - i[\phi, \bar{\phi}] = 0
\]

are Hitchin equations for Higgs bundles.

For Higgs bundles, the techniques are very similar as the ones for spectral covers and one needs to build a pair \((C_V, N_V)\) denoted \((C_V, N_V)^F\)
Heterotic - F Theory duality. The duality map is simply stated:

\[(C_V, N_V)^{\text{Het}} = (C_V, N_V)^{\text{F}}.\] (10)

The spectral data is used in both sides of the duality and is mapped under duality.

Avoids some subtleties related to del Pezzo fibrations usually used in discussing Heterotic-F theory duality.

Important to map the heterotic \( N_V = 0 \left( \frac{1}{2}r + \gamma \right) \) into F-theory

\( \gamma \) corresponds to four-form fluxes in F-theory.
Four Form Fluxes and Neutrino Masses

Right-handed neutrinos $\bar{N}$ are not charged under $SU(5)_{GUT}$

$$\Delta \mathcal{L} = \lambda^{(\nu)}_{ij} \bar{N}_i l_j h_u + h.c.$$  

$l_j$ are lepton doublets and $h_u$ the Higgs doublet.

Any moduli chiral multiplet in supersymmetric string compactification can be identified with chiral multiplets of right-handed neutrinos, as long as they have the trilinear interactions.

For measured value of atmospheric neutrino oscillation

$$\Delta m^2 \sim 2-3 \times 10^{-3}$$

the lightest right-handed neutrino is not heavier than about

$$\frac{(v \lambda_{\nu})^2}{\sqrt{\Delta m^2}} = \lambda_{\nu}^2 \times (5.5-6.7) \times 10^{14}$$

Here, $\lambda_{\nu}$ is the neutrino Yukawa couplings.
The complex structure moduli have interactions in the superpotential

\[ \Delta W = W_{GVW} = \int_X \Omega \wedge G. \]

A generic flux \( G \) determine a mass for all the complex structure moduli from the Gukov–Vafa–Witten superpotential.

In Type IIB string compactification on Calabi–Yau orientifolds, complex structure moduli acquire masses

\[ m_{cs}^2 \sim m_{KK}^6 l_s^4 = \left[ m_{KK} \times \left( \frac{l_s}{R_6} \right)^2 \right]^2. \]

The complex structure moduli of F-theory compactifications contain both complex structure moduli and D7-brane moduli of Type IIB orientifolds.

The 4-form fluxes of F-theory correspond both to the 3-form fluxes and to gauge bundles on D7-branes in Type IIB orientifolds.
By using \( i \frac{1}{g_{s,\text{IIB}}} = i \frac{\rho \beta}{\rho} \) \( M_* = \frac{1}{g_s l_s^4} = \frac{\rho^2}{l_{11}^6} \)

we find that \( m_{cs} \sim \frac{1}{R_6^3 M_*^2} \) is valid as an estimate of all the complex structure moduli masses in F-theory compactification.

Set

\[
\epsilon \equiv \left( \frac{R_{\text{GUT}}}{R_6} \right)^3 = \frac{\sqrt{4\pi} M_{\text{GUT}}}{\alpha_{\text{GUT}} c M_{\text{Pl}}} \sim 0.35 \times \left( \frac{M_{\text{GUT}}}{c \times 10^{16} \text{GeV}} \right)
\]

with \( R_6 \) the size of \( B_3 \) and \( R_{\text{GUT}}^4 \) the volume of the locus of \( A_4 \) singularity, the masses of complex structure moduli become
\[ m_{cs} \sim M_{GUT} \times \sqrt{\frac{\alpha_{GUT}}{c}} \times (\epsilon^\gamma=1)^{0-3} \]

with \( \frac{1}{\alpha_{GUT}} = 24 \), \( M_{GUT} = 10^{16} \text{GeV} \),
\( M_{Pl}^2 = 4\pi R_6^6 M_{*}^8 = (2.4 \times 10^{18} \text{GeV})^2 \).

The \( SU(5)_{GUT} \) singlet field in the Yukawa couplings - fluctuations from
the vacuum in \( H^{1,2}(X, C) \) and \( H^{3,1}(X, C) \)

The \( H^{1,2}(X, C) \) fluctuations are calculated by the overlap integral
\[ \int_S tr(\chi_U \wedge \psi_{adj(U)} \wedge \psi_{\bar{U}}) \] with \( \chi_U, \psi_U \) coming from the fluctuations of
the chiral matter multiplet min the \((U, \bar{5})\) of \( G \times SU(5)_{GUT} \)

The \( H^{3,1}(X, C) \) fluctuations are calculated by the overlap integral
\[ \int_S tr(\psi_U \wedge \chi_{adj(U)} \wedge \psi_{\bar{U}}). \]
Conclusions

1. Heterotic Strings: It is important to extend the spectral sequence constructions to include other representations of V besides the fundamental.

Singularities need to resolved by considering the covering matter curves

2. F-theory picture: based on the 8-dimensional field theory plus the uplift of S to covering spaces.

Extend the Heterotic - F theory duality beyond orientifold limit. Use Higgs Bundles.

4. Right-hand handed neutrinos - complex moduli space