SU(5) GUTs from compact Type IIB orientifolds

based on
R. Blumenhagen, V. Braun, T. Grimm, T. W. 0811.2936
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Motivation

GUT model building classic topos in string phenomenology since 1985
new impulses from recent GUT model building advances in F-theory

Donagi/Wijnholt and Beasley/Heckman/Vafa 2008

F-theory:

• IIB compactification with D-branes on fully backreacted manifold
• sensitive to non-perturbative effects beyond Type IIB

work on F-theory GUTs so far: properties of local quivers

except: Tatar et al.'08; Andreas, Curio; Donagi, Wijnholt; Marsano et al. ’09

Global consistency conditions are at the heart of string theory:
distinguish string landscape from swampland

at present consistency conditions better understood in Type IIB language
(esp. gauge flux)

most promising avenue for unification from F-theory is via SU(5) GUT
⇒ This is (and has been for a while) amenable to Type II methods
Motivation

This talk: Systematic analysis of SU(5) GUTs in IIB vacua

Aim: implementation of GUT quivers into actual string vacua as opposed to local quivers

- Explicit **Type IIB vacua serve as starting point for F-theory models upon uplifting**
  Do useful geometric properties of Type IIB Calabi-Yau 3-fold survive?
- Ultimate goal: model building in combination with moduli stabilisation
  ← required for satisfactory discussion of SUSY breaking, predictions...
  **Type IIB orientifolds on genuine (conformal) Calabi-Yau promising**
Outline

1) Motivation

2) Background on Type IIB orientifolds with D3/D7-branes
   • gauge flux
   • global consistency conditions
   • massless matter

3) SU(5) GUT model building in Type IIB orientifolds
   • GUT breaking
   • non-perturbative Yukawa couplings

4) Explicit construction of semi-realistic SU(5) vacua

5) Conclusions
Type IIB Orientifolds

- Compactification of Type IIB theory on Calabi-Yau $X$
- orientifold: divide by $\Omega(-1)^F L \sigma$, $\sigma$ : holomorphic involution of $X$

\[ \sigma^* J = J, \quad \sigma^* \Omega = -\Omega \]

split into even/odd cycles :

$h^1,1_+ Kähler moduli T_I = \int_{\Gamma^+} e^{-\phi} J \wedge J + iC_4,$

$h^1,1_- B$-field moduli $G_i = \int_{\gamma^-} -B + iC_2$

discrete $B$-field parameter $\frac{1}{2\pi} \int_{\gamma^+} B = 0, \frac{1}{2}$

$\Rightarrow$ fix-point set $O3/O7$-planes $\leftrightarrow$ spacetime-filling $D3/D7$-branes

- D3-brane: point on internal $X$
- D7-brane: wraps holomorphic 4-cycle (divisor) $D_a$

upstairs geometry: $D_a + \text{image } D'_a$

1) $D_a$ not invariant: $D_a \rightarrow D'_a \Rightarrow$ gauge group $U(N_a)$
2) $D_a$ invariant: $2N_a \Rightarrow SO(2N_a)/Sp(2N_a)$

Grimm, Louis ’05
Gauge Flux

Gauge flux on D-brane: \[ F_a = F_a + \iota^* B \]

focus on \( \langle F_a \rangle \neq 0 \) for abelian subgroups ⇔ line bundles \( L_a \)

- typical embedding: diagonal \( U(1) \subset U(N) \rightarrow SU(N) \times U(1) \)
  \( U(1) \) massive by Green-Schwarz mechanism
- can also switch on \( U(1) \subset SU(N) \)

gauge flux on divisor \( D \) ⇔ \( \langle F \rangle \in H^2(D) \) ⇔ 2-cycle \( \in H_2(D) \)

2 types of non-trivial 2-cycles on \( D \): \( \text{Lerche, Mayr, Warner'01/02; Jockers,Louis'05} \)

non-trivial also on \( X \) vs. boundaries of 3-chains on \( X \)

splitting \( \quad L_a = \iota^* L_a \otimes R_a \)

flux \( R_a \) does
- not affect chiral spectrum and
- not participate in GS mechanism \( \text{Buican et al.'06} \)
Global consistency conditions (I)

1) Freed-Witten quantisation condition on line bundles
   path-integral of open string worldsheet with boundary on single U(1) brane $D$ must be well-defined

Result: shift in Dirac quantisation condition

\[ c_1(L) - \iota^* B + \frac{1}{2} c_1(K_D) \in H^2(D, \mathbb{Z}) \]

\[ \Rightarrow L \text{ half-integer quantised} \]

- for discrete B-field $\int_{\gamma_+} B = \frac{1}{2}$
- for divisor $D$ not spin, i.e. $c_1(K_D) \in H^2(D, (2\mathbb{Z} + 1)/2)$

choice of B-field determines quantisation on several divisors at once!

Generalisation to more general embedding: 

\[ T_0 \left( c_1(L_a^{(0)}) - \iota^* B \right) + \sum_i T_i c_1(L_a^{(i)}) + \frac{1}{2} T_0 c_1(K_{D_a}) \in H^2(D_a, \mathbb{Z})_{N_a \times N_a} \]

\[ \Rightarrow \text{suitably fractional line bundles are allowed!} \]
Global consistency conditions (II)

2) Tadpole cancellation condition from CS action of D-brane and O-plane

- cancellation of D7-charge and induced D5-charge
  see also: Collinucci,Esole,Denef; Plauschinn’08

- D3:

\[ N_{D3} + \frac{N_{\text{flux}}}{2} - \sum_a \int_{D_a} \frac{\text{tr} F_a^2}{8\pi^2} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{12} + \sum_a N_a \frac{\chi_0(D_a)}{24} \]

\[ \chi(CY_4)/12 \text{ in } F\text{–theory} \]

constraint by integrality of \( N_{D3} \in \mathbb{Z}_0^+ \) ↔ Freed-Witten quantisation!

models with non-spin divisors tricky!

3) D-term supersymmetry

Fayet-Iliopoulos D-term: \( \xi \sim \int_D i^* J \wedge c_1(L) \)  
Marino et al.’99

\[ \xi_a = 0 \rightarrow -\frac{N_a}{2} \int_{D_a} c_1^2(L_a) \geq 0 \]  
Blumenhagen,Braun,Grimm,T.W.’08

⇒ SUSY bundles always contribute positively to D3-tadpole

↔ danger of overshooting
Massless Matter

adjoint chiral fields: \( h^{(0,2)}(D) \) deformation and \( h^{(0,1)}(D) \) Wilson moduli

charged chiral matter at intersection of D-branes along divisors \( D_a \) and \( D_b \)

open strings in

- \( a \rightarrow b \) sector: bifundamental matter \( (\square_a(-1), \square_b(1)) \)
- \( a' \rightarrow a \) sector: (anti-)symmetric matter \( \square(2) / \square(2) \)

- \( D_a = D_b \rightarrow \) matter localised on whole divisor \( D_a \)
  \( \exists \) generically vector-like pairs

- \( D_a \neq D_b \rightarrow \) matter localised on curve \( C_{ab} = D_a \cap D_b \)
  generically no vector-like pairs

either case: index \( I_{ab} = - \int_X [D_a] \wedge [D_b] \wedge (c_1(L_a) - c_1(L_b)) \)

relative flux only affects vector-like spectrum
SU(5) GUTs

starting point: $U(5)_a \times U(1)_b$ theory $U(1)_{a,b}$ massive

GUT brane: $(D_a, L_a)$, U(1) brane $(D_b, L_b)$

<table>
<thead>
<tr>
<th>sector</th>
<th>reps.</th>
<th>particle</th>
<th>sector</th>
<th>reps.</th>
<th>particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a', a)$</td>
<td>$10_{(2,0)}$</td>
<td>$(Q_L, u_R^c, e_R^c)$</td>
<td>$(b', b)$</td>
<td>$1_{(0,2)}$</td>
<td>$N_R^c$</td>
</tr>
<tr>
<td>$(a, b')$</td>
<td>$\overline{5}_{(-1,-1)}$</td>
<td>$(d_R^c, L)$</td>
<td>$(a, b)$</td>
<td>$5^H_{(1,-1)} + \overline{5}^H_{(-1,1)}$</td>
<td>$(T^u, H^u) + (T^d, H^d)$</td>
</tr>
</tbody>
</table>

matter localised on intersection loci

$10_{(2,0)} \iff H^*(D_a \cap D_{a'}, L_a^2)$

$\overline{5}^m_{(-1,-1)} \iff H^*(D_a \cap D_{b'}, L_a^{-1} \otimes L_{b'})$ no vectorlike exotics: $D_a \neq D_{a'}$

Two major challenges:

- complete description of GUT breaking:
  same solution exists in IIB/F-theory
  cannot be treated locally - depends on global data!

- Yukawa couplings:
  distinct approaches in IIB/F-theory
SU(5) GUT breaking

Idea: **Embed** \( U(1)_Y \subset U(5) \) [Beasley, Heckman, Vafa; Donagi, Wijnholt '08]

Approach very sensitive to global consistency:

1) \( U(1)_Y \) massless iff \( L_Y \) in relative cohomology

rigid GUT divisor must possess 2-cycles trivial on ambient space

global feature - depends on compactification details, not on local data!

2) Freed-Witten quantisation: [Blumenhagen, Braun, Grimm, T.W. '08]

\[
L_a \leftrightarrow T_a, \quad L_Y \leftrightarrow \frac{2}{5}T_a + \frac{1}{5}T_Y \quad T_a = 1_{5 \times 5}, T_Y = \text{diag}(-2, -2, -2, 3, 3)
\]

GUT breaking: \( U(5)_a \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)_a \)

From general quantisation condition:

\[
c_1(L_a) - \iota^* B + \frac{1}{2}K_{D_a} \in \mathbb{Z}, \quad c_1(L_a) + c_1(L_Y) - \iota^* B + \frac{1}{2}K_{D_a} \in \mathbb{Z}
\]

- for non-spin divisor \( D_a \) and \( B = 0 \):
  
  \( L_a \not= \mathcal{O} \) such that \( L_a \) half-integer quantised

- \( L_Y \) integer quantised
Top Yukawas

Yukawa couplings from triple intersection of 3 matter curves

Problem: $10^{(2,0)}$ $10^{(2,0)}$ $5_H^{(1,-1)}$ forbidden perturbatively in Type IIB

• Solution: Stringy D-brane instantons Blumenhagen, Cvetič, Weigand; Ibáñez, Uranga; Florea, Kachru, McGreevy, Saulina ’06

Euclidean D3-brane along divisor $\Xi$ with $\Xi \cap D_{a,b} \neq 0$
$\sim$ charged fermionic zero modes $\lambda^i_a$, $\lambda^j_b$ induce coupling

$W_{n.p.} \ni Y_\alpha Y_\beta \quad 10^\alpha 10^\beta 5_H e^{-\frac{Vol_E}{gs}}$ if $I_{a,\Xi} = 1 = I_{b,\Xi}$

Blumenhagen, Cvetič, Lüst, Richter, Weigand 2007

• Drawback: realistic GUT models in Type II require

$S_{\text{inst.}} \sim \frac{Vol_E}{gs} \rightarrow 0$

Philosophy: Search for setup where by classical D-terms $Vol_E = 0$
quantum corrections will resolve this at $Vol_E = O(l_s)$

Note: GUT brane can still be large!
Summary of approach

General requirements on compact CY:

- divisor $D_a$ with $h^{(0,1)}(D_a) = 0 = h^{(0,2)}(D_a)$ for GUT brane
- existence of relative two-cycles on $D_a$ for $\mathcal{L}_Y$
- additional divisor $D_b$ with intersection $D_a \cap D_b$
- define orientifold action, preferrably such that $D_a' \neq D_a$

<table>
<thead>
<tr>
<th>SU(5) property</th>
<th>mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>no vector-like matter</td>
<td>localisation on curves</td>
</tr>
<tr>
<td>1 vector-like of Higgs</td>
<td>choice of line bundles</td>
</tr>
<tr>
<td>3-2 splitting</td>
<td>Wilson lines on $g = 1$ curve</td>
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<tr>
<td>3-2 split + no dim=$5$ $p^+$-decay</td>
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<tr>
<td>$10 \mathbf{5}_H$ Yukawa</td>
<td>perturb. or D3-instanton</td>
</tr>
<tr>
<td>$10 10 5_H$ Yukawa</td>
<td>presence of appropriate D3-instanton</td>
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</table>
Explicit constructions

del Pezzo transitions of quintic $Q = \mathbb{P}^4[5], \quad (h^{1,1} = 1, \quad h^{2,1} = 101)$

see also: Grimm,Klemm ’08

1st step in chain of transitions: $Q \rightarrow Q^{dP_6}$

• create $dP_6$ singularity by fixing some complex structure moduli
• blow up singularity by pasting in a $dP_6$

$\Rightarrow h^{1,1}(Q^{dP_6}) = 2, \quad h^{2,1}(Q^{dP_6}) = 90$

$\Rightarrow$ no open moduli: $dP_6$ is rigid ✓

del Pezzo: $\mathbb{P}^2$ with $n$ points blown up to a $\mathbb{P}^1$ curve $E_i$,

$H^{1,1}(dP_n) = \langle l, E_1, \ldots E_n \rangle, \quad l \cdot l = 1 = -E_i \cdot E_i$

canonical class $K = \mathcal{O}(f), \quad f = -3l + \sum_i E_i$

$h^{1,1}(Q^{dP_6}) = 2 \Rightarrow$ only $f$ is non-trivial on $Q^{dP_6}$

trivial ones: those orthogonal to $f$: $\langle l - E_1 - E_2 - E_3, E_i - E_{i+1} \rangle$

$\Rightarrow$ ingredients for massless $U(1)_Y$ ✓

c1($\mathcal{L}_Y$) = $E_i - E_j$ leads to no vectorlike exotics from breaking of 24
Explicit constructions

toric description to analyse intersection form and topology of divisors

scaling relations: \( \{x_i\} \simeq \{\lambda Q_1(x_i) x_i\} \simeq \{\mu Q_2(x_i) x_i\} \)

<table>
<thead>
<tr>
<th></th>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
<th>(u_4)</th>
<th>(u_5)</th>
<th>(w)</th>
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<tr>
<td>(Q_1)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(Q_2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>class</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H +X</td>
<td>X</td>
</tr>
</tbody>
</table>

\(Q^{dP_6} : P_{(5,2)}(u_i, v, w) = 0\)

sequence of transitions: fix more compl. structure and blow-up

- new del Pezzos intersect in \(\mathbb{P}^1\) \(\Rightarrow\) matter curves
- \(E_6\) sublattice of each higher \(dP_n\) is trivial on Calabi-Yau \(\Rightarrow U(1)_Y\)
Explicit constructions

2 types of involutions: inversion: \( x_i \rightarrow -x_i \) or exchange: \( x_i \leftrightarrow x_j \)

Focus on exchange involution on \( Q^{dP_9}_4 \):

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( z )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
</tr>
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<tbody>
<tr>
<td>( Q_1 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Q_4 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( Q_5 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>class</td>
<td>( D_5 )</td>
<td>( D_5 + D_9 )</td>
<td>( D_5 + D_6 )</td>
<td>( D_5 + D_8 )</td>
<td>( D_5 + D_7 )</td>
<td>( D_7 )</td>
<td>( D_8 )</td>
<td>( D_6 )</td>
<td>( D_9 )</td>
</tr>
</tbody>
</table>

Involution: \( \sigma : v_1 \leftrightarrow v_2, \quad w_1 \leftrightarrow w_2 \)

invariant: \( v_1 v_2, \quad w_1 w_2, \quad v_1 w_1 + v_2 w_2, \quad \text{anti-inv.:} \quad v_1 w_1 - v_2 w_2 \)

\( h^{1,1}_+ = 4, \quad h^{1,1}_- = 1 \)

O7-plane: \( v_1 w_1 - v_2 w_2 = 0, \quad [O7] = [D_5 + D_7 + D_8], \quad \chi(O7) = 37 \)

further: \( N_{03} = 3 \Rightarrow N_{03} + \chi(O7) = 40 \)
Explicit constructions

SU(5) GUT stack on $dP_9$:

$U(5)$: $D_a = D_7$, $D'_a = D_8$, $D7-TAD \checkmark$

$U(1)$: $D_b = D_5$, $D'_b = D_5$

$U(3)$: $D_c, = D_5 + D_7$, $D'_c = D_5 + D_8$

matter curves:

10: $D_7 \cap D_8 = \mathbb{P}^1$  5$_m$: $D_7 \cap D_5 = T^2$  5$_H + \overline{5}_H$: $D_8 \cap D_5 = T^2$

find line bundles + B-field that are quantised properly

(Freed-Witten: divisors are non-Spin!)

possible to obtain exactly 3 generations, 1 vectorlike Higgs pair, no exotics

Drawbacks:

• Overshooting of D3-TAD by 3 units
• uncertainty of one K-theory constraint
Semi-realistic global example

Example on manifold $Q^{dP_9^4}$: GUT brane on $dP_9$

[Blumenhagen, Braun, Grimm, Weigand 0811.2936]

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<tr>
<td>globally consistent</td>
<td>tadpoles + K-theory</td>
<td>✓∗∗∗</td>
</tr>
<tr>
<td>D-term susy</td>
<td>vanishing FI-terms inside Kähler cone</td>
<td>✓</td>
</tr>
<tr>
<td>gauge group $SU(5)$</td>
<td>$U(5) \times U(1)$ stacks</td>
<td>✓</td>
</tr>
<tr>
<td>3 chiral generations</td>
<td>choice of line bundles</td>
<td>✓</td>
</tr>
<tr>
<td>no vector-like matter</td>
<td>localisation on $g = 0, 1$ curves</td>
<td>✓</td>
</tr>
<tr>
<td>5 vector-like Higgs</td>
<td>choice of line bundles</td>
<td>✓</td>
</tr>
<tr>
<td>no adjoints</td>
<td>rigid 4-cycles, del Pezzo</td>
<td>✓</td>
</tr>
<tr>
<td>GUT breaking</td>
<td>$U(1)_Y$ flux on trivial 2-cycles</td>
<td>✓</td>
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<td>–∗∗∗</td>
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</table>
Another class of geometries

Second type of backgrounds:
Elliptic fibration over del Pezzo $dP_n$ and flop transitions thereof

$$\pi : M_r \rightarrow B = dP_r$$

divisors of $M_n$:
- base $B$
- ell. fibration over 2-cyles on base intersecting along elliptic fibre

pullback divisors are $dP_9$
flops to $dP_8$ describable in toric language

Swiss cheese type intersection form: $Vol = \Gamma^3_0 - \Gamma^3_B - \sum_i \Gamma^3_i$
$\Rightarrow$ D-term constraint forces cycles on boundary of Kähler cone

involution of type $x \rightarrow -x$ leads to 3-generation examples with instanton generated Yukawas
Conclusions

Type IIB orientifolds suitable arena for SU(5) GUT model building

Recent technological input:
- GUT breaking by $U(1)_Y$ flux as in F-theory
- $10\ 10\ 5_H$ couplings in Type IIB by exotic D-brane instantons

Realisation of many, but not all phenomenologically desirable features in globally consistent Type IIB orientifolds

Open issues include:
- models with Higgs localised on different curves
- SUSY breaking?

Extension to proper F-theory models on the way via uplift

[Blumenhagen,Grimm,Jurke,T.W. 0906.0013] cf. Talk by R. Blumenhagen

Structure of used manifolds promising for moduli stabilisation:
'swiss cheese' form as in large volume scenario [Conlon, Quevedo et al.]
Dream: Explicit examples with all moduli stabilised!