Using the identity $\rho_1 - \rho_2 = x(\mathcal{M}_{12}^2 - \mathcal{M}_{1'2'}^2)$, one arrives at the expression

$$f_{LR}\,\frac{(\mathcal{M}_{12}^2 - \mathcal{M}_{1'2'}^2)^2}{\rho_1\rho_2}\,\frac{\eta^\mu\eta^\nu}{4P^{+2}}\,f^r_{\overline{1},\underline{1}g}\,f^r_{\overline{2},\underline{2}g}\;, \tag{135}$$

which means that the interaction is entirely free from the small-x divergences, as promised. Hence, it is not necessary to introduce the counterterm identified in Eq. (36) of [2] in the heavy-quark QCD including the gluon mass m_g but not accounting for the third polarization state. The third polarization is here accounted for using the auxiliary field ϕ.

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$$H_I = f_{LR}\,\mathcal{H}_0^{(2)} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\,\mathcal{H}_0^{(1)}\mathcal{H}_0^{(1)} \tag{136}$$

$$\mathcal{H}_0^{(2)} = \delta\Sigma + \delta m^2 + S_{\eta\eta} \tag{137}$$

$$H_I = \delta\Sigma + \delta m^2 + f_{LR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\,\mathcal{H}_0^{(1)}\mathcal{H}_0^{(1)} \tag{138}$$

$$H_I = \delta\Sigma + \delta m^2 + f_{LR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;(\Delta_{LIR}\Sigma + \Delta_{LIR}E) \tag{139}$$

$$E = E_g + E_{\eta\eta} \tag{140}$$

$$H_I = \delta\Sigma + \delta m^2 + (f_{LR} - f_{LI}f_{IR})\,S_{\eta\eta} + f_{LI}f_{IR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;(\Delta_{LIR}\Sigma + \Delta_{LIR}E) \tag{141}$$

$$\begin{aligned} H_I &= \delta\Sigma + \delta m^2 + [f_{LR} - f_{LI}f_{IR}]\,[S_{\eta\eta} + \Delta_{LIR}(E_g + E_{\eta\eta})] \\ &+ f_{LI}f_{IR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\Sigma \end{aligned} \tag{142}$$

$$\begin{aligned} H_I &= \delta\Sigma + \delta m^2 + (f_{LR} - f_{LI}f_{IR})\,[S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta} + \Delta_{LIR}E_g] \\ &+ f_{LI}f_{IR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\Sigma \end{aligned} \tag{143}$$

$$\begin{aligned} H_I &= \delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma + (f_{LR} - f_{LI}f_{IR})\,[S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta} + \Delta_{LIR}E_g] \\ &+ f_{LI}f_{IR}S_{\eta\eta} - f_{LI}f_{IR}\Delta_{LIR}\Sigma \end{aligned} \tag{144}$$

$$\begin{aligned} H_I &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} + \{f_{LR}\Delta_{LIR}E_g\}_C && \text{selfinteraction plus Coulomb} \\ &+ (f_{LR} - f_{LI}f_{IR})\,(S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta}) && \text{finite cancellation} \\ &- f_{LI}f_{IR}\,(\Delta_{LIR}E_g - S_{\eta\eta} + \Delta_{LIR}\Sigma) \end{aligned} \tag{145}$$

So far I was going for the cancellation according to the sequence of equations

$$H_I = f_{LR}\,\mathcal{H}_0^{(2)} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\,\mathcal{H}_0^{(1)}\mathcal{H}_0^{(1)} \tag{146}$$

$$\mathcal{H}_0^{(2)} = \delta\Sigma + \delta m^2 + S_{\eta\eta} \tag{147}$$

$$H_I = \delta\Sigma + \delta m^2 + f_{LR}S_{\eta\eta} + (f_{LR} - f_{LI}f_{IR})\;\Delta_{LIR}\,\mathcal{H}_0^{(1)}\mathcal{H}_0^{(1)} \tag{148}$$

$$H_I = \delta\Sigma + \delta m^2 + f_{LR}S_{\eta\eta} + f_{LR}(\Delta_{LIR}\Sigma + \Delta_{LIR}E) - f_{LI}f_{IR}(\Delta_{LIR}\Sigma + \Delta_{LIR}E) \tag{149}$$

$$\begin{aligned} H_I &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} \\ &+ f_{LR}\,(S_{\eta\eta} + \Delta_{LIR}E) - f_{LI}f_{IR}(\Delta_{LIR}\Sigma + \Delta_{LIR}E_g + \Delta_{LIR}E_{\eta\eta}) \end{aligned} \tag{150}$$

$$\begin{aligned} H_I &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} \\ &+ f_{LR}\,(S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta} + \Delta_{LIR}E_g) - f_{LI}f_{IR}(\Delta_{LIR}\Sigma + \Delta_{LIR}E_g + \Delta_{LIR}E_{\eta\eta}) \end{aligned} \tag{151}$$

$$\begin{aligned} &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} + \{f_{LR}\Delta_{LIR}E_g\}_C \\ &+ f_{LR}\,(S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta}) - f_{LI}f_{IR}(\Delta_{LIR}\Sigma + \Delta_{LIR}E_g + \Delta_{LIR}E_{\eta\eta}) \end{aligned} \tag{152}$$

$$\begin{aligned} H_I &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} + \{f_{LR}\Delta_{LIR}E_g\}_C && \text{selfinteraction plus Coulomb} \\ &+ \{(f_{LR})\,(S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta})\}_{\rm den} && \text{finite cancellation} \\ &- f_{LI}f_{IR}\Delta_{LIR}\,(\Sigma + E_g + E_{\eta\eta}) \end{aligned} \tag{153}$$

For comparison

$$\begin{aligned} H_I &= \left\{\delta\Sigma + \delta m^2 + \Delta_{LIR}\Sigma\right\}_{\rm self} + \{f_{LR}\Delta_{LIR}E_g\}_C && \text{selfinteraction plus Coulomb} \\ &+ (f_{LR} - f_{LI}f_{IR})\,(S_{\eta\eta} + \Delta_{LIR}E_{\eta\eta}) && \text{finite cancellation} \\ &- f_{LI}f_{IR}\,(\Delta_{LIR}E_g - S_{\eta\eta} + \Delta_{LIR}\Sigma) \end{aligned} \tag{154}$$