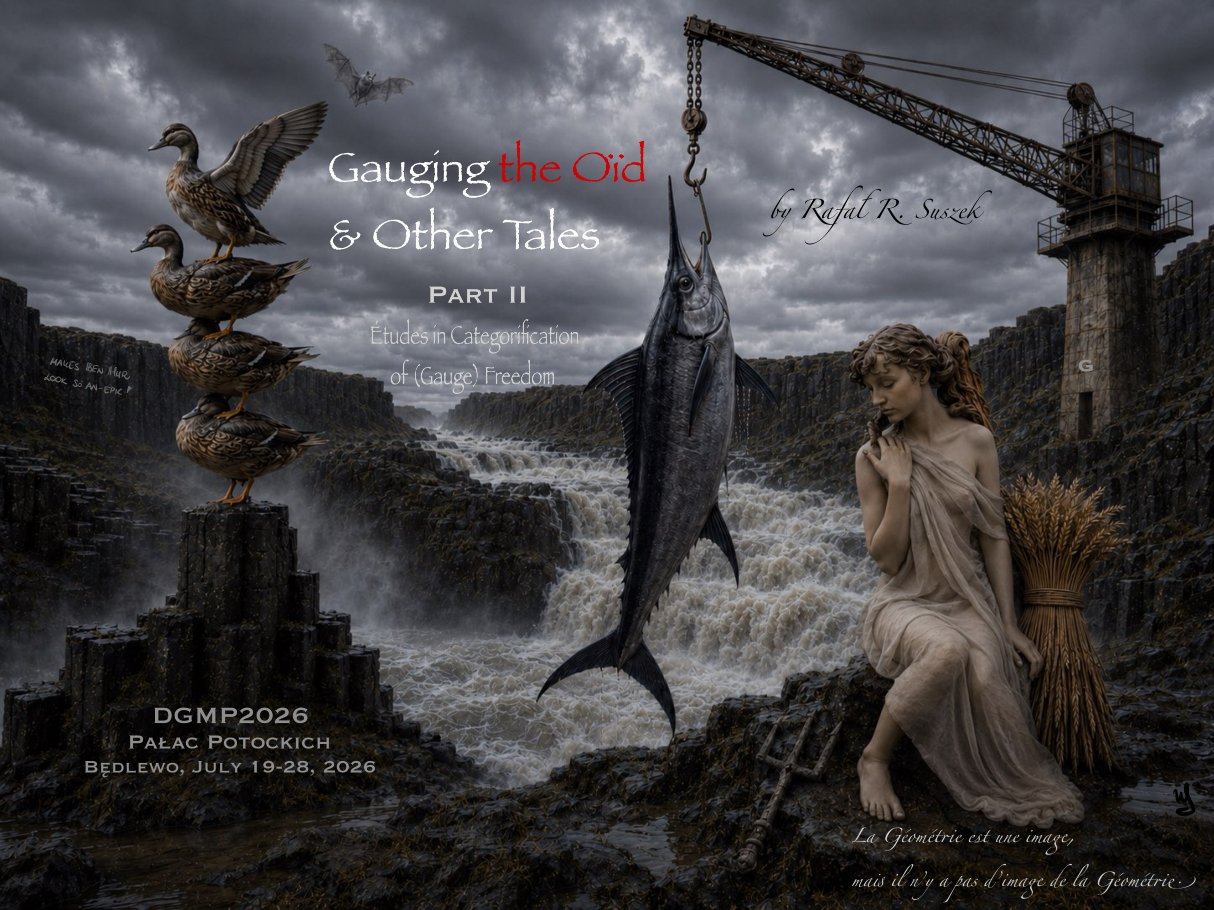
Gauging the Oïd
& Other Tales
Part II
Études in Categorification
of (Gauge) Freedom
by Rafał R. Suszek
MAKES BEN HUR LOOK SO AN-EPIC!
G
DGMP2026
Pałac Potockich
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La Géométrie est une image,
mais il n'y a pas d'image de la Géométrie.

IN THE LAST EPISODE :

(*) GEOMETRIC FORMULATION of FIELD THEORY

- CONFIGURATION BUNDLES & DF AMPLITUDES

(**) CATEGORIFICATION/FIBRATION of CONFIGURATIONAL SYMMETRY

- LIE GROUPOIDS & THEIR BISECTIONS

(***) PROTOTYPE of GAUGE SYMMETRY with TYPICAL FIBRE

- EHRESMANN–CARTAN : MATTER FIELDS through ASSOCIATION,

YET TO BE DONE $\left\{ \begin{array}{l} \text{GAUGE FIELDS through CONNECTION,} \\ \text{MINIMAL COUPLING through COVARIANT DERIVATIONS} \end{array} \right.$

THE OVERARCHING GOAL:

(MECHANISM)

GAUGE A GIVEN RIGID SYMMETRY $\mathcal{G} \Rightarrow M$!

(E.G., PAG σ-MODEL)

OR

(FEATURE)

EXPLAIN/EXTEND A KNOWN GAUGE SYMMETRY

(E.G., POISSON σ-MODEL)

OR

CONSTRUCT CURVED YANG-MILLS-HIGGS

& MORE GENERAL $\mathcal{E}$-CHARGED-MATTER GAUGE DYNAMICS

(INTRINSIC DYNAMICS)

WE HAVE

(*) BUNDLE of GAUGES
(PRINCIPAL G-BUNDLE)

$$\begin{tikzcd} & G \arrow[d, hook] & \\ & P \arrow[d, "\pi_P"'] \arrow[dr, dashed] & G \arrow[l, bend right, "e"'] \arrow[d, Rightarrow, dashed] \\ P/G \simeq \Sigma & \Sigma & \bullet \end{tikzcd}$$

with G-EQUIVARIANT CONNECTION 1-FORM

$$\chi^{(e)-1} \circ \Theta = A \in \Omega^1(P) \otimes \mathrm{Lie}\,G$$

(**) EXTENSION

$$\begin{tikzcd} G \times M \arrow[d, hook] & & \\ P \times M \arrow[d, "\pi_P \circ pr_1"'] \arrow[dr, "pr_2"] & & G \times M \arrow[ll, bend right, "e"'] \arrow[ull, bend right, "r"'] \arrow[dl, Rightarrow] \\ \Sigma & M & \end{tikzcd}$$

r (LOCAL MODEL)

with $G \ltimes_\lambda M$-EQUIVARIANT CONNECTION 1-FORM $\widetilde{\Theta} = pr_1^*\Theta + id_{pr_2^*TM}$

(***) CHARGED-MATTER BUNDLE ≡ CONFIGURATION BUNDLE
(ASSOCIATED BUNDLE)

$$\begin{tikzcd} & P \times M \arrow[d, "\pi_\sim"] \\ M \arrow[d, hook] & \\ P \times_\lambda M \equiv (P \times M)/G \arrow[d, "{[\pi_P]}"'] & \\ \Sigma & \end{tikzcd}$$

with CRITTENDEN CONNECTION $\Theta_C \circ T\pi_\sim := T\pi_\sim \circ \widetilde{\Theta}$

& SECTIONS ≡ FIELDS $\phi \in \Gamma(P \times_\lambda M) \xrightarrow{loc.} \phi_i : \mathcal{O}_i \to M$

GAUGE TRANSFORMATIONS : START from $\mathrm{Aut}_{\mathrm{Bun}_G(\Sigma)/\Sigma}(P) \subset \mathrm{Aut}_{\mathrm{Bun}_G(\Sigma)}(P)$

$$\begin{tikzcd} P \arrow[r, "\Phi"] \arrow[d, "\pi_P"'] & P \arrow[d, "\pi_P"] \\ \Sigma \arrow[r, equal] & \Sigma \end{tikzcd} \in \mathrm{Gau}(P) \xleftrightarrow{1:1} \Gamma(\mathrm{Ad}P) \ni \gamma$$

$$\mathrm{Ad}P = P \times_{\mathrm{Ad}} G$$

ADJOINT BUNDLE

$$\begin{tikzcd} \mathrm{Ad}P \arrow[r, hook] \arrow[d, Rightarrow] & \mathrm{At}P \arrow[r, two heads] \arrow[d, Rightarrow] & \Sigma \times \Sigma \arrow[d, Rightarrow] \\ \Sigma & \Sigma & \Sigma \end{tikzcd}$$

ATIYAH SEQUENCE

WE HAVE

$$\begin{tikzcd} \mathrm{Ad}P \arrow[r, bend left, "\tilde{\ell}"] \arrow[dr, Rightarrow] & P \arrow[d] \arrow[dr, dashed] & G \arrow[l, bend right, "r"'] \arrow[d, Rightarrow] \\ & \Sigma & \bullet \end{tikzcd}$$

$$\tilde{\ell}([(p,g)], p) = r_{g^{-1}}(p)$$

$\xRightarrow{\text{EXTENSION}}$

$$\begin{tikzcd} \mathrm{Ad}P \arrow[r, bend left, "\tilde{\ell}_1"] \arrow[dr, Rightarrow] & P \times M \arrow[d] \arrow[dr] & G \times M \arrow[l, bend right, "\tilde{r}"'] \arrow[d, Rightarrow] \\ & \Sigma & M \end{tikzcd}$$

$\xRightarrow{\text{INDUCTION}}$

$$\begin{tikzcd} \mathrm{Ad}P \arrow[r, bend left, "\tilde{\lambda}"] \arrow[dr, Rightarrow] & P \times_\lambda M \arrow[d] \\ & \Sigma \end{tikzcd}$$

$$\tilde{\lambda}([(p,g)], [(p,x)]) = [(p, \lambda_g(x))]$$

UPON RESTRICTION TO (BI)SECTIONS,

$$\begin{tikzcd} P \times M \arrow[r, "\Phi \times \mathrm{id}_M"] \arrow[d, "\pi_\sim"'] & P \times M \arrow[d, "\pi_\sim"] \\ P \times_\lambda M \arrow[r, "{[\Phi]}"] & P \times_\lambda M \end{tikzcd} \in \mathrm{Gau}(P \times_\lambda M)$$

LOCALLY :

$$[\tau_i]^{\mathrm{Ad}} \circ \gamma = (\mathrm{id}_{O_i}, \gamma_i), \quad \gamma_i : O_i \to G$$

$$\phi_i \longmapsto \lambda_{\gamma_i}(\phi_i)$$

CONNECTIONS GAUGE-TRANSFORM AS

$$\forall \Phi \in \mathrm{Gau}(P): \quad \begin{tikzcd} TP \arrow[r, "\Theta"] \arrow[d, "T\Phi"'] & VP \arrow[d, "T\Phi"] \\ TP \arrow[r, "\Theta^{\Phi}"'] & VP \end{tikzcd} \quad \Rightarrow \quad \Theta_c^{\Phi} = T[\Phi] \circ \Theta_c \circ T[\Phi]^{-1}$$

THIS ENSURES $\mathrm{Gau}(P\times_\lambda M)$-COVARIANCE of COVARIANT DERIVATIVE

$$\nabla^{\Theta} : \Gamma(P\times_\lambda M) \ni \phi \longmapsto \Theta_c \circ T\phi \in \Gamma(T^*\Sigma \otimes \phi^* V(P\times_\lambda M)),$$

i.e., $\nabla^{\Theta^{\Phi}} \phi^{\Phi} = T[\Phi] \circ \nabla^{\Theta} \phi$ for $(\phi^{\Phi}, \Theta^{\Phi}) = ([\Phi]\circ\phi, T\Phi \circ \Theta \circ T\Phi^{-1})$

GAUGE TRANSFORMS

LOCAL PICTURE: $\Theta_i(\sigma, g) := T\tau_i \circ \Theta \circ T\tau_i^{-1}\big|_{T_{(\sigma,g)}(\mathcal{O}_i\times G)} = \mathrm{id}_{T_gG} + T_e r_g \circ \underline{A}_i(\sigma)$

$$\Rightarrow \quad \Theta_{ci}(\sigma, x) = \mathrm{id}_{T_xM} - \mathcal{K}^{(\lambda)}(x) \circ \underline{A}_i(\sigma)$$

$\underline{A}_i(\sigma) \in T_\sigma^*\Sigma \otimes \mathrm{Lie}\, G$

LOCAL GAUGE FIELD

So LOCAL COVARIANT DERIVATIVES: $\mathcal{D}^{A_i}\phi_i := T[\tau_i] \circ \nabla^{\Theta}\phi = T\phi_i - \mathcal{K}^{(\lambda)}(\phi_i)\circ \underline{A}_i$

SIMPLE GAUGING SCHEME for G-INVARIANT TENSORIAL COUPLINGS:

MINIMAL COUPLING: GIVEN $T \in \Gamma(T^*M^{\otimes n})^G$ in $\mathcal{A}_{GFT}$,

(E.g., g_M in $\mathcal{A}_{PAG}$ for $\lambda(G) \subset \mathrm{Iso}(M, g_M)$)

REPLACE $T \rightsquigarrow \underline{T[A_i]} := [\tau_i]^*(T \circ \Theta_{C_i}^{\otimes n}) \in \Gamma(T^*\pi_{P\times_\lambda M}^{-1}(\mathcal{O}_i)^{\otimes n})$

& NOTE: $T[A_i]\restriction_{\pi_{P\times_\lambda M}^{-1}(\mathcal{O}_{ij})} = T[A_j]\restriction_{\pi_{P\times_\lambda M}^{-1}(\mathcal{O}_{ij})}$

$\Longrightarrow \exists\, \widetilde{T} \in \Gamma(T^*(P\times_\lambda M)^{\otimes n}) : \widetilde{T}\restriction_{\pi_{P\times_\lambda M}^{-1}(\mathcal{O}_i)} = T[A_i]$

UPSHOT: PULL BACK $\widetilde{T}$ along $\phi \in \Gamma(P\times_\lambda M)$

VITAL EXERCISE: INTEGRATE OUT A_i in THE PAG σ-MODEL ASSUMING $M/G \in \mathrm{Man}$ (with $M \twoheadrightarrow M/G$ PRINCIPAL)!

GOALS: (i) CONSTRUCT BUNDLE of FRAMES / OBSERVERS / GAUGES MODELLED on $\mathcal{G} \Longrightarrow M$, MAKING EQUIVARIANCE w.r.t. SPACETIME-GLOBAL CHANGE of GAUGE AN ORGANISING PRINCIPLE

(ii) EXTRACT GAUGE FIELD from EQUIVARIANT CONNECTION

(iii) MODEL CHARGED MATTER FIELDS on SECTIONS of BUNDLE with TYPICAL FIBRE M, $\mathcal{G}$-CLUTCHING & -AUTOMORPHISMS

(iv) ESTABLISH ELEMENTARY GAUGE-TO-MATTER COUPLING through GAUGE-COVARIANT DERIVATIVES

(v) 'DESCEND' MINIMALLY GAUGE-TO-MATTER-COUPLED SPACETIME-LOCAL DYNAMICS to MATTER BUNDLE

Ad (i) Def$^{\underline{n}}$ 7. PRINCIPALOID $\mathcal{G}$-BUNDLE IS A FIBRE-BUNDLE OBJECT $(\mathcal{P}, \Sigma, \mathcal{G}, \pi_{\mathcal{P}})$ in $\mathbf{Man}_{\mathrm{gop}}$ with TRIVIAL ACTION on Σ & THAT on $\mathcal{P}$ MODELLED on r through $\mathcal{G}$-EQUIVARIANT LOCAL TRIVIALISATIONS over $\mathcal{O} = \bigsqcup_{i \in I} \mathcal{O}_i$

[arXiv: 2503.09886 [math.DG]]

$$\begin{tikzcd}
\mathcal{G} & & & & \\
\mathcal{O} \times \mathcal{G} \arrow[u, "pr_2"] \arrow[d, "pr_1"'] & \mathcal{O} \times_\Sigma \mathcal{P} \arrow[l, "\mathcal{P}\tau"', "\simeq"] \arrow[r, "pr_2"] \arrow[d, "pr_1"'] & \mathcal{P} \arrow[d] \arrow[dr, "\mu_{\mathcal{P}}"] & & \mathcal{G} \arrow[ll, bend right, "e"'] \arrow[dl, Rightarrow] \arrow[ullll, bend right, "r"'] \\
\mathcal{O} \arrow[r, equal] & \mathcal{O} \arrow[r] & \Sigma & M &
\end{tikzcd}$$

$\mathcal{G}$ – STRUCTURE GROUPOID

$B(\mathcal{G})$ – STRUCTURE GROUP

Cor. 1. $\mathcal{P}$ HAS TRANSITION MAPS

$$\left(\mathcal{P}\tau_i \circ \mathcal{P}\tau_j^{-1}\right)(\sigma, g) = \left(\sigma, L_{\beta_{ij}(\sigma)}(g)\right)$$

for $\beta_{ij} : \mathcal{O}_{ij} \to B(\mathcal{G})$. TRANSITION 1-COCYCLE

P: By Prop$^{\underline{n}}$ 3. $\square$

$\Rightarrow$ CLUTCHING MODEL $\mathcal{P} \simeq \bigsqcup_{i \in I} \mathcal{O}_i \times \mathcal{G} / \sim_{L_{\beta_{..}}}$ $\quad (\sigma, g, j) \sim (\sigma, L_{\beta_{ij}(\sigma)}(g), i)$

E.g., (*) $\mathcal{G} = \mathrm{B}G$: $\mathscr{P} = P$ (PRINCIPAL G-BUNDLE)

(**) $\mathcal{G} = G \ltimes_\lambda M$ with REDUCED STRUCTURE GROUP $\mathrm{L}(\iota(G))$

(see: Ex.(***)/p. 3)

: $\mathscr{P} = P \times M$ (MODULE-EXTENDED PRINCIPAL G-BUNDLE)

(***) $\mathcal{G} = \mathrm{Pair}(M)$: $\mathscr{P} = E \times M$, E - ARBITRARY BUNDLE with TYPICAL FIBRE M

NB: GIVEN PRINCIPAL $\mathcal{G}$-BUNDLE $(\mathscr{P}, \Sigma, \mathcal{G}, \pi_{\mathscr{P}})$ & (LEFT) $\mathcal{G}$-MODULE

$$\begin{tikzcd} \mathcal{G} \arrow[rr, bend left, "\lambda"] \arrow[dr, shift left] \arrow[dr, shift right] & & X \arrow[dl, "\mu_X"] \\ & M & \end{tikzcd}$$

, WE MAY CONSTRUCT AN ASSOCIATED BUNDLE $(\mathscr{P} {}_{\mu_{\mathscr{P}}}\!\times_{\mu_X} X)/\mathcal{G}$, WHICH ... TURNS OUT TO BE A PRINCIPALOID BUNDLE.

Ex.: WHAT ARE: THE STRUCTURE GROUPOID & GROUP of $(\mathscr{P} {}_{\mu_{\mathscr{P}}}\!\times_{\mu_X} X)/\mathcal{G}$?

Ad (ii) **Def$^{\underline{n}}$ 8.** (COMPATIBLE) CONNECTION on $\mathcal{P}$ is a $\mathcal{G}$-INVARIANT EHRESMANN CONNECTION $T\mathcal{P} \simeq V\mathcal{P} \oplus H\mathcal{P}$, i.e., s.th.

$$V\mathcal{P} = \operatorname{Ker} T\pi_{\mathcal{P}}$$

(CC1) $H\mathcal{P} \subset \operatorname{Ker} T\mu_{\mathcal{P}}$

(CC2) $\forall (p,g) \in \mathcal{P}\,{}_{\mu_{\mathcal{P}}}\!\times_t \mathcal{G} : \ T_p\varrho_g(H_p\mathcal{P}) = H_{\varrho_g(p)}\mathcal{P}$

Prop$^{\underline{n}}$ 5. A COMPATIBLE CONNECTION on $\mathcal{P}$ IS EQUIVALENTLY GIVEN by A VERTICAL VECTOR-BUNDLE MORPHISM

$$\begin{array}{ccc} T\mathcal{P} & \xrightarrow{\ \Theta\ } & V\mathcal{P} \\ {\scriptstyle \pi_{T\mathcal{P}}}\downarrow & & \downarrow{\scriptstyle \pi_{T\mathcal{P}}} \\ \mathcal{P} & = & \mathcal{P} \end{array}$$

s.th.

(CF1) $\Theta|_{V\mathcal{P}} = \mathrm{id}_{V\mathcal{P}}$

(CF2) $T\mu_{\mathcal{P}} \circ \Theta = T\mu_{\mathcal{P}}$

(CF3) $\forall (p,g) \in \mathcal{P}\,{}_{\mu_{\mathcal{P}}}\!\times_t \mathcal{G} :$

$$T_p\varrho_g(\operatorname{Ker} \Theta_p) = \operatorname{Ker} \Theta_{\varrho_g(p)}$$

CONNECTION FORM

Proof: $H\mathcal{P} = \operatorname{Ker} \Theta$ $\square$

Prop$^{\underline{n}}$ 6. CONNECTION FORM DETERMINES & IS DETERMINED by

LOCAL CONNECTION DATA $\quad \underline{A}_i \in \Gamma\left(pr_1^* T^*\mathcal{O}_i \otimes pr_2^* \mathcal{E}\right)$ over $\mathcal{O}_i \times M$

with GLUING : $\quad \underline{A}_i(\sigma, t_*(\beta_{ij}(\sigma))(x)) = T_{Id_x} C_{\beta_{ij}(\sigma)} \circ \underline{A}_j(\sigma, x) + \Theta \circ T_\sigma \tilde{\beta}_{ij}(\cdot, x)$

for $\tilde{\beta}_{ij} : \mathcal{O}_{ij} \times M \longrightarrow \mathcal{G}, \ (\sigma, x) \longmapsto \beta_{ij}(\sigma)(x)$

Proof: $(T\mathcal{P}\tau_i \circ \Theta \circ T\mathcal{P}\tau_i^{-1})(\sigma, g) = id_{T_g\mathcal{G}} - T_{Id_{t(g)}} r_g \circ \underline{A}_i(\sigma, t(g))$ by DEFINITION. $\square$

NB: $\underline{A}_i$ is AN $\mathcal{E}$-VALUED $T\mathcal{O}_i$-FOLIATED 1-FORM on $\left((id_\Sigma \times t) \circ \mathcal{P}\tau_i\right)(\pi_{\mathcal{P}}^{-1}(\mathcal{O}_i))$

PHYSICALLY : THEY ARE THE (LOCAL) GAUGE FIELDS.

E.G.: (*) $\mathcal{G} = BG$: PRINCIPAL G-CONNECTION 1-FORM $\mathcal{K}^{(e)} \circ \mathcal{A}$

(**) $\mathcal{G} = G \rtimes_\lambda M$ w/ RSG $L(\lambda(G))$: DESCENDABLE M-FLATLY EXTENDED

$$\widetilde{\Theta} = pr_1^* (\mathcal{K}^{(e)} \circ \mathcal{A}) + id_{pr_2^* TM}$$

(***) $\mathcal{G} = \mathrm{Pair}(M)$: $pr_1^* \Theta + id_{pr_2^* TM}$, Θ – ARBITRARY

CONCLUSION: ARBITRARY BUNDLES w/ CONNECTION ARE COVERED by OUR CONSTRUCTION.

(Th^m 1. $\forall P \; \exists \; \Theta$ $\mathfrak{I}$: PARTITIONS of UNITY for $\mathcal{O}$.)

WE MAY NEXT LOOK for CHARGED-MATTER FIELDS...

AD (iii) RECALL

Thm 2. [CLUTCHING THEOREM]

FIBRE BUNDLE E OVER Σ WITH TRIVIALISATION $\mathcal{O}\times_{\Sigma} E \simeq \mathcal{O}\times F$ DETERMINES & IS DETERMINED (UP TO ISO) BY 1-COCYCLE $g_{ij} : \mathcal{O}_{ij} \to \mathrm{Aut}(F)$.

ANALOGOUSLY, A VERTICAL AUTOMORPHISM OF E DETERMINES & IS DETERMINED BY $h_i : \mathcal{O}_i \to \mathrm{Aut}(F)$, $g_{ij}\cdot h_j\restriction_{\mathcal{O}_{ij}} = h_i \cdot g_{ij}\restriction_{\mathcal{O}_{ij}}$.

Q: $E \simeq \coprod_{i\in I} \mathcal{O}_i \times F / \sim_{g_{..}}$ $\square$

& NOTE : RIGHT-FIBRED ACTION $r : \mathcal{G}\,{}_s\!\times_t \mathcal{G} \longrightarrow \mathcal{G}$ IS PRINCIPAL

w.r.t. M along t, with $\phi_{\mathcal{G}} : \mathcal{G}\,{}_t\!\times_t \mathcal{G} \longrightarrow \mathcal{G}$, $(g_1, g_2) \longmapsto g_1^{-1}\cdot g_2$ s.th. $(pr_1, \phi_{\mathcal{G}}) = (pr_1, r)^{-1}$

DIVISION MAP

WE THEN OBTAIN...

Thm 3. DEFINING ACTION $\varrho : \mathcal{P}\,{}_{\mu_{\mathcal{P}}}\!\times_t \mathcal{G} \longrightarrow \mathcal{P}$ IS FIBREWISE PRINCIPAL wrt. FIBRE BUNDLE $(\mathcal{F}, \Sigma, M, \pi_{\mathcal{F}})$ along VERTICAL $\mathcal{D} : \mathcal{P} \twoheadrightarrow \mathcal{F}$ MODELLED on t.

Pf: $\mathcal{D}_i : \pi_{\mathcal{P}}^{-1}(\mathcal{O}_i) \longrightarrow \pi_{\mathcal{F}}^{-1}(\mathcal{O}_i)$, $\mathcal{P}\tau_i^{-1}(\sigma, g) \longmapsto \mathcal{F}\tau_i^{-1}(\sigma, t(g))$ GLUE for $\mathcal{O} \times_{\Sigma} \mathcal{F} \xrightarrow{\mathcal{F}\tau} \mathcal{O} \times M$

with $(\mathcal{F}\tau_i \circ \mathcal{F}\tau_j^{-1})(\sigma, x) = (\sigma, t_*(\beta_{ij}(\sigma))(x))$. $\square$

Defn 9. THE QUOTIENT BUNDLE $\mathcal{F} \equiv \mathcal{P}/\mathcal{G} \simeq \bigsqcup_{i \in I} \mathcal{O}_i \times M / \sim_{t_*\beta}$ IS CALLED THE SHADOW BUNDLE (of $\mathcal{P}$), & THE QUOTIENT MAP $\mathcal{D}$ IS TERMED THE SITTING DUCK.

FOR GAUGE SYMMETRIES, WE CONSIDER

Propn 7. VERTICAL $\mathcal{G}$-EQUIVARIANT AUTOMORPHISMS $\Phi \in \mathrm{Aut}_{\mathbf{Bun}(\Sigma)/\Sigma}(\mathcal{P})$ ARE MODELLED on $L(B(\mathcal{G}))$, $(\mathcal{P}\tau_i \circ \Phi \circ \mathcal{P}\tau_i^{-1})(\sigma, g) = (\sigma, L_{\gamma_i(\sigma)}(g))$ for $\gamma_i : \mathcal{O}_i \longrightarrow B(\mathcal{G})$

s.th. $\forall \sigma \in \mathcal{O}_{ij} : \ \beta_{ij}(\sigma) \cdot \gamma_j(\sigma) = \gamma_i(\sigma) \cdot \beta_{ij}(\sigma)$.

Pf: FOLLOWS from Propn 3. $\square$

Def$^{\underline{n}}$ 10. GROUP $\mathrm{Gau}(\mathscr{P}) = \mathrm{Aut}_{\mathbf{Bun}(\Sigma)/\Sigma}(\mathscr{P}) \cap \mathrm{Diff}_{\mathfrak{e}(\mathcal{G})}(\mathscr{P})$ IS CALLED THE GAUGE GROUP of $\mathscr{P}$.

$\mathcal{G}$-EQUIVARIANCE ENSURES INDUCTION along SITTING DUCK...

Prop$^{\underline{n}}$ 8. $\exists\ \mathscr{F}_* : \mathrm{Gau}(\mathscr{P}) \xrightarrow[\mathbf{Grp}]{} \mathrm{Aut}_{\mathbf{Bun}(\Sigma)/\Sigma}(\mathscr{F})$ s.th.

$$\begin{tikzcd} \mathscr{P} \arrow[r, "\Phi"] \arrow[d, twoheadrightarrow, "\mathscr{D}"'] & \mathscr{P} \arrow[d, twoheadrightarrow, "\mathscr{D}"] \\ \mathscr{F} \arrow[r, "\mathscr{F}_*(\Phi)"'] & \mathscr{F} \end{tikzcd}$$

$\mathfrak{P}$: FOLLOWS from $t \circ L_\beta = t_* \beta \circ t$, $\beta \in B(\mathcal{G})$. $\square$

Def$^{\underline{n}}$ 11. GROUP $\mathrm{Gau}(\mathscr{F}) = \mathrm{Im}\,\mathscr{F}_*$ IS CALLED THE SHADOW GAUGE GROUP.

GIVEN $\phi \in \Gamma(\mathscr{F})$, ITS SHADOW GAUGE TRANSFORM is $\phi^{\Phi} := \mathscr{F}_*(\Phi) \circ \phi$.

E.g., $\mathcal{G} = G \rtimes_\lambda M$ w/ RSG $L(\mathfrak{r}(G))$ & $\mathrm{Gau}(\mathscr{P})$ MODELLED on $\mathfrak{r}(G)$ YIELDS $\mathscr{F} = P \times_\lambda M$

TEASER for LECTURE III :

THE EXTENSIVE REVIEW of THE CLASSIC GAUGE-ING SCHEME (TOGETHER with THE ACCOMPANYING EXERCISES) CONVINCES US THAT THE GAUGE-ING IS A METHOD of CONSTRUCT-ING A GFT with THE REDUCED CONFIGURATION FIBRE $M//G$ (EVER – OR ESPECIALLY – WHEN $M//G \notin \mathrm{Man}$). THIS INDICATES THE CONCEPTUAL GOAL of THE GAUGE-ING of THE MORE GENERAL GROUP-OIDAL SYMMETRIES WE ARE AFTER. In ORDER to ATTAIN IT, WE STILL NEED ONE MISSING PIECE of THE GEOMETRIC MACHINERY, WHICH IS THE MATTER-to-GAUGE FIELD COUPLING MODELLED by THE COVARIANT DERIVATIVE on $\Gamma(\mathcal{F})$. Upon INTRODUCING IT, WE SHALL – at LONG LAST – DO THE GAUGE-ING in THE NONTRIVIAL PAG SETTING, WHEREBY WE ALSO GET INSIGHTS into ... THE KNOWN GAUGE SYMMETRY of THE PσM ...

VERB!