

Gauging the Oïd
& Other Tales

PART V

Making Poisson Sigma Again
& Livin' with Defects

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*La Géométrie est une image,
mais il n'y a pas d'image de la Géométrie*

IN THE LAST EPISODE :

WE GAUGED A SYMMETRY MODELLED ON $G \Rightarrow M$ IN THE PAC σ -MODEL WITH TARGET (M, g_M, G) BY DESCENDING THE QFT TO THE SHADOW OF AN ARBITRARY PRINCIPALOID G -BUNDLE WITH REDUCED STRUCTURE GROUP

$$\begin{array}{ccc} (P, \pi) & \longrightarrow & (F, \pi_f) \\ \downarrow & & \downarrow \\ \Sigma & \xlongequal{\quad} & \Sigma \end{array}$$

$$B_o(G) \swarrow \searrow B(G)$$

$$\& \text{ GAUGE SYMMETRY } \text{Gau}_o(P) \xrightarrow{F_*} \text{Gau}_o(F)$$

(*) IDENTIFICATION OF EXTENSION OF $H^3_{dR}(M)$ & $H^2(M, D(2))$ NATURAL FOR $M//G$

$\Rightarrow$ ω -TWISTED G -EQUIVARIANT STRUCTURE ON G_j

$$\Upsilon: t^*G_j \xrightarrow{\cong} s^*G_j \otimes I_\omega$$

$$\gamma: (pr_2^* \Upsilon \otimes id_{I_{pr_1^* \omega}}) \circ pr_1^* \Upsilon \xrightarrow{\cong} m^* \Upsilon, \quad d_2^{(3)*} \gamma \cdot (id \circ d_o^{(3)*} \gamma) = d_1^{(3)*} \gamma \cdot ((d_3^{(3)*} \gamma \otimes id) \circ id),$$

GEOMETRISING $[(H, \omega, 0, 0)] \in H^3_{BSS}(G \Rightarrow M)$ FOR $m^* \omega = pr_1^* \omega + pr_2^* \omega$

MULTIPLICATIVITY

(**) IDENTIFICATION of RIGID-SYMMETRY GROUP $B_\omega(G) = \{ \beta \in B(G) \mid \beta^* \omega = 0 \}$
 for A_{DF}^{top} ω -HOLONOMIC GROUP

(***) EXTRACTION of $B_\omega(G)$ -EQUIVARIANT 1-COMOMENTUM

with PROPERTIES: $\forall E_1, E_2 \in \Gamma(E) : \alpha_E(E_1) \lrcorner K(E_2) + \alpha_E(E_2) \lrcorner K(E_1) = 0$

$$\begin{array}{ccc} E & \xrightarrow{K} & T^*M \\ \downarrow & & \downarrow \\ M & \xlongequal{\quad} & M \end{array}$$

$$\forall \beta \in B_\omega(G) : \omega|_{\beta(M)} = \langle k \circ \theta \circ P_{\ker TS}, T\tau \circ (\text{id} - P_{\ker TS}) \rangle + \frac{1}{2} \langle k \circ \theta \circ P_{\ker TS}, \overset{T\tau}{\parallel} \alpha_E \circ \theta \circ P_{\ker TS} \rangle$$

RELATIVE to $TG|_{\beta(M)} = T(\beta(M)) \oplus \ker TS|_{\beta(M)}$

RIGHT-INVARIANT
MC FORM on G

(****) CONSTRUCTION of $B_\omega(G)^{\sigma_i}$ -INVARIANT LOCAL $\underline{A}_i$ -AUGMENTED GERBES

$$G_j[A_i] := \mathcal{F}c_i^* (p_{\Sigma}^* G_j \otimes I_{\omega[A_i]})$$

$$\omega[A_i] = (\text{id}_2 \times \text{Id})^* (\omega \circ (\mathcal{U}_i \otimes \mathcal{U}_i))$$

(*****) EXTENSION of $\{G_j[A_i]\}_{i \in I}$ to DESCENT DATUM USING $(\mathcal{I}, \gamma)$

$\Rightarrow$ DESCENDED $\text{Gau}_\omega(\mathcal{F})$ -INVARIANT GERBE $G_{\mathcal{F}}$ on $\mathcal{F}$.

AND NOW FOR SOMETHING COMPLETELY DIFFERENT...



...THE FISH / FISH / RYBKA / РИЗ / ΨΑΡΙ / РИБА / ... **Poisson**

TRYING to MAKE SENSE-OJD of the POISSON σ -MODEL (q.3):

$$\mathcal{L}_{\text{PSM}}(X, TX; a) \Big|_{\text{loc.}} = \left(a_{\mu a} \partial_b X^\mu + \frac{1}{2} \Pi^{\mu\nu}(X) a_{\mu a} a_{\nu b} \right) d\sigma^a \wedge d\sigma^b$$

$$\Pi \Big|_{\text{loc.}} = \Pi^{\mu\nu} \partial_\mu \wedge \partial_\nu$$

$$\begin{array}{ccc} T\Sigma & \xrightarrow{a} & T^*M \\ \downarrow \pi_{T\Sigma} & & \downarrow \pi_{T^*M} \\ (\sigma^a) \in \Sigma & \xrightarrow{X} & (M, \Pi) \end{array}$$

& ITS TANGENTIAL SYMMETRIES (q.4): $\langle X^* Z_{\text{dR}}^1(M) \rangle_{C^\infty(\Sigma, \mathbb{R})} \ni \epsilon_\mu(\cdot) dX^\mu(\cdot)$

$$X^\mu \mapsto X^\mu - \Pi^{\mu\nu}(X) \epsilon_\nu; \quad a_\mu \mapsto a_\mu + d\epsilon_\mu - \partial_\mu \Pi^{\lambda\nu}(X) \epsilon_\lambda a_\nu$$

OBSERVATIONS à PROPOS :

(*) 'CONFIGURATION SPACE' $T^*M \longrightarrow M$ for (M, Π) POISSON
CARRIES CANONICAL STRUCTURE of LIE ALGEBROID:

- POISSON ANCHOR : $\Pi^\# : T^*M \longrightarrow TM, \quad \eta \mapsto \eta \lrcorner \Pi$
- KOSZUL BRACKET : $[\eta_1, \eta_2]_K := \Pi^\#(\eta_1) \lrcorner d\eta_2 - \Pi^\#(\eta_2) \lrcorner d\eta_1 + d(\Pi^\#(\eta_1) \lrcorner \eta_2)$

WHENEVER INTEGRABLE, IT ARISES AS $\simeq$ TANGENT LIE ALGEBROID of

Defⁿ 21. SYMPLECTIC LIE GROUPOID $(G, \omega) \Rightarrow (M, \Pi)$, with ARROW
MANIFOLD ENDOWED WITH MULTIPLICATIVE (!) SYMPLECTIC FORM ω ,
 $(m^* - p_1^* - p_2^*)\omega = 0$

IDEA of CONSTRUCTION: USE DECOMPOSITION $T_x \text{Id}(T_x M) \oplus \text{Ker } T_{\text{Id}_x} s \simeq T_{\text{Id}_x} G \simeq T_x \text{Id}(T_x M) \oplus \text{Ker } T_{\text{Id}_x} t$
& BEHAVIOUR $\omega(e)|_{T_{\text{Id}_x} G} = \omega(e) \circ T_{\text{Id}_x} (\text{Id} \circ t)|_{T_{\text{Id}_x} G}$, $e \in \text{Ker } T_{\text{Id}_x} s$ of MULTIPLICATIVE ω to VERIFY
LAGRANGIANITY of $\text{Id}(M)$ & OBTAIN $\mathcal{E} \xrightarrow{\simeq} T^*M$, $e \mapsto \text{Id}^*(e \lrcorner \omega)$ (COMOMENTUM!)

$\Rightarrow$ PoM FIELD (X, α) HAS DOFS of CONNECTION on TRIVIAL SITTING DUCK

THAT COVERS SECTION $\text{graph}(X)$ of $\mathcal{F}_0$...

$$\mathcal{P}_0 \simeq \Sigma \times G \xrightarrow{\text{id}_\Sigma \times t} \Sigma \times M \simeq \mathcal{F}_0$$

(**) INFINITESIMAL GAUGE SYMMETRIES MODELLED on

$$Z_{dr}^1(M)_c \simeq \text{Lie}(\mathbb{B}_\omega(\mathfrak{g})) \text{ [Pybidi]} \quad \text{LAGRANGIAN BISECTIONS}$$

⇒ GAUGE TRANSFORMATIONS

$$X^\mu \mapsto X^\mu - \Pi^{\mu\nu}(X) \epsilon_\nu; \quad a_\mu \mapsto a_\mu + d\epsilon_\mu - \partial_\mu \Pi^{\lambda\nu}(X) \epsilon_\lambda a_\nu$$

$\Pi^\# \equiv \alpha_{T^*M} \equiv T\mathbb{E}$ $C_\mu^\lambda(X)$ STRUCTURE FUNCTIONS

DISPLAY STRUCTURE of LINEARISED ω -KOLONOMIC SHADOW AUTOMORPHISMS

IDEA: (*) REINTERPRET PoM AS GAUGE FIELD THEORY for $\Gamma(\mathcal{F}_0)$ COUPLED to CONNECTION

$\Theta(\sigma|g) = \text{id}_{T_g G} + T_e \Gamma_g \circ A_i(\sigma) \text{ on } \mathcal{P}_0$

(**) DERIVE OBSTRUCTION against $\text{Gau}_\omega(\mathcal{F}_0) \simeq \mathbb{B}_\omega(\mathfrak{g})^\Sigma$ -INVARIANCE & ... KILL IT!

(***) EXTEND THE OBSTRUCTION KILLER with VIEW to GENERALISING PoM to SYMPLECTIC PRINCIPALOID BUNDLES of ARBITRARY TOPOLOGY

CASE STUDY: LINEAR POISSON σ -MODEL (EXERCISE)

IN THE CONVENIENT PRESENTATION $(\varepsilon, \alpha_\varepsilon) \equiv (T^*M, \Gamma^\#)$, WE HAVE (TAUTOLOGICALLY)

$$\kappa \equiv \text{id}_{T^*M} \stackrel{\text{loc.}}{=} \partial_\mu \otimes dx^\mu,$$

& WE MAY ASK A SEEMINGLY SILLY...

QUESTION: HOW MANY GAUGE-THEORETICALLY MEANINGFUL QUADRATIC POLYNOMIALS IN T^*M -VALUED GAUGE FIELD, MADE OF κ & α_ε AS ABOVE, ARE THERE ON $\Sigma \times M \equiv \mathcal{F}_0 \leftarrow \mathcal{P}_0 \equiv \Sigma \times G$, WITH $\Theta(\sigma, g) = \text{id}_{T_g G} + T_{\text{id}_{T_g G}} \tau_g \circ \underline{A}(\sigma, t(g))$?

ANSWER: ONE - LO WIT: $(\text{id}_\Sigma, X)^*(\text{id}_\Sigma \times \text{Id})^*(\omega \circ (\Theta \otimes \Theta)) = -\mathcal{L}_{\text{PSM}}(X, TX; a)$!

for $a(\sigma) = \underline{A}(\sigma, X(\sigma))$

Cor. 4. THE POISSON σ -MODEL HAS LARGE $B_\omega(G)^\Sigma$ -GAUGE ANOMALY:

$$\mathcal{A}_{\text{PSM}}[t_* \gamma(X), a^\gamma] = \mathcal{A}_{\text{PSM}}[X, a] \cdot \exp\left(-i \int \omega_{\tilde{\gamma} \circ (\text{id}_\Sigma, X)(\Sigma)}\right), \text{ WHERE}$$

$$\gamma: \Sigma \rightarrow B_\omega(G)$$

$$\tilde{\gamma}: \Sigma \times M \rightarrow G, (\sigma, x) \mapsto \gamma(\sigma)(x).$$

$$a^\gamma(\sigma) = \underline{A}^\gamma(\sigma, t_*(\gamma(\sigma))(X(\sigma)))$$

by MASTER FORMULA (Lemma 2.): $\omega[A] - (p_{\Gamma_1}, t \circ \tilde{\gamma})^* \omega[A^\gamma] = \tilde{\gamma}^* \omega$

Η ΤΗΣ ΑΝΩΜΑΛΙΑΣ ΣΦΑΓΗ
- Η ΤΡΙΤΤΗ ΟΔΟΣ

I THE BLOODLESS SCENARIO

$\omega \in B_{\text{dr}}^2(G) \Rightarrow$ NO ANOMALY for $\Sigma \in \partial^{-1}\phi$

E.G., LINEAR POISSON σ -MODEL, with $\omega = -d\theta_{\text{Liouville}}$

II ELIEBOTOMIA

$\text{Per}(\omega) < 2\pi\mathbb{Z} \Rightarrow \exists$ PREQUANTISATION:

$$\begin{array}{c} U(1) \hookrightarrow P_\omega, \vartheta \\ \downarrow \pi_{P_\omega}, \quad \pi_{P_\omega}^* \omega = d\vartheta \\ G \end{array}$$

WE SHALL STUMBLE upon XTRA STRUCTURE on P_ω WHEN TRYING
to GENERALISE THE PSM to ARBITRARY $P \rightarrow F$.

III ΣΞΑΓΗ

$\text{Per}(\omega) \not< 2\pi\mathbb{Z} \Rightarrow \neg \exists$ PREQUANTISATION, BUT... REVERSE THE LOGIC:

$$\exists G \in \text{Grb}_D(M)_0 \exists \mathbb{I}: t^*G \xrightarrow{\sim} s^*G \otimes I_\omega \Rightarrow \text{CO-AUGMENT } I_{\omega[A]} \nearrow (I_{\omega[A]} \otimes p_2^*G) =: G[A] \quad (69)$$

Defⁿ 22. THE TRIVIAL CO-AUGMENTED POISSON $\mathcal{G}$ -MODEL :

$$\left(\overline{\mathcal{F}}_0 \equiv \Sigma \times M, \Sigma, M, p_G, A_{\text{tcPSM}}^g \right) \text{ with GAUGE SYMMETRY } B_\omega(\mathcal{G})^\Sigma$$

HAS Σ : $\dim \Sigma = 2$, ORIENTABLE, $\partial \Sigma = \emptyset$, COMPACT

(M, Π) : POISSON OBJECT MANIFOLD of SYMPLECTIC GROUPOID $(\mathcal{G}, \omega) \Rightarrow (M, \Pi)$

& GERBE $\mathcal{G}_j$ with $\mathcal{G}$ -EQUIVARIANT CURVATURE $H \in \ker \Delta_{(1)} \cap \mathcal{Z}_{\text{dR}}^3(M)$

$$\& \quad \Upsilon : t^* \mathcal{G}_j \xrightarrow{\sim} s^* \mathcal{G}_j \otimes I_\omega$$

$(\mathcal{P}_0 \xrightarrow{\text{id}_\Sigma \times t} \overline{\mathcal{F}}_0) :$ SHADOW of TRIVIAL PRINCIPALOID BUNDLE $\mathcal{P}_0 \equiv \Sigma \times \mathcal{G} \xrightarrow{p_G} \Sigma$

with ω -HOLONOMICALLY REDUCED STRUCTURE GROUP $B_\omega(\mathcal{G})$

& DF AMPLITUDE

$$\Gamma(\overline{\mathcal{F}}_0) \times \text{Conn}(\mathcal{P}_0) \ni \left(\phi \equiv (\text{id}_\Sigma, X), \binom{H}{0} \right) \longmapsto \text{Hol}_{\mathcal{G}[\mathbb{A}]}(\phi(\Sigma))^{-1} = A_{\text{tcPSM}}^g[\phi, \binom{H}{0}]$$

CONNECTIONS on $\mathcal{P}_0$

THE 2-STACK ANALYSIS THAT LED TO DEFINITION OF THE $\mathfrak{g}$ -GAUGED
 PAQ $\mathfrak{g}$ -MODEL (Defⁿ 20.) YIELDS STRAIGHTFORWARD GENERALISATION OF TRIVIAL
 CO-AUGMENTED POM from Defⁿ 22. TO ARBITRARY SITTING DUCK
 $\mathcal{P} \twoheadrightarrow \mathcal{F}$ WITH SUITABLY REDUCED STRUCTURE GROUP, TO WIT:

Defⁿ 23. THE CO-AUGMENTED POISSON $\mathfrak{g}$ -MODEL:

$$(\mathcal{F}, \Sigma, M, \mathcal{P}_{\mathcal{F}}, \mathcal{A}_{\text{cPSM}}^{\mathfrak{g}}) \text{ with GAUGE SYMMETRY } \text{Gau}_{\omega}(\mathcal{F})$$

HAS Σ & (M, Π) AS BEFORE, STRUCTURE GROUP OF SITTING DUCK $\mathcal{P} \xrightarrow{\mathcal{D}} \mathcal{F}$
 REDUCED TO LAGRANGEAN GROUP $B(\mathfrak{g}, \omega) \equiv B_{\omega}(\mathfrak{g})$, GERBE $\mathcal{G}_{\mathcal{F}}$ ENDOWED WITH
 $\mathfrak{g}$ -EQUIVARIANT STRUCTURE $(\mathcal{I}, \gamma)$, & DF AMPLITUDE

$$\Gamma(\mathcal{F}) \times \underset{\text{CONNECTIONS on } \mathcal{P}}{\text{Conn}(\mathcal{P})} \ni (\phi, \textcircled{u}) \longmapsto \text{Hol}_{\mathcal{G}_{\mathcal{F}}}(\phi(\Sigma))^{-1} = \mathcal{A}_{\text{cPSM}}^{\mathfrak{g}}[\phi, \textcircled{u}],$$

WHERE $\mathcal{G}_{\mathcal{F}}$ IS AS IN Cor. 2.

NB: EULER-LAGRANGE EQUATIONS of PoM HAVE STRUCTURAL INTERPRETATION in TERMS of LIE-ALGEBROID MORPHISMS (SEE: STROBL'S LECTURES)

⇒ MIGHT PREFER to KEEP THEM UNAFFECTED by CO-AUGMENTATION, & THE LATTER CONTRIBUTES CORRECTIONS $\propto \text{curv}(G_j)$

⇒ FLAT CO-AUGMENTATIONS, $[G_j] \in H^2(M, U(1))$, ARE DISTINGUISHED

⊃ TRIVIAL FLAT CO-AUGMENTATIONS $G_j \simeq I_0$ (E.G., for $M: H_2(M)=0$), with CORRESPONDING 1-ISOMORPHISM of G -EQUIVARIANT STRUCTURE:

$$\Upsilon: I_0 = I_{t^*0} = t^* I_0 \xrightarrow{\simeq} s^* I_0 \otimes I_\omega = I_{s^*0} \otimes I_\omega = I_0 \otimes I_\omega = I_\omega$$

LOCAL DATA $(P_i, k_{ij}) \in \Omega^1(\mathcal{O}_i) \oplus C^\infty(\mathcal{O}_{ij}, U(1))$ of Υ SATISFY (SEE: §.44.)

$$(dP_i, (P_j - P_i)|_{\mathcal{O}_{ij}} - \text{id} \log k_{ij}, (k_{jk}^{-1} k_{ik} k_{ij}^{-1})|_{\mathcal{O}_{ijk}})|_{\mathcal{O}_{ij}} = (\omega|_{\mathcal{O}_i}, 0, 1)$$

SCENARIO II from p. 68.

CONCLUSION: 1-COCHAIN (P_i, k_{ij}) DETERMINES PREQUANTISATION $(P_\omega, \mathcal{V})$ of ω ! (72)

THE REMAINING COMPONENT of G -EQUIVARIANT STRUCTURE ACQUIRES INTERPRETATION of G -MULTIPLICATIVE STRUCTURE on (P_ω, ϑ) :

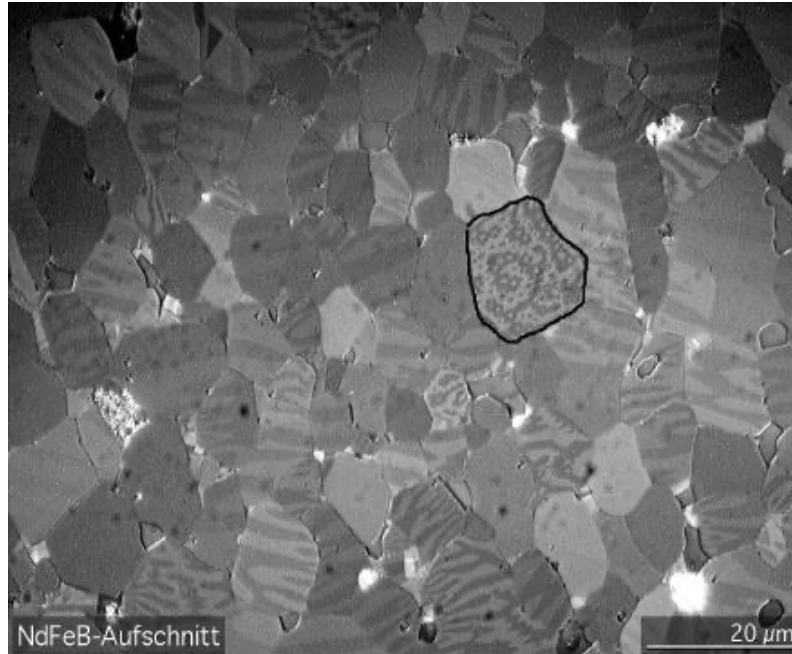
$$\gamma: pr_1^* P_\omega \otimes pr_2^* P_\omega \xrightarrow{\sim} m^* P_\omega, \quad \Delta_{(13)} \gamma = \text{id}$$

Defⁿ 24.

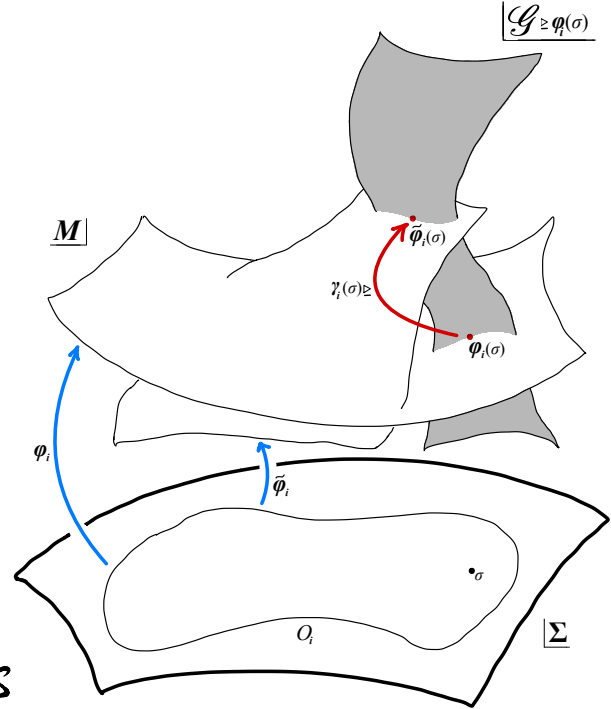
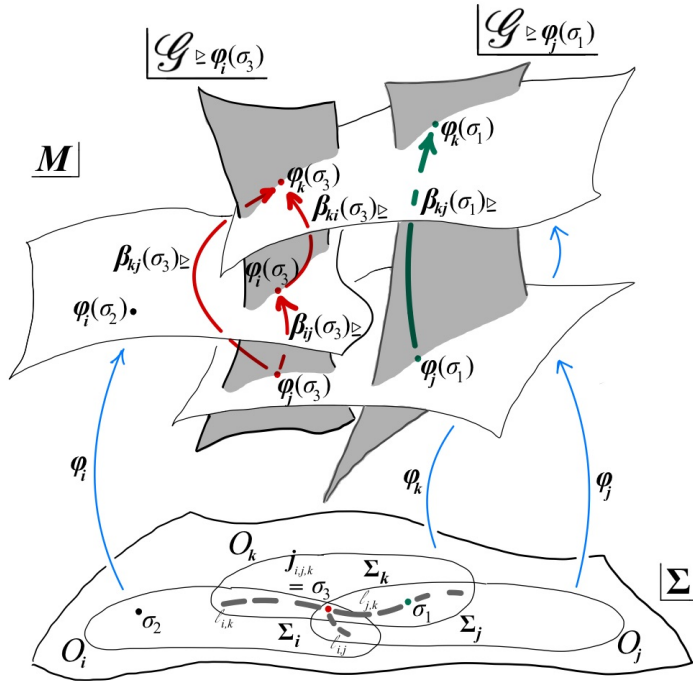
SUMMARY:

- (*) GENERIC (NON-FLAT) CO-AUGMENTATION INTRODUCES G -EQUIVARIANT TWIST $\text{curv}(G)$ to PoM;
- (**) FLAT CO-AUGMENTATION ALLOWS for NON-PREQUANTISABLE SYMPLECTIC STRUCTURE (g, ω) OR NON-MULTIPLICATIVE PREQUANTISATION;
- (***) TRIVIAL FLAT CO-AUGMENTATION CAPTURES MULTIPLICATIVE PREQUANTISATION of SYMPLECTIC INTEGRATION (g, ω) of POISSON TARGET (M, Π)

LIFE with DEFECTS (is POSSIBLE)



$\mathcal{T}_M^{\text{un}} \ 8. \ (\mathcal{T}_M^{\text{un}} \ 7.)$ SWEEPS UNDER THE RUG THE PASSAGE FROM THE LOCAL PICTURE - WHERE THE $\mathcal{G}[A_i]$ LIVE - TO THE GLOBAL ONE - WHERE $\mathcal{G}_{\mathcal{F}}$ DWELLS, WHEREAS THE GEOMETRY OF $\mathcal{P} \rightarrow \mathcal{F}$ (& - ON A MORE FUNDAMENTAL LEVEL - THAT OF THE HOMOTOPY QUOTIENT) INVITES US TO THINK IN TERMS OF Σ -LOCAL FAMILIES OF STRUCTURES ON M SUBJECT TO TRANS-LOCAL CONSTRAINTS (GLUINGS BY $B_{\infty}(\mathcal{G})$ -ACTIONS)...



... & LOCAL IDENTIFICATIONS

(GAUGE AUTOS, ALSO MODELLED ON $B_{\infty}(\mathcal{G})$).

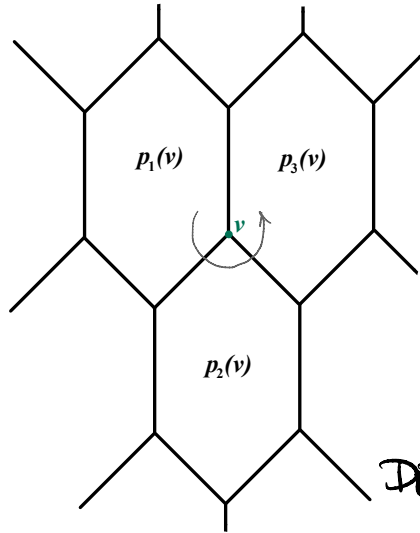
QUESTIONS: CAN WE MAKE SENSE OF THE 2d QFT DIRECTLY IN TERMS OF THESE LOCAL STRUCTURES, I.E., HOW DO WE FORMULATE THE LOCAL QFTs (over $\Sigma_i \subset \mathcal{O}_i$, $\partial \Sigma_i \neq \emptyset$) & GLUE THEM INTO A GLOBAL ONE? WHAT IS THE ROLE OF THE EQUIVARIANT STRUCTURE $(\mathbb{T}, \chi)$ ON G_j IN THIS?

ANSWERS: WE CAN—INDEED, WE HAVE BEEN DOING IT ALL ALONG...
...USING THE INTRINSIC STRUCTURE OF QFT WITH DEFECTS IN 2d PAC σ -MODEL,
BASED ON THE ELEMENTARY 2-STACK STRUCTURE—& $(\mathbb{T}, \chi)$ DECORATES
THE ENSUING GAUGE-SYMMETRY DEFECT...

THE DF AMPLITUDE A_{TAG}
RECEIVES CONTRIBUTIONS:

$$A_p[x_p] = \exp\left(-\frac{i}{2} \|TX_p\|_{\eta_p, g_M|_{\sigma_{\mathcal{U}(p)}}}^2\right) \cdot \exp\left(i \int_{X(p)} B_{\mathcal{U}(p)}\right)$$

from LOCAL LAGRANGIAN
PHASES $(\mathcal{O}_{\mathcal{U}(p)}, g_M|_{\sigma_{\mathcal{U}(p)}}, I_{B_{\mathcal{U}(p)}})$

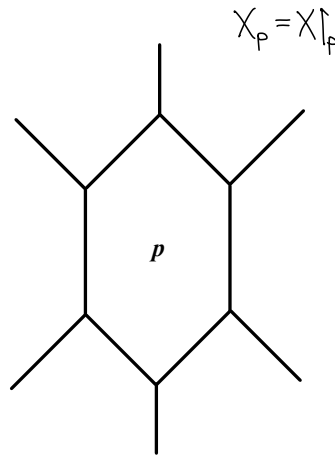


$$A_v[x_v] = g_{\mathcal{U}(p_1(v)), \mathcal{U}(p_2(v)), \mathcal{U}(p_3(v))}(X(v))$$

from JUNCTIONS

$$v \in V_{\Delta_\Sigma}$$

DECORATED by $\varphi_{\mathcal{U}(p_1(v)), \mathcal{U}(p_2(v)), \mathcal{U}(p_3(v))}$ over $\mathcal{O}_{\mathcal{U}(p_1(v)), \mathcal{U}(p_2(v)), \mathcal{U}(p_3(v))}$



over DOMAINS

$$p \in P_{\Delta_\Sigma}$$

SEPARATED by...

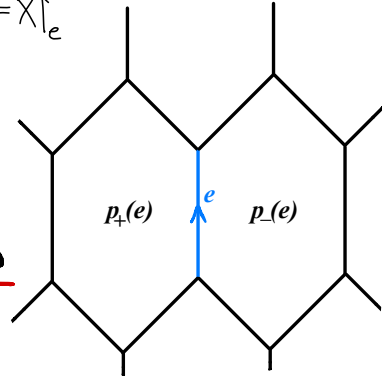
$$X_e = X|_e$$

$$A_e[x_e] = \exp\left(i \int_{X(e)} A_{\mathcal{U}(p_+(e)), \mathcal{U}(p_-(e))}\right)$$

from DOMAIN WALLS

$$e \in E_{\Delta_\Sigma}$$

DECORATED by $\Phi_{\mathcal{U}(p_+(e)), \mathcal{U}(p_-(e))}$ over $\mathcal{O}_{\mathcal{U}(p_+(e)), \mathcal{U}(p_-(e))}$
& INTERSECTING at...



THE ABOVE IS A MANIFESTATION OF A MORE GENERAL CONSTRUCTION, with XTRA MODALITY:

- JUNCTIONS MAY HAVE HIGHER VALENCE;
- GERBE BIMODULES MAY HAVE CURVATURE $\varpi \in \Omega^2(Q)$.

[Runkel-S]
 DEFINITION of POLY-PHASE PAQ σ -MODEL USES
 ARBITRARY STRATIFICATION $\mathcal{D} \sqcup \mathcal{L} \sqcup \bigsqcup_{n \geq 3} \mathcal{I}_n$ of Σ
 with EMBEDDED DEFECT QUIVER $\Gamma \hookrightarrow \bar{\Sigma}$ (& ADAPTED Δ_Σ).

IT WORKS with (STRATIFIED)

(*) BULK TARGET (M, g_M, G)
 over $\mathcal{D}$

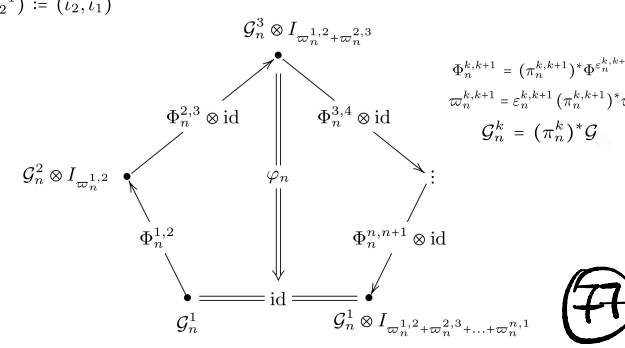
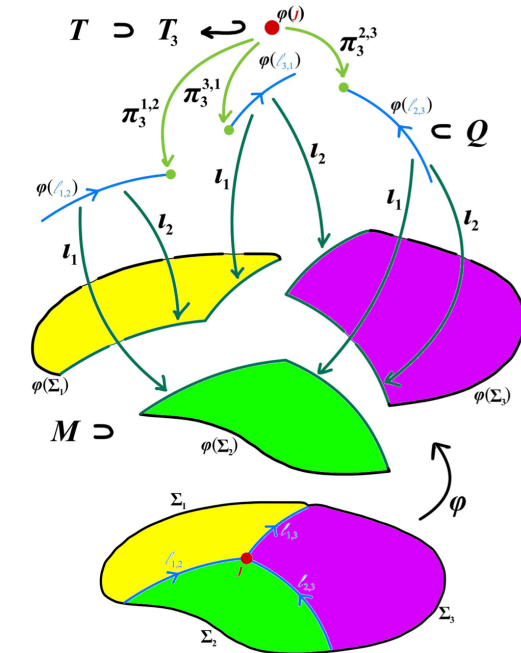
(**) BI-BRANE $(Q, \iota_1, \iota_2, \varpi, \Phi : \iota_1^* G \xrightarrow{\sim} \iota_2^* G \otimes I_\varpi)$
 over $\mathcal{L}$

(***) INTER-BI-BRANE $\bigsqcup_{n \geq 3} (T_n, \pi_n^{k,k+1}, \varepsilon_n^{k,k+1}, \varphi_n | k-1 \in \mathbb{Z}/n\mathbb{Z})$
 over $\bigsqcup_{n \geq 3} \mathcal{I}_n$
 (ORIENTATION MAPS) $T_n \rightarrow \{-1, 1\}$

CONSTRAINED AS (*)

$$\varepsilon_2^{k-1,k} \circ \pi_n^{k-1,k} = \varepsilon_1^{k,k+1} \circ \pi_n^{k,k+1} =: \pi_n^k$$

$$(\iota_1^{-1}, \iota_2^{-1}) := (\iota_1, \iota_2) \wedge (\iota_1^{-1}, \iota_2^{-1}) := (\iota_2, \iota_1)$$



FOR THE ABOVE, ONE WRITES A STRATIFIED CHARACTER [Runkel-S]

$$\text{Hd}_{(g, \mathbb{F}, \varphi)}(X(\Sigma) | \Gamma),$$

WHICH HAS THE PRODUCT STRUCTURE :

$$\prod_{d \in \mathcal{D}} \text{Hd}_{g_j}^{\text{cut}}(X(d)) \cdot \prod_{l \in L} \text{Hd}_{\mathbb{F}}^{\text{cut}}(X(l)) \cdot \prod_{n \in \mathbb{N}_{\geq 3}} \prod_{j \in \mathcal{I}_n} \text{Hd}_{\varphi_n}(X(j))$$

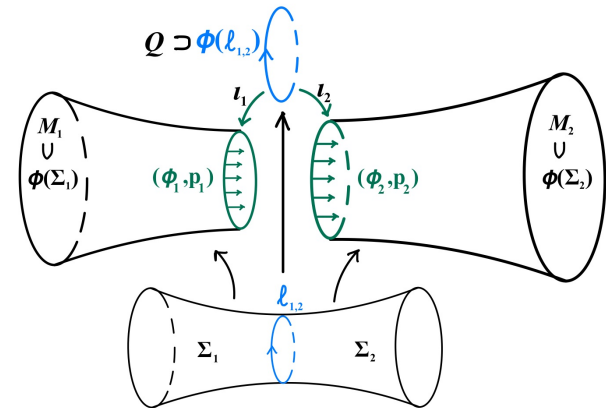
SC MODELS PATCHWISE SMOOTH

PROPAGATION of CHARGED LOOPS in $M = M_1 \sqcup M_2 \sqcup \dots$

within CONNECTED COMPONENTS $M_i \subset M$,

GLUED along DEFECT LINES $\in L$

by CROSS-DEFECT STATE CORRESPONDENCES

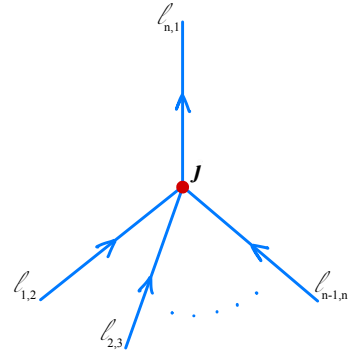


FAMILIAR EXAMPLE of STRING BACKGROUND: RIGID G-SYMMETRY DEFECT

(*) BULK TARGET (M, g_M, G) - METRIC OBJECT MANIFOLD of $G \Rightarrow M$,
with ω -TWISTED G -EQUIVARIANT GERBE G

(**) BI-BRANE (G, t, s, ω, I)

(***) TRINION INTER-BI-BRANE $(G_2, \pi_1, \pi_2, m, \gamma)$
 $\varepsilon_3^{1,2} = 1 = \varepsilon_3^{2,3} = -\varepsilon_3^{3,1}$



(****) $(n-1)$ - $\mathbb{P}_0-1$ INTER-BI-BRANE G_{n-1} with MAPS

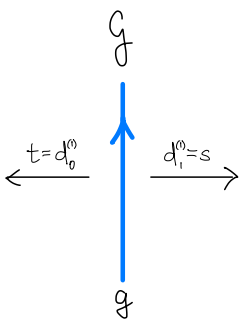
$$\pi_n^{1,2} = d_0^{(1)} \circ d_0^{(2)} \circ \dots \circ d_0^{(n-1)}; \quad \pi_n^{l, l+1} = d_2^{(1)} \circ d_2^{(2)} \circ \dots \circ d_2^{(l)} \circ d_0^{(l+1)} \circ d_0^{(l+2)} \circ \dots \circ d_0^{(n-1)}, \quad l \in \overline{2, n-1}; \quad \pi_n^{n,1} = d_1^{(1)} \circ d_2^{(2)} \circ d_2^{(3)} \circ \dots \circ d_2^{(n-1)}$$

& FUSION 2-ISOMORPHISM $\varphi_n = (\text{id} \circ \hat{\pi}_n^{12 \dots n-2, n-1} * \gamma) \cdot (\text{id} \circ \hat{\pi}_n^{12 \dots n-3, n-2} * \gamma) \cdot \dots \cdot (\text{id} \circ \hat{\pi}_n^{12, 3} * \gamma) \cdot (\text{id} \circ \hat{\pi}_n^{1, 2} * \gamma)$

$$\hat{\pi}_n^{1,2} = d_0^{(3)} \circ d_0^{(4)} \circ \dots \circ d_0^{(n-1)}; \quad \hat{\pi}_n^{12 \dots l, l+1} = d_2^{(3)} \circ d_3^{(4)} \circ \dots \circ d_l^{(l+1)} \circ d_0^{(l+2)} \circ d_0^{(l+3)} \circ \dots \circ d_0^{(n-1)}, \quad l \in \overline{2, n-3}; \quad \hat{\pi}_n^{12 \dots n-2, n-1} = d_2^{(3)} \circ d_3^{(4)} \circ \dots \circ d_{n-2}^{(n-1)}$$

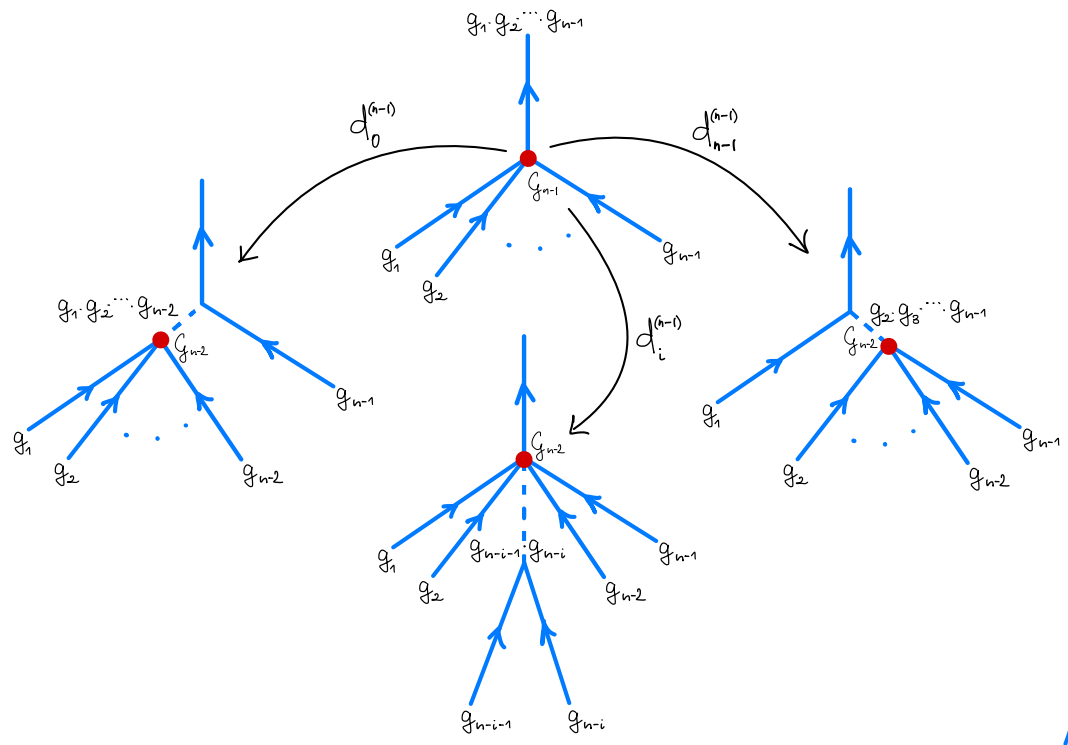
THE 'GRAPHICAL' DEFECT CALCULUS

WALL-TO-DOMAIN FACE MAPS

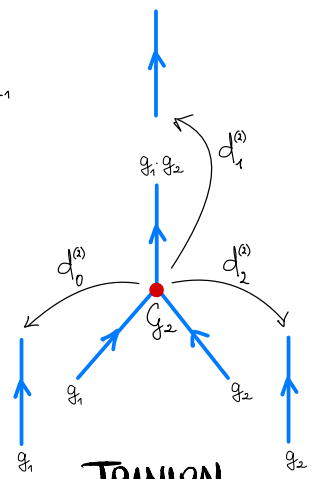


DEFECT
LINE

JUNCTION FACE MAPS

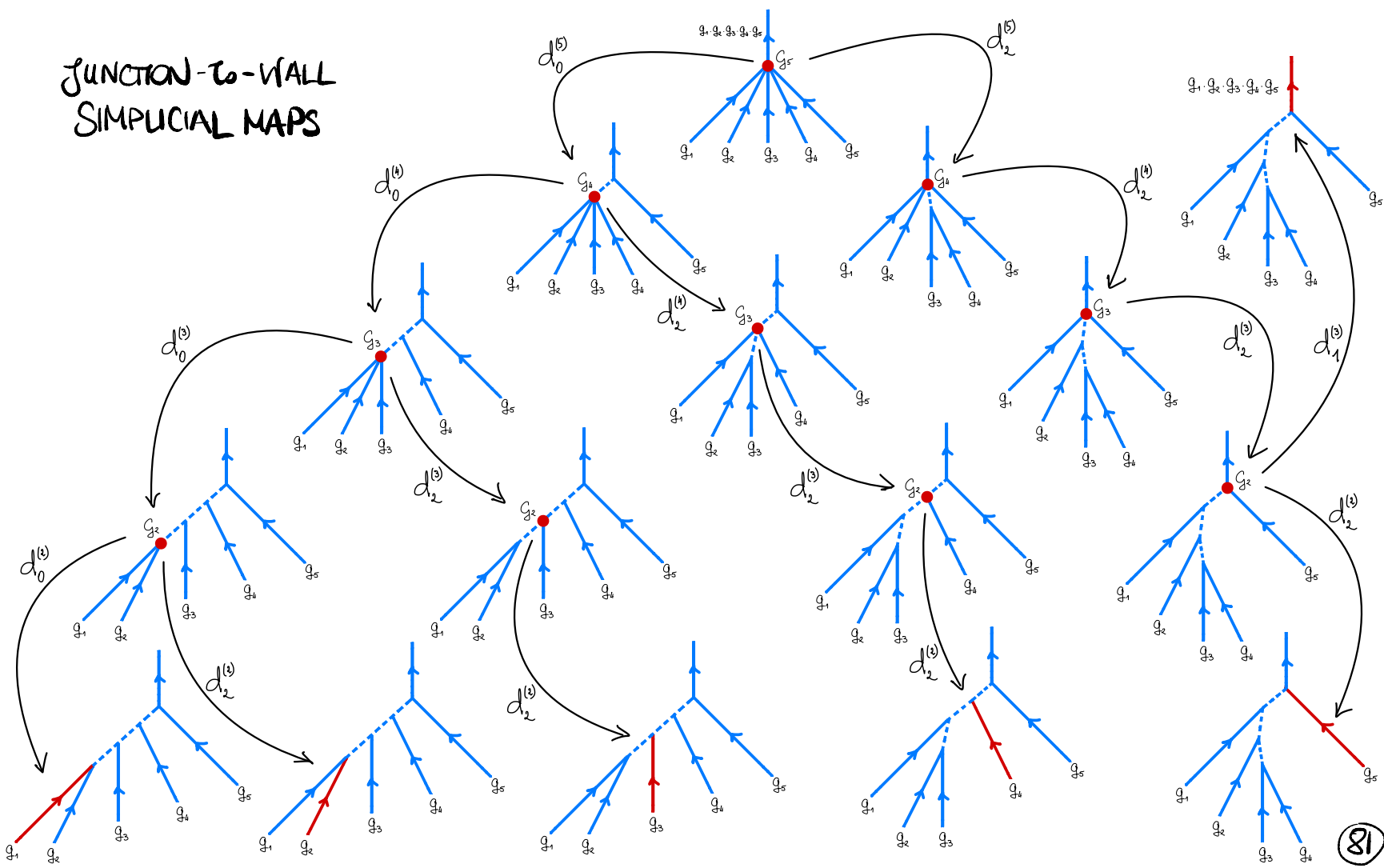


JUNCTION-TO-WALL FACE MAPS

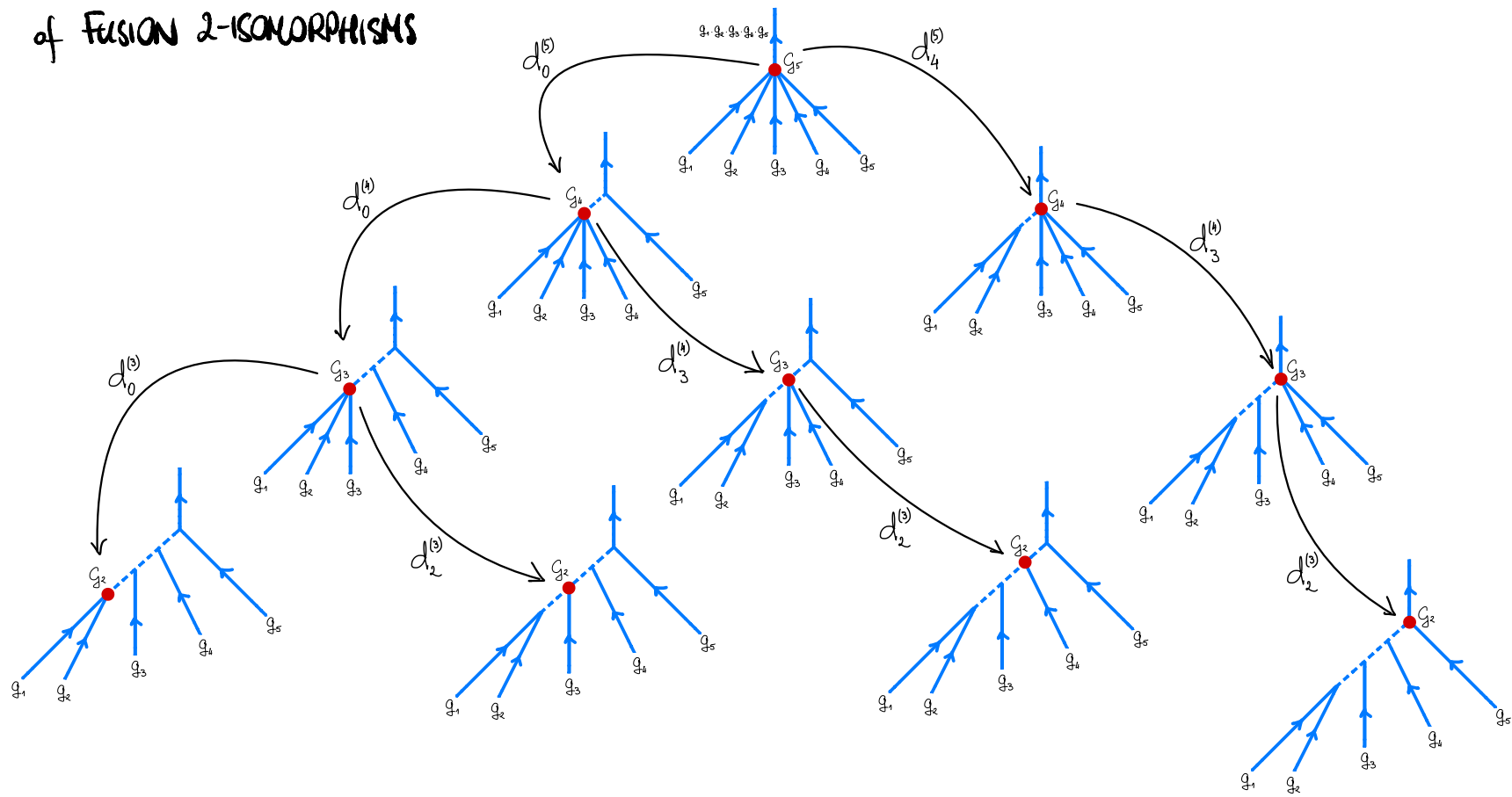


TRINION
JUNCTION

JUNCTION-TO-WALL SIMPLICIAL MAPS



SIMPLICIAL INDUCTION of FUSION 2-ISOMORPHISMS

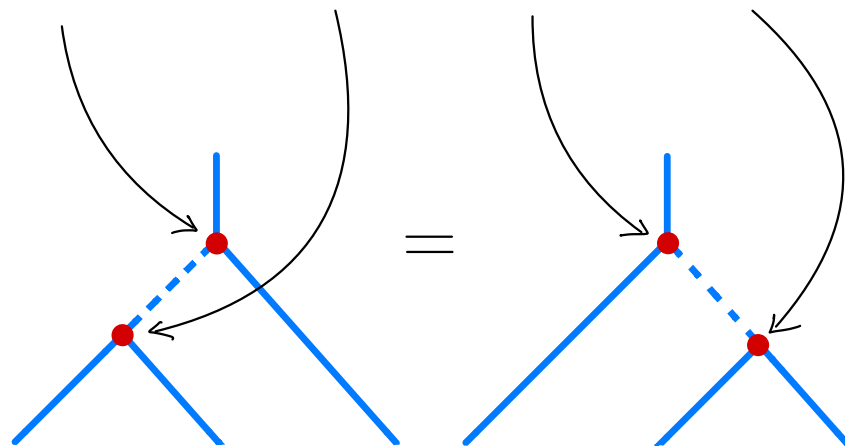


PROBLEM: WHILE DEFINITION OF THE $\mathcal{T}_n^{k,l+1}$ IS UNIQUE, THAT OF THE φ_n MAY SEEM HIGHLY NON-UNIQUE, BUT...

COHERENCE IDENTITY (SEE: 9.51.)

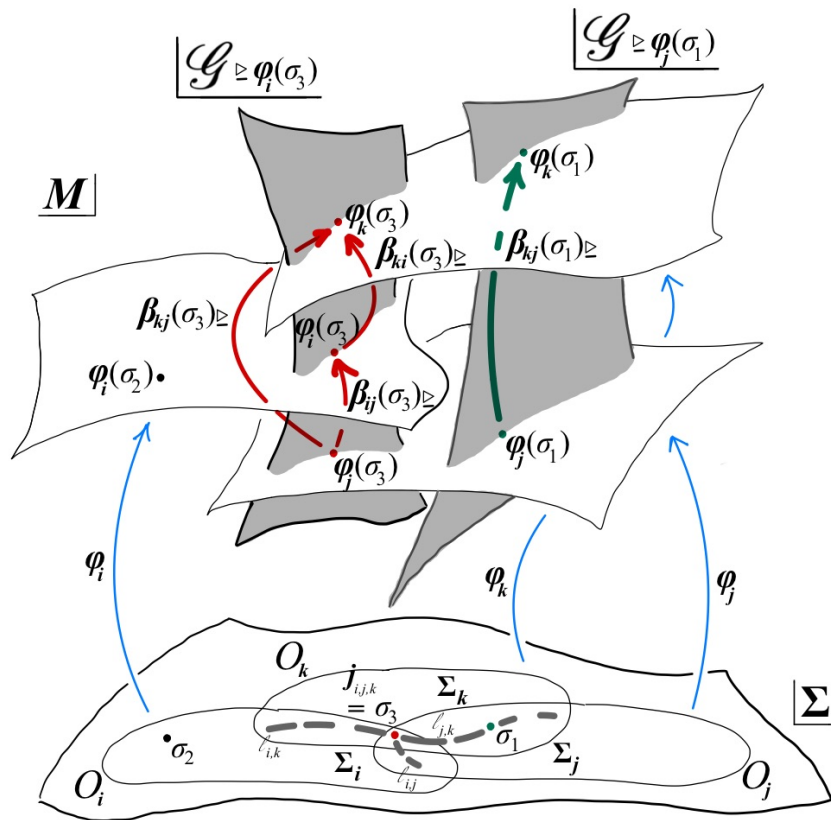
$$d_2^{(3)*} \gamma \cdot (\text{id} \circ d_0^{(3)*} \gamma) = d_1^{(3)*} \gamma \cdot ((d_3^{(3)*} \gamma \otimes \text{id}) \circ \text{id})$$

TRANSLATES into
ASSOCIATOR MOVE



CONCLUSION: AS LONG AS WE INDUCE FUSION 2-ISOMORPHISMS for $n > 3$ FROM THE ELEMENTARY ONES, γ , PROVIDED BY G -EQUIVARIANT STRUCTURE, THE RESULT IS UNIQUE.

THE (TOPOLOGICAL) GAUGE DEFECT



(D) PHASES over DOMAINS $\Sigma_i \subset \mathcal{O}_i$ for $\pi_j^{-1}(\mathcal{O}_i) \simeq \mathcal{O}_i \times \mathcal{G}$:

$$M_i := \mathcal{O}_i \times M \longleftarrow \Sigma_i : (id_{\Sigma_i}, \varphi_i) = \mathcal{F}\mathcal{T}_i \circ \varphi, \varphi \in \Gamma(\mathcal{F})$$

with BACKGROUND $g_M \circ \left(\bigoplus_{\mathcal{F}_i} \right) \Big|_{M_i} =: g_{M_i}$ & $p_2^* \mathcal{G} \otimes I_{\omega[\mathcal{A}_i]} =: \widetilde{\mathcal{G}}_i$

SEPARATES Σ_i from Σ_j

(L) $\mathcal{B}_\sigma(\mathcal{G})$ -CORRESPONDENCES over DEFECT LINES $\ell_{ij} \subset \mathcal{O}_{ij}$:

$$Q_{ij} := \widehat{\beta}_{ij}(\mathcal{O}_{ij} \times M) \longleftarrow \ell_{ij} : (id_{\Sigma}, \bar{\beta}_{ij}(\cdot, \varphi_j(\cdot))) =: \bar{\beta}_{ij} \circ \mathcal{F}\mathcal{T}_j \circ \varphi$$

$$\widehat{\beta}_{ij}(\sigma, x) = (\sigma, \beta_{ij}(\sigma)(x))$$

$$\mathcal{O}_{ij} \overset{\wedge}{\times} \mathcal{G}$$

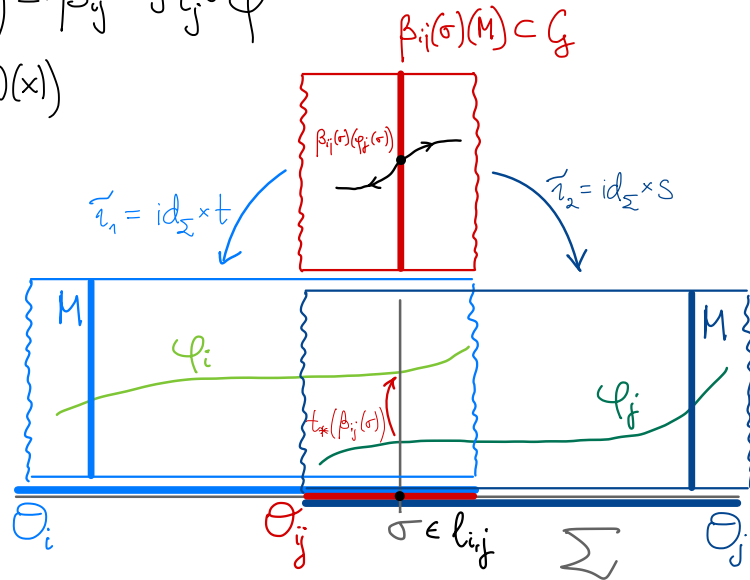
SMOOTH FIBRATION of BISECTIONS over $\mathcal{O}_{ij}$

with BI-BRANE MAPS

$$\widetilde{\pi}_1 = id_{\Sigma} \times t \quad \& \quad \widetilde{\pi}_2 = id_{\Sigma} \times s$$

& **FLAT** BIMODULES

$$\widetilde{\mathcal{G}}_{ij} := p_2^* \mathcal{I} \otimes id_{I_{\omega[\mathcal{A}]}} \Big|_{Q_{ij}} : \widetilde{\pi}_1^* \widetilde{\mathcal{G}}_i \xrightarrow{\simeq} \widetilde{\pi}_2^* \widetilde{\mathcal{G}}_j \otimes I_0$$



(J) FUSIONS of $B_\sigma(g)$ -CORRESPONDENCES over DEFECT FUNCTIONS $j_n \in \mathcal{O}_{i_1 i_2 \dots i_n}$:

$$T_{i_1 i_2 \dots i_n} := \hat{\beta}_{i_1 i_2 \dots i_n}(\mathcal{O}_{i_1 i_2 \dots i_n} \times M) \longleftarrow \{j_n\} : (id_\Sigma, \beta_{i_1 i_2}(\cdot, \varphi_{i_2}(\cdot)), \beta_{i_2 i_3}(\cdot, \varphi_{i_3}(\cdot)), \dots, \beta_{i_{n-1} i_n}(\cdot, \varphi_{i_n}(\cdot))) = \hat{\beta}_{i_1 i_2 \dots i_n} \circ F\tau_{i_n} \circ \varphi$$

$\cap$
 $\mathcal{O}_{i_1 i_2 \dots i_n} \times G_{n-1}$

SMOOTH FIBRATION of M -FIBRED $(n-1)$ -TUPLES of BISECTIONS over $\mathcal{O}_{i_1 i_2 \dots i_n}$

with INTER-BI-BRANE MAPS AS in THE RIGID G -SYMMETRY EXAMPLE (1.79)

& FUSION 2-ISOMORPHISM INDUCED from THE TRINION

with TARGET $T_{ijk} := \hat{\beta}_{ijk}(\mathcal{O}_{ijk} \times M) \longleftarrow \{j_3\} : (id_\Sigma, \beta_{ij}(\cdot, \varphi_j(\cdot)), \beta_{jk}(\cdot, \varphi_k(\cdot))) = \hat{\beta}_{ijk} \circ F\tau_k \circ \varphi$

$\cap$
 $\mathcal{O}_{ijk} \times G_2$

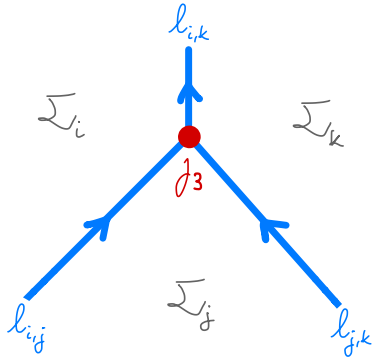
MAPPING to THE BI-BRANE by

(in THE 2→1 SCENARIO)

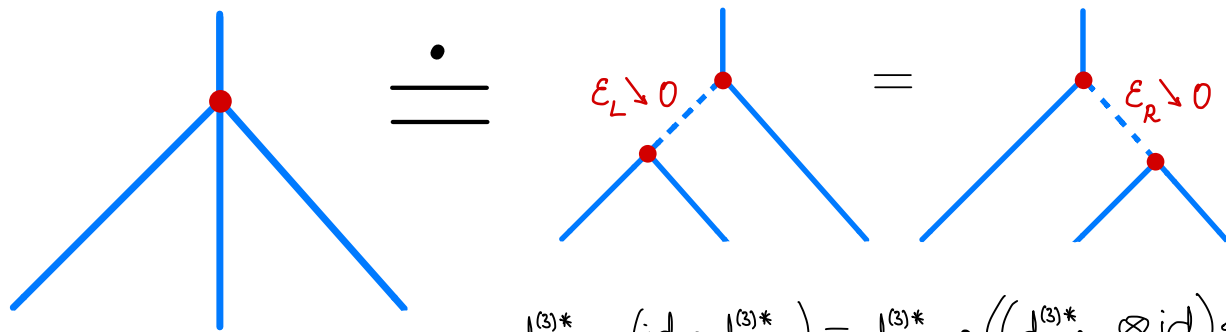
$$\begin{aligned} \pi_3^{1,2} &:= id_\Sigma \times pr_1 \\ \pi_3^{2,3} &:= id_\Sigma \times pr_2 \\ \pi_3^{3,1} &:= id_\Sigma \times m \end{aligned}$$

& COMING with THE FUSION 2-ISOMORPHISM

$$\varphi_{++} = pr_2^* \gamma \otimes id|_{T_{ijk}}$$



INDUCTION: enough ASSOCIATIVE VERTICAL FUSION of TRINIIONS



$$d_2^{(3)*} \gamma \cdot (\text{id} \circ d_0^{(3)*} \gamma) = d_1^{(3)*} \gamma \cdot ((d_3^{(3)*} \gamma \otimes \text{id}) \circ \text{id})$$

THE FUSION IS WELL-DEFINED DUE TO THE FACT THAT

THE GAUGE DEFECT IS TOPOLOGICAL,

I.E., A HOMOTOPIC DEFORMATION OF THE DEFECT QUIVER PRESERVES THE VALUE OF THE DF AMPLITUDE.

UPSHOT: POLY-PHASE PAC / COAUGMENTED POISSON σ -MODEL
for ARBITRARY NETWORK-FIELD CONFIGURATIONS

$$\varphi: \mathcal{D} \sqcup \mathcal{L} \sqcup \bigsqcup_{n \geq 3} \mathcal{I}_n \longrightarrow \mathcal{M} \sqcup \mathcal{Q} \sqcup \bigsqcup_{n \geq 3} \mathcal{T}_n$$

with TOPOLOGICAL GAUGE-SYMMETRY DEFECT.

$$\begin{cases} \mathcal{M} = \bigsqcup_i \mathcal{M}_i \\ \mathcal{Q} = \bigsqcup_{ij} \mathcal{Q}_{ij} \\ \mathcal{T}_n = \bigsqcup_{i_1, \dots, i_n} \mathcal{T}_{i_1 \dots i_n} \end{cases}$$

SUMMARY:

$$\text{'GAUGE' IS A VERB } M \rightrightarrows M//_G = \|G.\|$$

* 'UNIVERSAL' STRUCTURE for the GAUGE-ING of SYMMETRY $G \Rightarrow M$:

$$\begin{array}{ccc} \text{THE PRINCIPALOID } G\text{-BUNDLE } (P, \pi) & \longrightarrow & (F, \pi_f) \\ \downarrow \Sigma & \xlongequal{\quad} & \downarrow \Sigma \\ \mathcal{C} \quad (P_{\mu_P} \times_{\mu_X} X)/G & & \text{IMPLEMENTED by } B(G) \end{array}$$

** SHEAFY DESCENT of MINIMALLY COUPLED $B_*(G)$ -INVARIANT TENSORIAL COUPLINGS

*** STACKY DESCENT of (H) -AUGMENTED MULTIPLICATIVELY-TWISTED EQUIVARIANT COHOMOLOGICAL COUPLINGS

**** IMPLEMENTATION of the GAUGING by TOPOLOGICAL DEFECTS

***** (G, ω) -EQUIVARIANTLY CO-AUGMENTED POISSON σ -MODEL AS GAUGE(D) TQFT

OUTLOOK:

- * PROPER DYNAMICS of THE HE-ALGEBROIDAL & MCGSÖID FIELDS.
- ** PROPAGATION of PROBES in EXTERNAL HE-ALGEBROIDAL FIELDS.
HOLONOMY on PRINCIPALOID BUNDLES.
- *** THE FULLY FLEDGED BICATEGORY of THE COAUGMENTED POISSON σ -MODEL
(PREQUANTISABLE BOUNDARY & MAXIMALLY SYMMETRIC DEFECT STATES etc.).
- **** DIRAC σ -MODELS AS GROUPOIDAL GAUGE QFTs.
- ***** CONCRETE EXAMPLES of NON-ACTION STRUCTURE GROUPOIDS
of IMMEDIATE PHYSICAL INTEREST — E.G., THE LU-WEINSTEIN
INTEGRATORS of POISSON-LIE GROUPS, & THE ASSOCIATED DUALITIES..
- ***** GAUGING of NON-ISOMETRIES in THE FAG σ -MODEL.
- ***** THE ¹ROLE of THE COAUGMENTING CERBE in DEFORMATION QUANTISATION
CAPTURED by THE POISSON σ -MODEL...

Gauging the Oid
& Other Tales

CECI (N')EST
(PAS) LA FIN!

by Rafał R. Suszek

DGMP2026

PAŁAC POTOCKICH

BĘDLEWO, JULY 19-28, 2026

MAKES BEN HUB
LOOK SO AM-EPIC!

*La Géométrie est une image,
mais il n'y a pas d'image de la Géométrie.*