

# Nanostructures



PHYSICAL REVIEW B **84**, 165319 (2011)

## **Magnetophotoluminescence study of intershell exchange interaction in CdTe/ZnTe quantum dots**

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## **Probing the Spin State of a Single Magnetic Ion in an Individual Quantum Dot**

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# THE ARTICLE

## Dr Tomasz Kazimierczuk

1. What type of the quantum dots is described in the paper? What is expected energy range for their luminescence?
2. How the quantum dots were obtained? How dense they are (e.g., how many dots there is within the laser spot)?
3. How many eigenstates neutral exciton has? How many spectral lines?
4. Which of the two exhibit larger anisotropy: neutral exciton or biexciton?
5. Why the experiments are carried out at low temperatures? What is the relevant energy scale?

Questions for paper 2:

6. Why the exciton is splitted into 6 lines? Which of them corresponds to Mn spin of  $+5/2$ , and which corresponds to spin of  $-5/2$ ?
7. In the magnetic field, the photoluminescence lines shift due to Zeeman effect (see Fig. 2). How this figure would have changed, if the g-factor of the manganese had been 2 times larger?
8. What is the origin of the anticrossing marked in Fig. 2?

## ARTIFICIAL ATOMS

The charge and energy of a sufficiently small particle of metal or semiconductor are quantized just like those of an atom. The current through such a quantum dot or one-electron transistor reveals atom-like features in a spectacular way.

Marc A. Kastner

M A Kastner, *Phys. Today* , **46**, 24 (1993)

REVIEW ARTICLE

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## Electrons in artificial atoms

**R. C. Ashoori**

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Progress in semiconductor technology has enabled the fabrication of structures so small that they can contain just one mobile electron. By varying controllably the number of electrons in these 'artificial atoms' and measuring the energy required to add successive electrons, one can conduct atomic physics experiments in a regime that is inaccessible to experiments on real atoms.

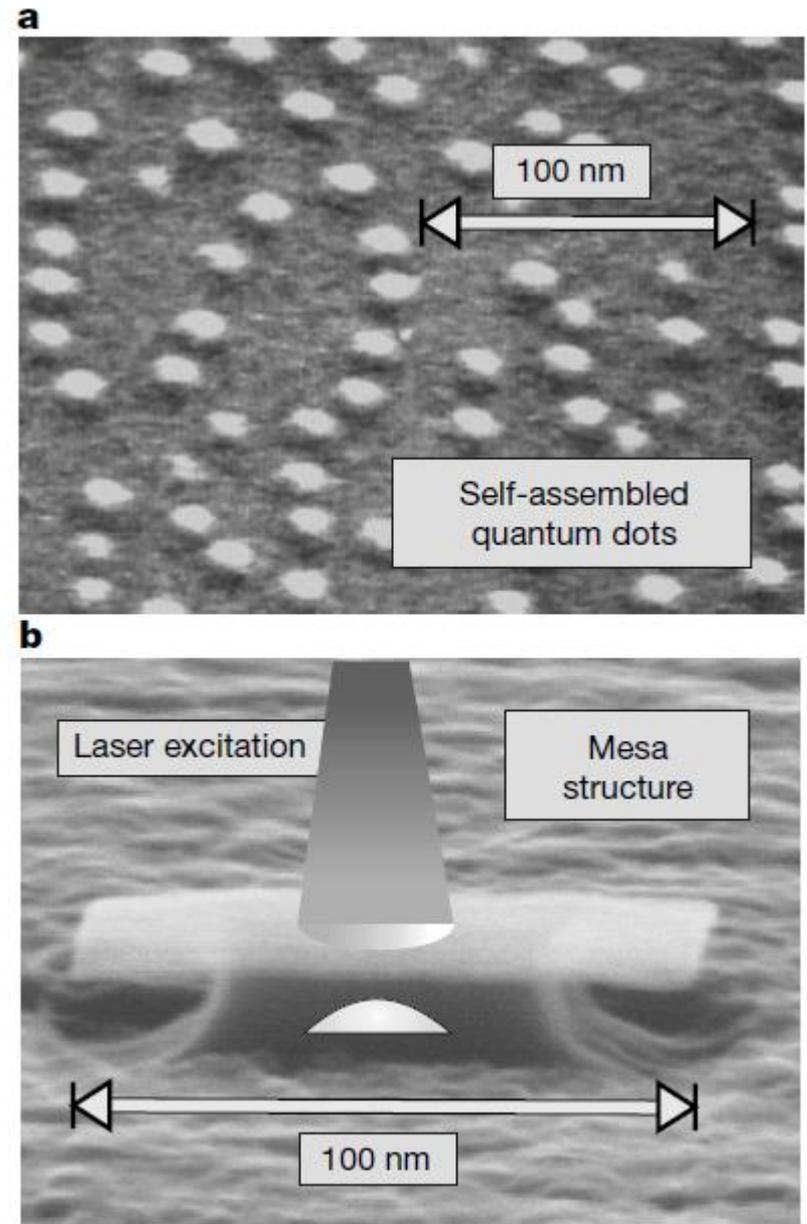
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R C Ashoori, *Nature* , **379**, 413 (1996)

# THE ARTICLE

NATURE | VOL 405 | 22 JUNE 2000 | www.nature.com

**Figure 1** Scanning electron micrographs illustrating the experimental technique used for studying single self-assembled quantum dots. **a**, Scanning electron micrograph of a GaAs semiconductor layer on which  $\text{In}_{0.60}\text{Ga}_{0.40}\text{As}$  self-assembled quantum dots with a density of about  $10^{10} \text{ cm}^{-2}$  have been grown by molecular beam epitaxy. To permit their microscopic observation these dots—unlike those used for spectroscopy—have not been covered by a GaAs cap layer. To a good approximation, all quantum dots have the same shape exhibiting rotational symmetry. However, their size varies by a few nanometres around an average diameter of 15 nm. This inhomogeneity results in a considerable broadening of the emission lines in spectroscopic studies. **b**, To avoid this broadening we have studied the emission of a single quantum dot. Lithographic techniques were used to fabricate small mesa structures on samples capped by a GaAs layer. The lateral mesa size was reduced to such an extent ( $<100 \text{ nm}$ ) that only a single dot is contained in it. These mesa structures have been studied by photoluminescence spectroscopy at low temperature. A laser beam (shown schematically as a truncated cone above the mesa) injects a controlled number of electrons and holes into the dot indicated by the lens shape, and the emission spectrum of this complex is recorded. To reduce sample heating under optical excitation, the structures are held in superfluid helium at about 1.2 K. After dispersion by a monochromator, the emission is detected by a CCD (charge-coupled device) camera.



# Harmonic potential 2D

$$E_n^x = \hbar\omega_0 \left( n_x + \frac{1}{2} \right) \text{ in } x \text{ direction and the same in } y$$

$$E_n^y = \hbar\omega_0 \left( n_y + \frac{1}{2} \right)$$

$$E_n = E_n^x + E_n^y = \hbar\omega_0(N + 1)$$

Degeneracy?

$$N = n_x + n_y$$

$$g_N = N + 1$$

$N$	$(n_x, n_y)$
0	(0,0)
1	(1,0) (0,1)
2	(2,0) (1,1) (0,2)
3	(3,0) (2,1) (1,2) (0,3)

2D disk shaped dot

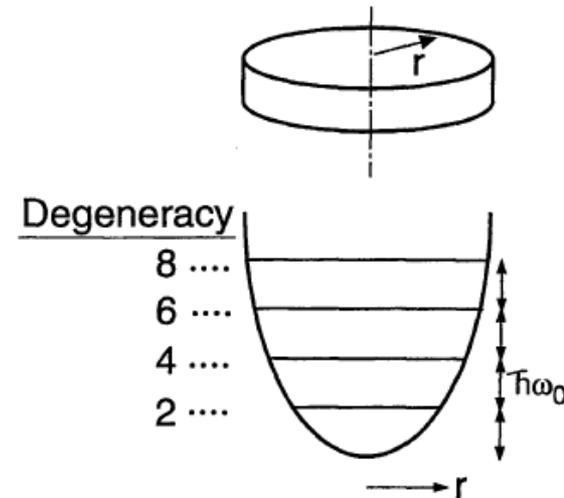


Fig. 5. Schematic model for the vertical dot with a harmonic lateral potential. The single-particle states are laterally confined into discrete equidistant 0D levels whose degeneracies are 2, 4, 6, 8, ... including spin degeneracy from the lowest level.

Jpn. J. Appl. Phys. Vol. 36 (1997) pp. 3917-3923  
Part 1, No. 6B, June 1997

# Harmonic potential 3D

$$E_n^x = \hbar\omega_0 \left( n_x + \frac{1}{2} \right) \text{ in } x, y \text{ i } z$$

$$E_n = E_n^x + E_n^y + E_n^z = \hbar\omega_0 \left( N + \frac{3}{2} \right)$$

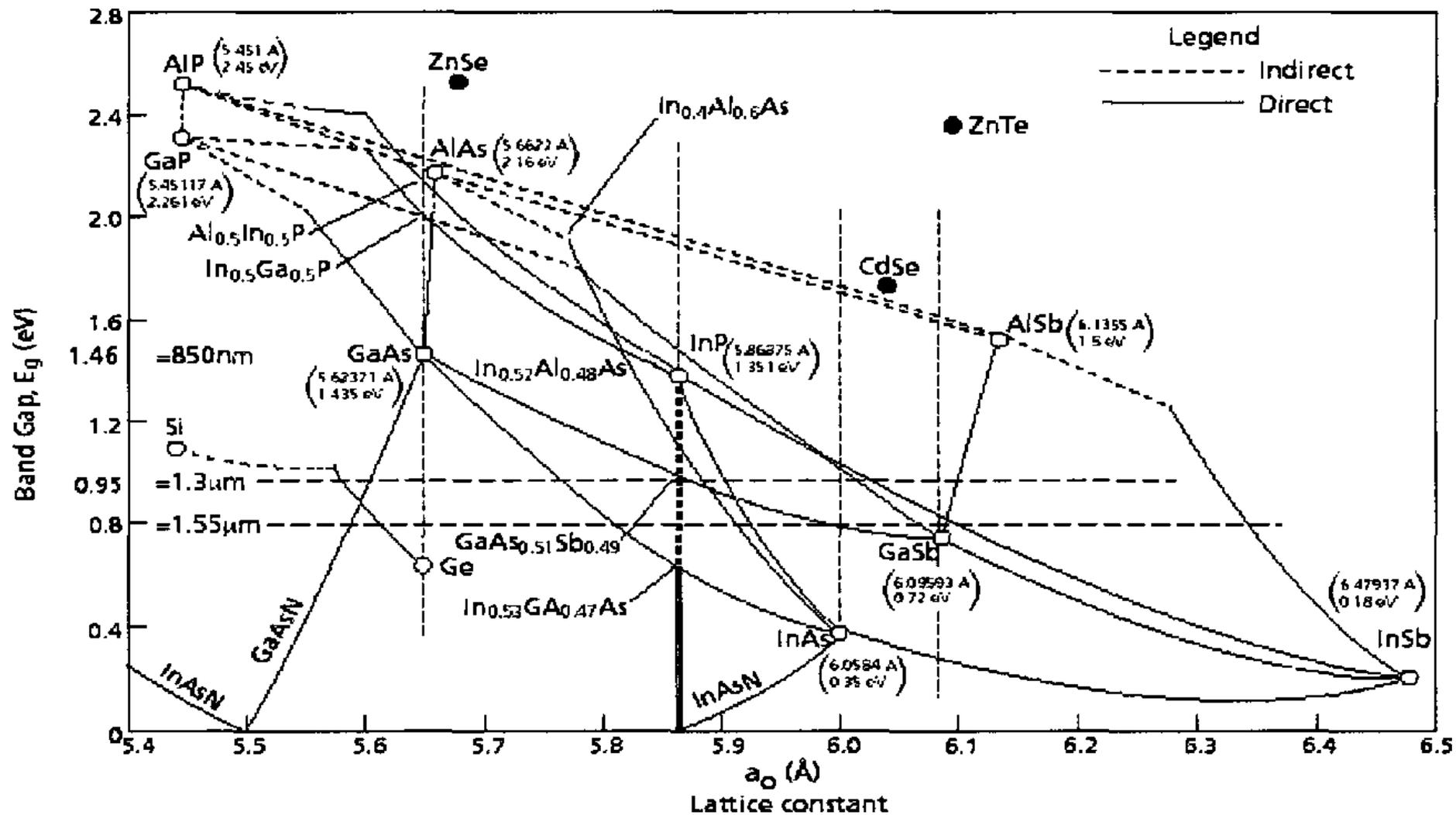
Degeneracy?

$$N = n_x + n_y + n_z$$

$$g_N = \frac{(N+1)(N+2)}{2}$$

$N$	$(n_x, n_y, n_z)$
0	(0,0,0)
1	(1,0,0) (0,1,0) (0,0,1)
2	(2,0,0) (0,2,0) (0,0,2) (1,1,0) (1,0,1) (0,1,1)
3	3x(3,0,0) 1x(1,1,1) 6x(2,0,1)

# Semiconductor heterostructures



Investigation of high antimony-content gallium arsenic nitride-gallium arsenic antimonide heterostructures for long wavelength application

# Pasma energetyczne

Do optoelektroniki potrzebna jest przerwa prosta.

$$E_g^{InGaAs} = 0.4105 + 0.6337x + 0.475x^2 \text{ eV @ 2.0K}$$

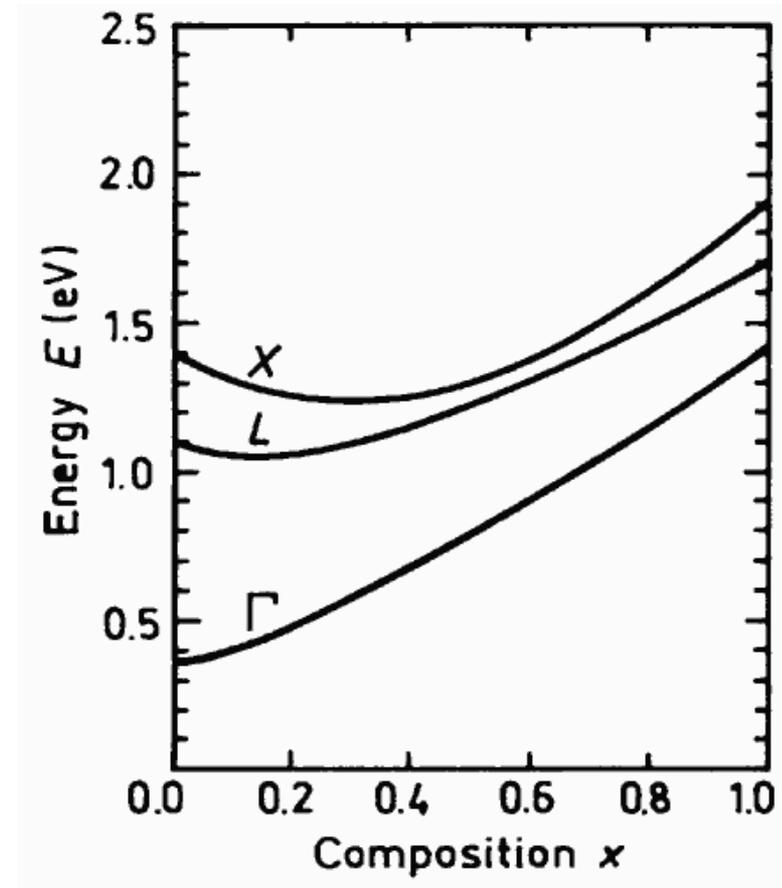
$$\hbar\omega_n = \varepsilon_{e,n_e} - \varepsilon_{h,n_h} =$$

$$= E_g^{InGaAs} + \frac{\hbar^2 \pi^2 n^2}{2m_0 a^2} \left( \frac{1}{m_e} + \frac{1}{m_h} \right) =$$

$$= E_g^{InGaAs} + \frac{\hbar^2 \pi^2 n^2}{2m_0 m_{eh} a^2}$$

$$m_e = (0.023 - 0.037x + 0.003x^2)m_0$$

$$m_h = (0.41 - 0.1x)m_0$$



NAPRĘŻENIA! – nie da się dobrać grubości, ważny czynnik to naprężenie warstwy i kropek!

# Pasma energetyczne

Do optoelektroniki potrzebna jest przerwa prosta.

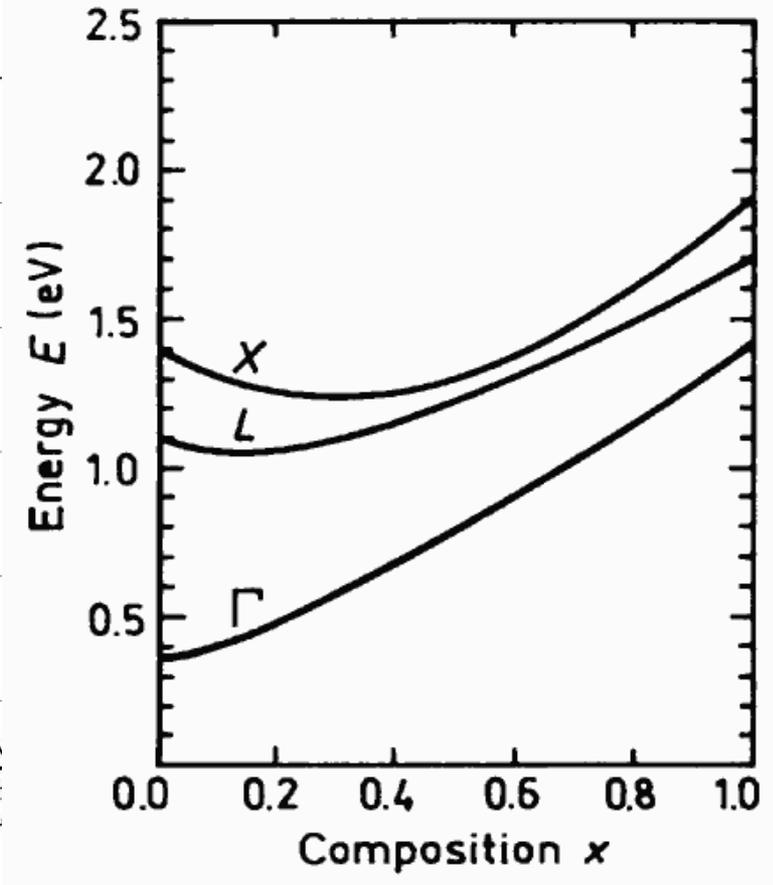
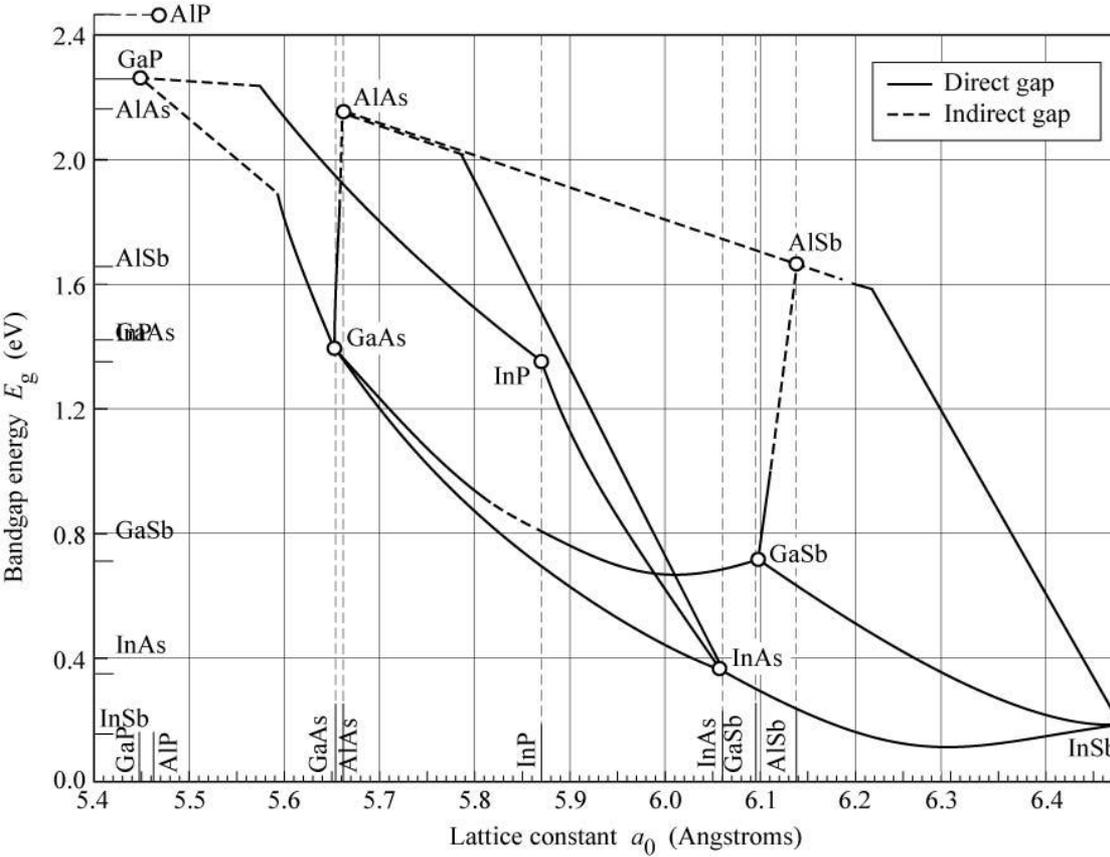
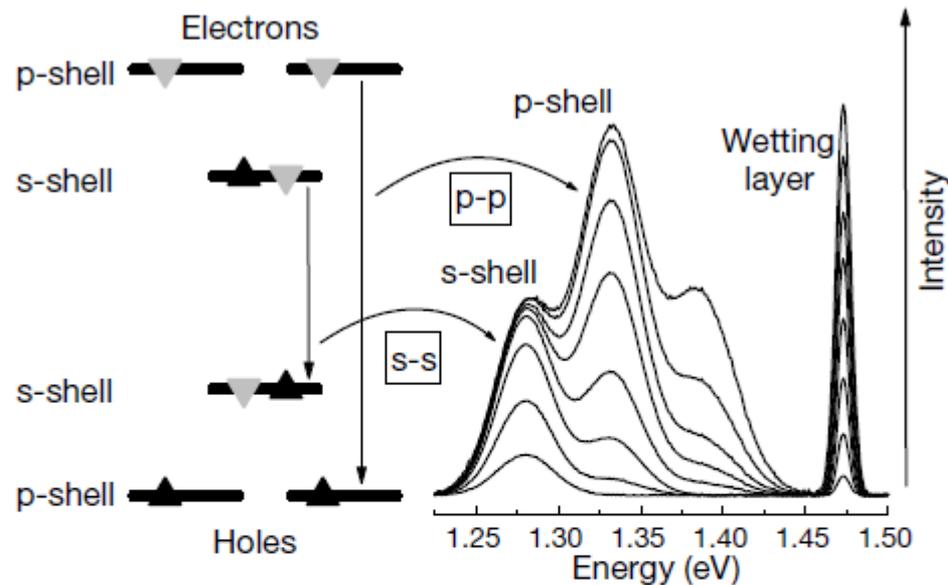


Fig. 12.6. Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).

E. F. Schubert  
 Light-Emitting Diodes (Cambridge Univ. Press)  
[www.LightEmittingDiodes.org](http://www.LightEmittingDiodes.org)

$$(0.4105 + 0.6337x + 0.475x^2)$$

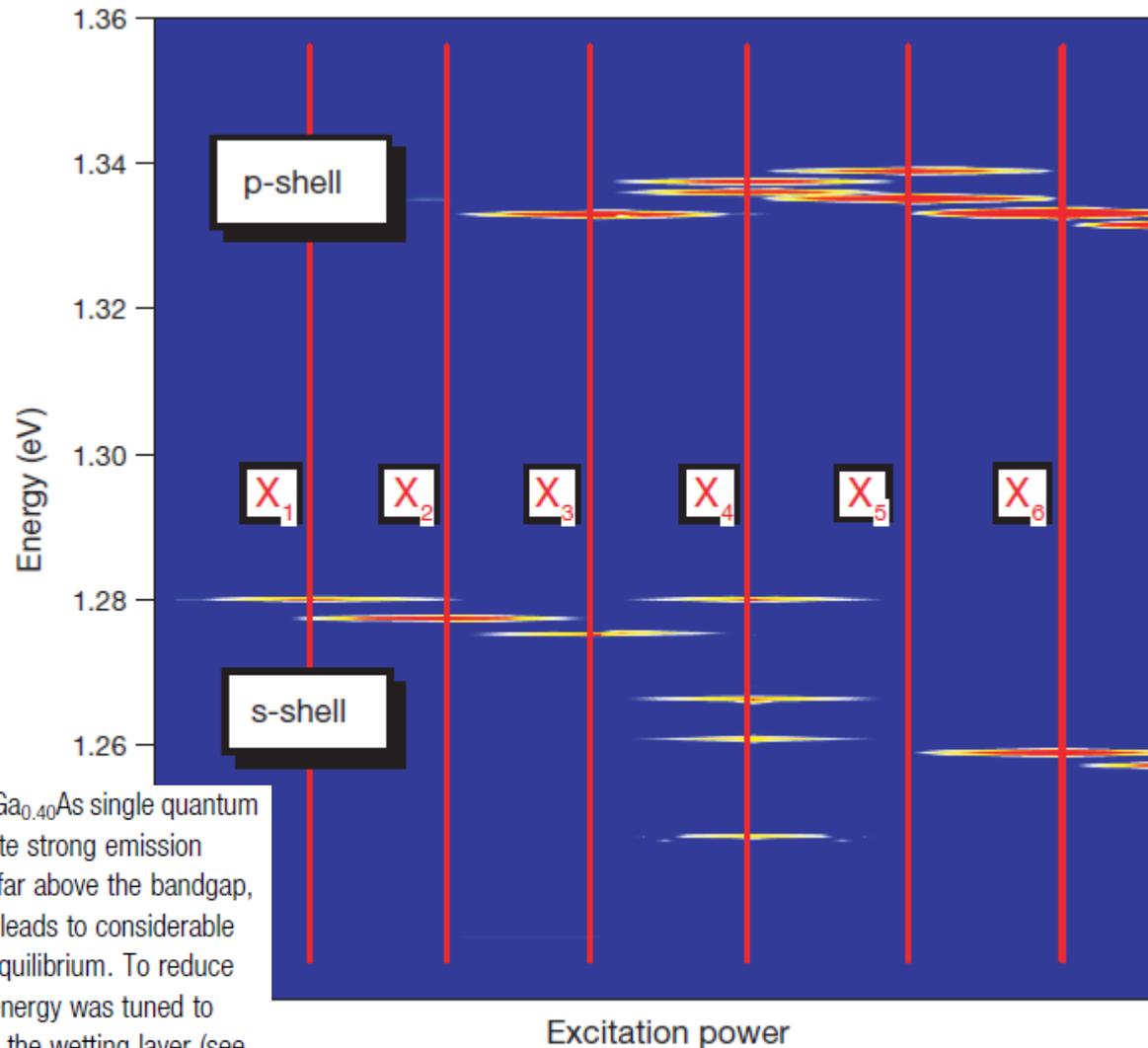
<http://www.ioffe.rssi.ru/SVA/NSM/Semicond/GaInAs/bandstr.html>



**Figure 2** State filling spectroscopy on quantum dots. On the left is a scheme of the dot energy levels, their occupation by carriers and the radiative transitions. Spin orientations of electrons and holes: grey triangles, spin-down; black triangles, spin-up. On the right are typical emission spectra resulting from these transitions for an ensemble of  $\text{In}_{0.60}\text{Ga}_{0.40}\text{As}$  quantum dots; these spectra were recorded at different excitation powers (an Ar-ion laser was used).

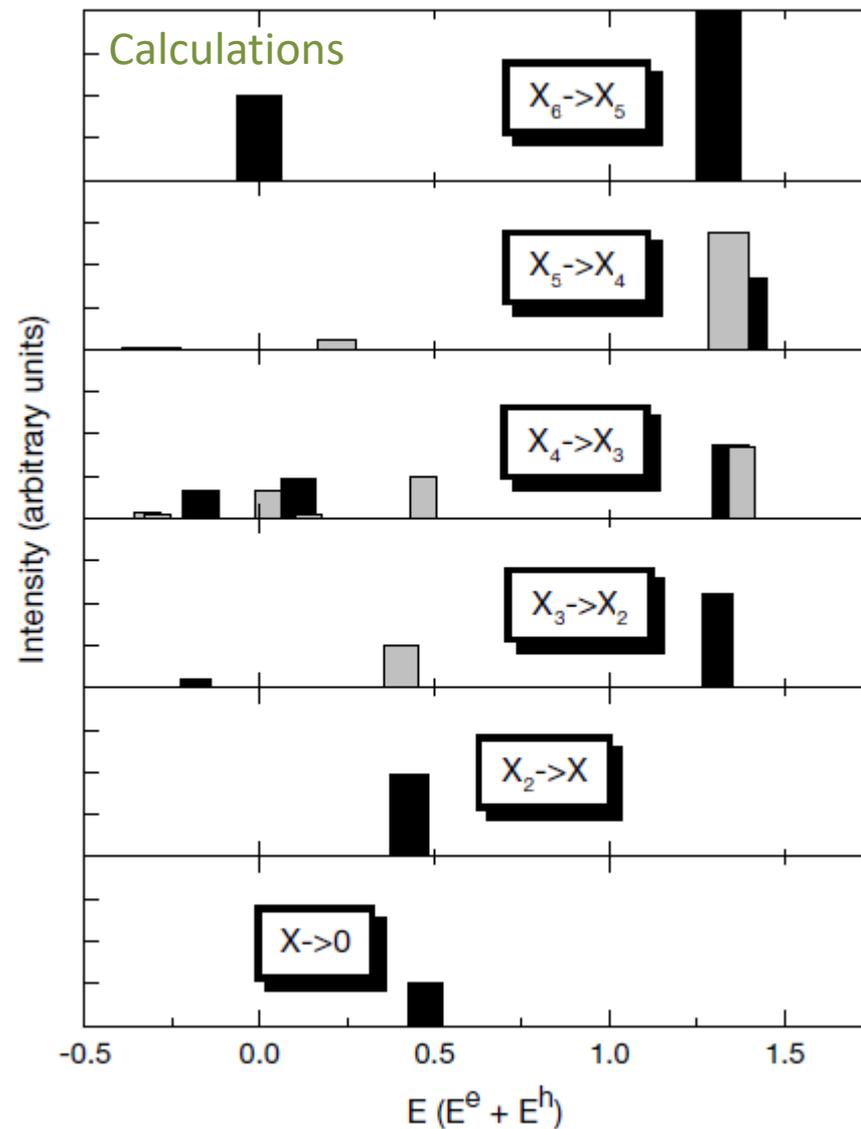
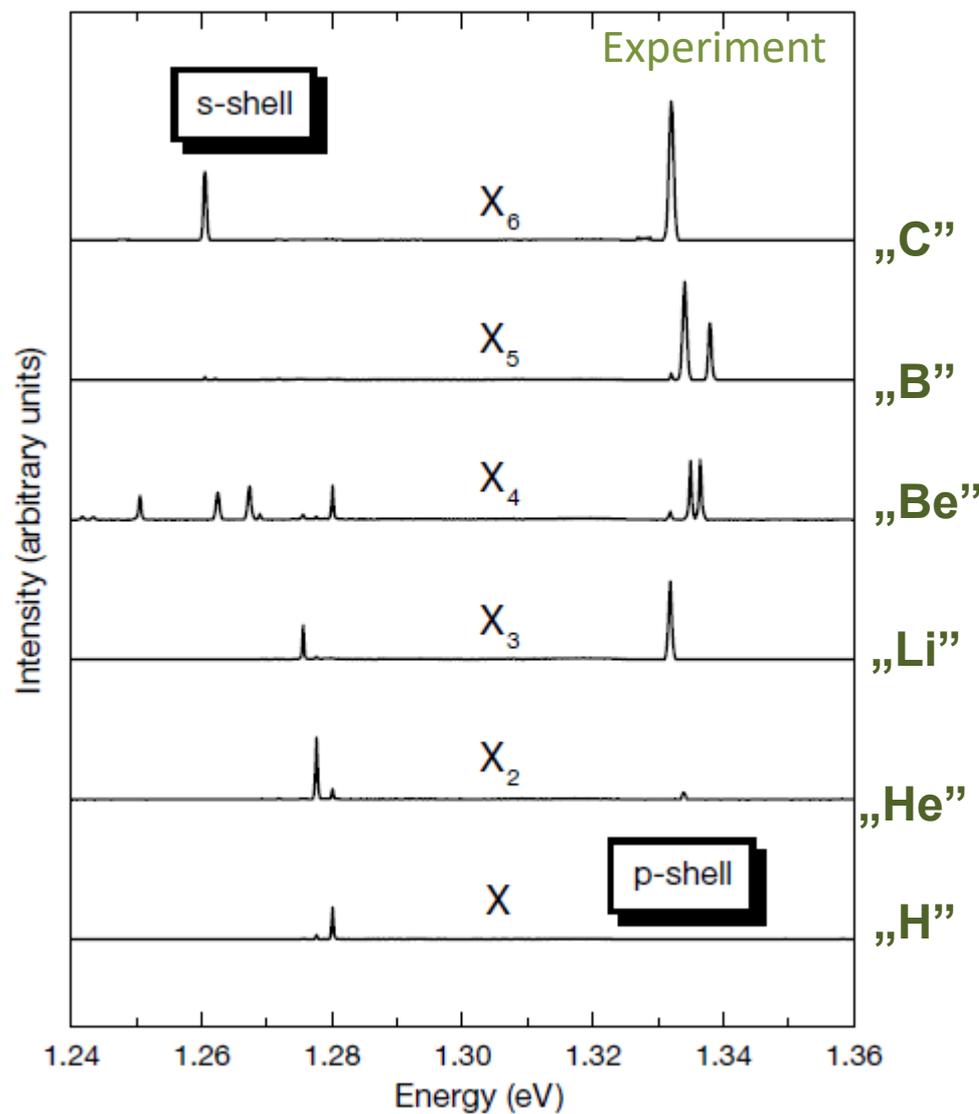
# THE ARTICLE

NATURE | VOL 405 | 22 JUNE 2000 | www.nature.com



**Figure 3** Contour plot of the variation of the emission of an  $\text{In}_{0.60}\text{Ga}_{0.40}\text{As}$  single quantum dot with excitation power and with energy. Bright regions indicate strong emission intensities, blue regions low intensities. When optically exciting far above the bandgap, carrier relaxation involving multiple phonon emission processes leads to considerable sample heating, which causes the system to be in strong non-equilibrium. To reduce heating, a Ti-sapphire laser was used as excitation source. Its energy was tuned to  $E = 1.470$  eV, corresponding to emission close to the bottom of the wetting layer (see Fig. 2). The excitation power  $P_{\text{ex}}$  was varied between 50 nW and 5 mW.

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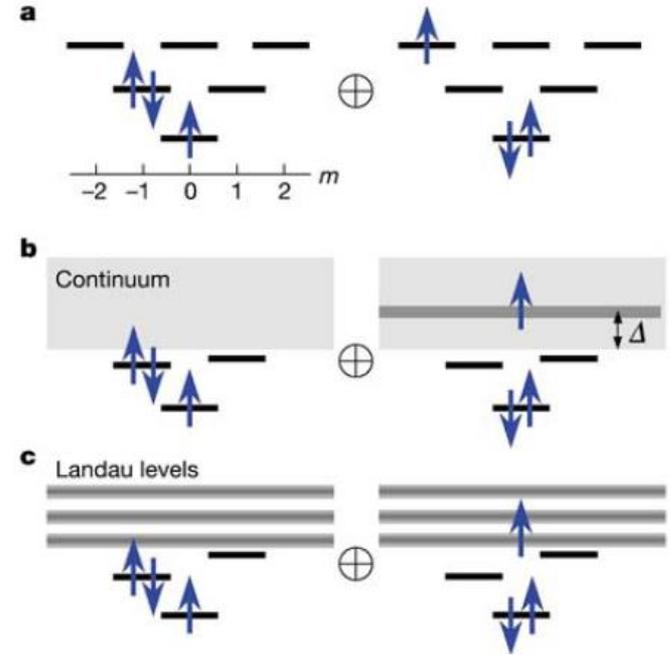
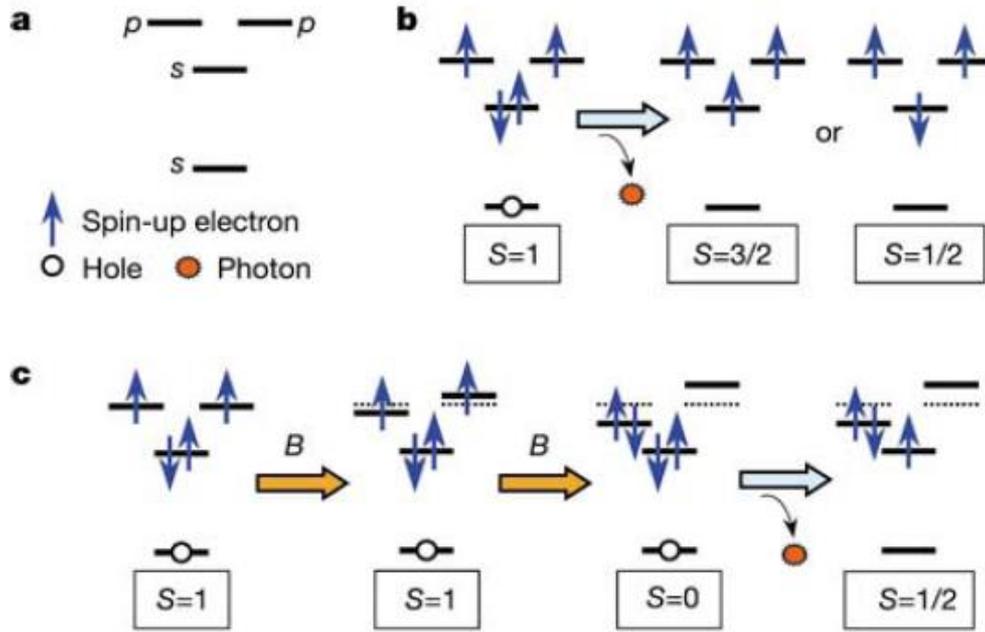


$$\begin{aligned}
 H = & \sum_i E_i^e c_i^\dagger c_i + \sum_i E_i^h d_i^\dagger d_i - \sum_{ijkl} \langle ij | V_{eh} | kl \rangle c_i^\dagger d_j^\dagger d_k c_l \\
 & + \frac{1}{2} \sum_{ijkl} \langle ij | V_{ee} | kl \rangle c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{2} \sum_{ijkl} \langle ij | V_{hh} | kl \rangle d_i^\dagger d_j^\dagger d_k d_l
 \end{aligned}$$

where  $c_i^\dagger$  and  $d_i^\dagger$  ( $c_i$  and  $d_i$ ) are the creation (annihilation) operators for electrons and holes.  $E_i^{e/h}$  are the electron/hole single particle energies and  $V_{mn}$ ,  $m, n = e, h$  are the interparticle Coulomb interactions.

The interband optical processes are described by the polarization operator  $P^+ = \sum_i c_i^\dagger d_i^\dagger$ , where  $P^+$  annihilates a photon and creates an electron–hole pair. The main question arises when populating

$$L_N(\omega) = \sum_f |\langle N-1, f | P^- | i, N \rangle|^2 \cdot \delta(E_N^i - E_{N-1}^f - \hbar\omega)$$



K.Karrai et al., *Nature* **427**, 135 (2004)

# Magnetic field and spin

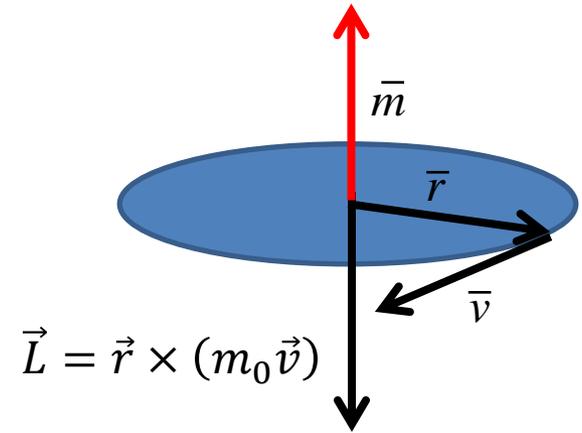
Magnetic field:

$$H' = -\vec{m}\vec{B}$$

Here  $\vec{m}$  is the magnetic moment

classically:

$$|\vec{m}| = |I\vec{S}|$$



$$\hat{L} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$$

# Magnetic field and spin

Magnetic field:

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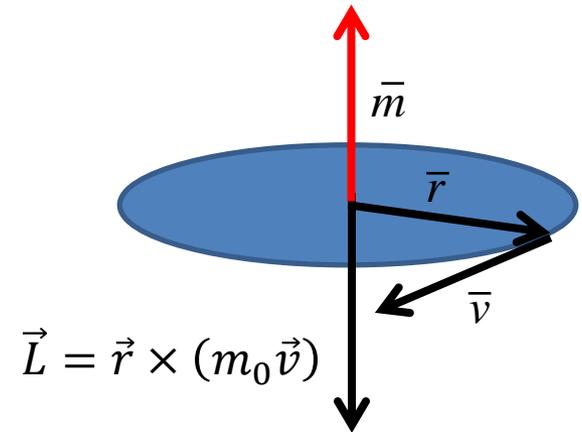
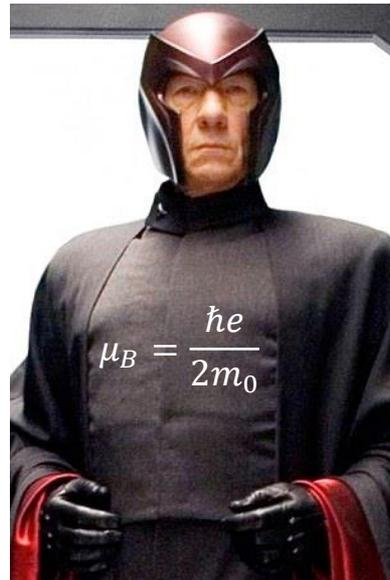
$$|\vec{m}| = |I\vec{S}| = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e}{2}rv \quad [\text{Am}^2]$$

$$\text{thus: } \vec{m} = -\frac{e}{2m_0}\vec{L} = -\frac{\mu_B}{\hbar}\vec{L}$$

$$\text{Bohr magneton } \mu_B = \frac{\hbar e}{2m_0}$$

$$\mu_B = 9,274009994(57)\times 10^{-24} \text{ J/T}$$

$$H' = -\vec{m}\vec{B} = \frac{\mu_B}{\hbar}\hat{L}\vec{B}$$



$$\vec{L} = \vec{r} \times (m_0 \vec{v})$$

$$\hat{L} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$$

# Magnetic field and spin

Magnetic field:

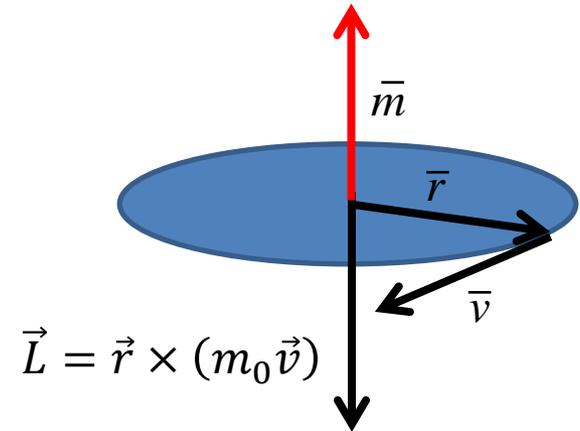
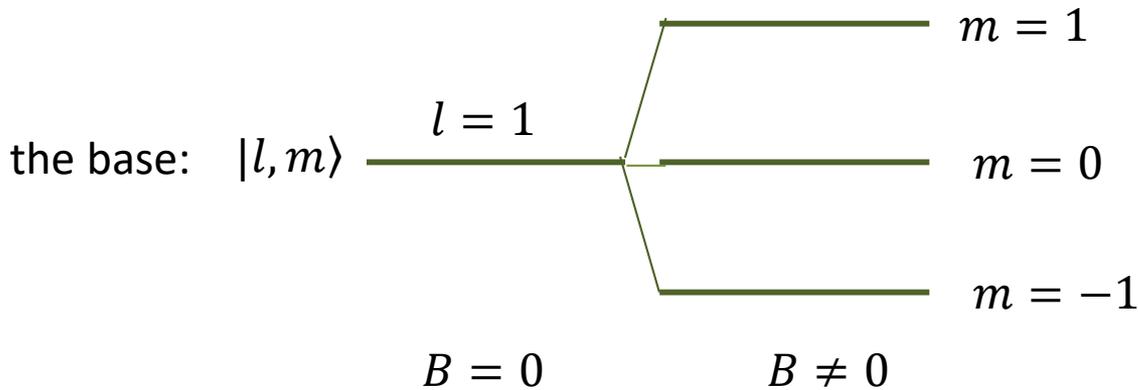
$$H' = -\vec{m}\vec{B} = \frac{\mu_B}{\hbar} \hat{L}\vec{B}$$

for  $\vec{B} = (0,0,B_z)$

Here  $\vec{m}$  is the magnetic moment

we have:  $H' = \frac{\mu_B}{\hbar} \hat{L}_z B_z = \mu_B B_z m$  where  $m = -l, -l+1, \dots, l-1, l$

Here  $m$  is the quantum number  $|n, l, m\rangle$



# Magnetic field and spin

Magnetic field:

$$H' = -\vec{m}\vec{B} = \frac{\mu_B}{\hbar} \hat{L}\vec{B}$$

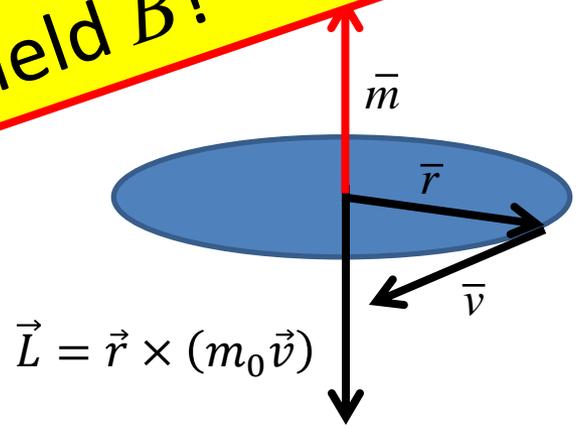
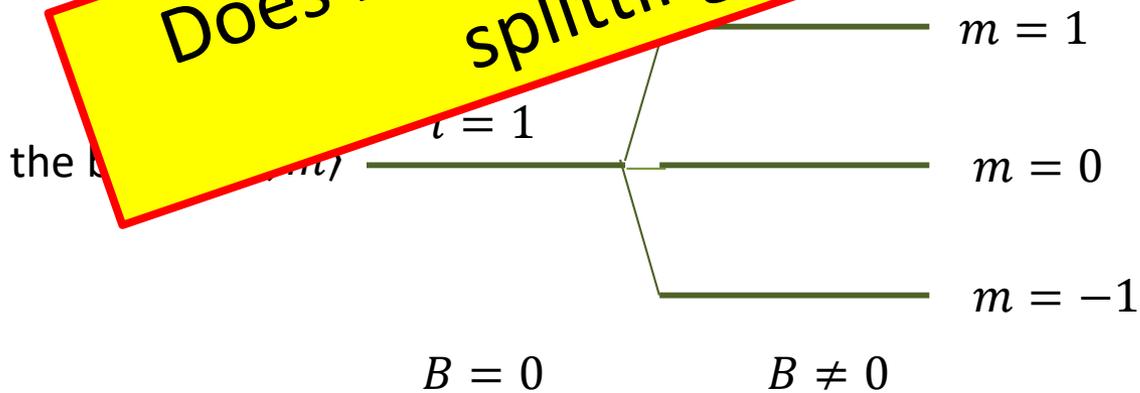
for  $\vec{B} = (0,0,B_z)$

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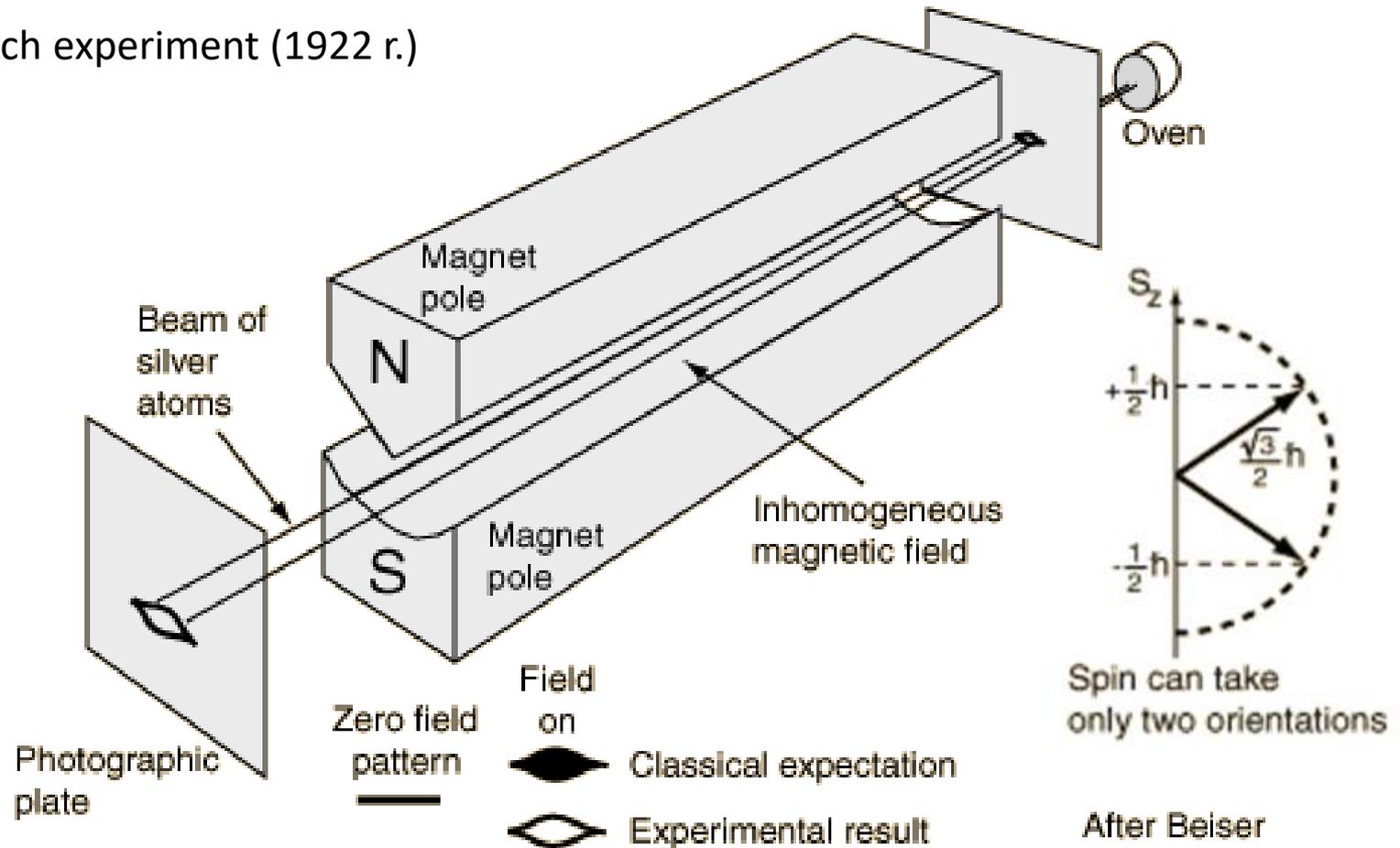
Here

Does it mean that for  $s$ -state ( $l=0$ ) there is no splitting in magnetic field  $B$ ?



# Magnetic field and spin

Stern-Gerlach experiment (1922 r.)



# Magnetic field and spin

## Spin, spin-orbit interaction

Spin operators  $\hat{S}_x, \hat{S}_y, \hat{S}_z, \hat{S}^2$        $\psi(\vec{r}, S_z) = \psi(\vec{r})\chi(S_z)$

Spinor

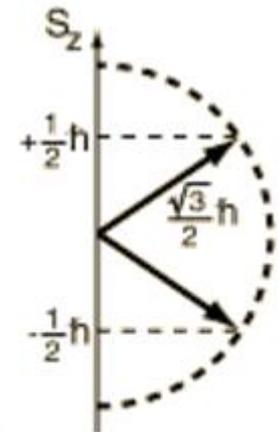
$$[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z, \text{ etc.}$$

Pauli matrices:  $\sigma_x, \sigma_y, \sigma_z$

$$\hat{S}_x = \frac{1}{2}\hbar\sigma_x = \frac{1}{2}\hbar \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\hat{S}_y = \frac{1}{2}\hbar\sigma_y = \frac{1}{2}\hbar \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\hat{S}_z = \frac{1}{2}\hbar\sigma_z = \frac{1}{2}\hbar \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Spin can take only two orientations

projections of the spin on the axis  $z$

$$\chi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Magnetic field and spin

## Spin, spin-orbit interaction

Spin operators  $\hat{S}_x, \hat{S}_y, \hat{S}_z, \hat{S}^2$

$$H' = \frac{\mu_B}{\hbar} (\hat{L} + g\hat{S})\vec{B}$$

$$[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z, \text{ etc.}$$

$g$ -factor for the agreement with experiments

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$g$ -factor for the agreement with experiments

Pauli matrices:  $\sigma_x, \sigma_y, \sigma_z$

$$\hat{S}_x = \frac{1}{2}\hbar\sigma_x = \frac{1}{2}\hbar \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$g = -2.00231930436182 \pm 0.000000000000052$$

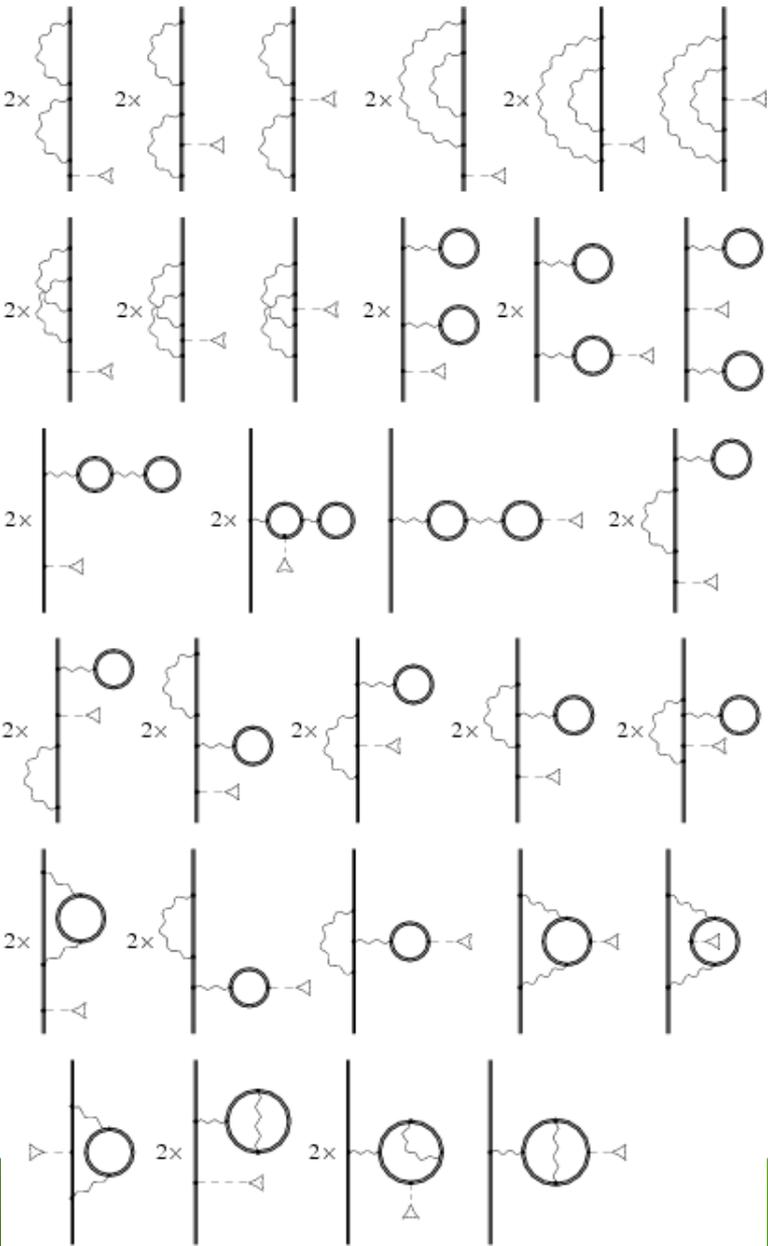
$$\hat{S}_y = \frac{1}{2}\hbar\sigma_y = \frac{1}{2}\hbar \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\hat{S}_z = \frac{1}{2}\hbar\sigma_z = \frac{1}{2}\hbar \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

projections of the spin on the axis  $z$

$$\chi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# QED – Quantum ElectroDynamics



$$g = -2.00231930436182 \pm 0.000000000000052$$

# Magnetic field and spin

## Spin, spin-orbit interaction

Spin operators  $\hat{S}_x, \hat{S}_y, \hat{S}_z, \hat{S}^2$

$$H' = \frac{\mu_B}{\hbar} (\hat{L} + g\hat{S})\vec{B}$$

$g$ -factor for the agreement with experiments

Total angular momentum operator  $\hat{J} = \hat{L} + \hat{S}$ , the base  $|j, m_j\rangle$

$$\text{Total magnetic moment } \hat{M} = \hat{M}_L + \hat{M}_S = -g_L \frac{\mu_B}{\hbar} \hat{L} - g_S \frac{\mu_B}{\hbar} \hat{S}$$

$\uparrow$                        $\uparrow$   
 $=1$                        $=2$

$\hat{M} \neq \hat{J}$  - magnetic anomaly of spin

# Magnetic field and spin

Spin-orbit interaction  $\hat{H}_{SO} = \lambda \hat{L} \hat{S}$  with the base  $|n, l, s, m_l, m_s\rangle$

For s-states  $\hat{L} = 0 \Rightarrow \hat{L} \hat{S} = 0$

Total angular momentum operator  $\hat{J} = \hat{L} + \hat{S}$ , the base  $|j, m_j\rangle$

$$\hat{H}_{SO} = \lambda \hat{L} \hat{S} = \lambda \frac{1}{2} (J^2 - L^2 - S^2) = \lambda \left( L_z S_z + \frac{1}{2} (L_+ S_- + L_- S_+) \right)$$

fine-structure constant

$$\lambda = hc A = \frac{Z \alpha^2}{2} \left\langle \frac{1}{r^3} \right\rangle$$
$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137.037}$$

$$Ry = hcR_\infty$$

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^3 c}$$

$$R_\infty = 1,097 \times 10^7 \text{ m}^{-1}$$

$$E_{SO} = \int \psi^* H_{SO} \psi dV = \frac{Z}{2(137)^2} \int \psi^* \frac{\hat{L} \hat{S}}{r^3} \psi dV$$

# Magnetic field and spin

Spin-orbit interaction  $\hat{H}_{SO} = \lambda \hat{L} \hat{S}$  with the base  $|n, l, s, m_l, m_s\rangle$

For s-states  $\hat{L} = 0 \Rightarrow \hat{L} \hat{S} = 0$

Total angular momentum operator  $\hat{J} = \hat{L} + \hat{S}$ , the base  $|j, m_j\rangle$

$$\begin{aligned}\hat{H}_{SO} &= \lambda \hat{L} \hat{S} = \lambda \frac{1}{2} (J^2 - L^2 - S^2) = \lambda \left( L_z S_z + \frac{1}{2} (L_+ S_- + L_- S_+) \right) \\ &= \frac{1}{2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right) \left( \frac{g_s}{2m^2 c^2} \right) \frac{\hat{L} \hat{S}}{r^3} \\ \left\langle \frac{1}{r^3} \right\rangle &= \frac{Z^3}{n^3 a_B^3} \frac{1}{l(l + \frac{1}{2})(l + 1)} \\ \langle \hat{L} \hat{S} \rangle &= \frac{\hbar^2}{2} [j(j + 1) - l(l + 1) - s(s + 1)]\end{aligned}$$

e.g. for  $\psi_{210}$  we get  $\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{24} \left( \frac{Z}{a_0} \right)^3$  and for general  $n$  (principal quantum number)

$$E_{SO} = \frac{Z^4}{2(137)^2 a_0^3 n^3} \left( \frac{j(j + 1) - l(l + 1) - s(s + 1)}{2l(l + 1/2)(l + 1)} \right)$$

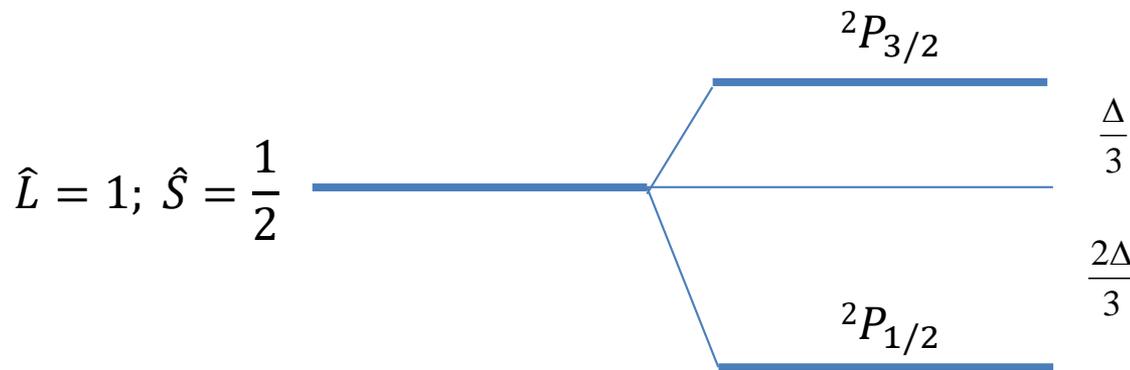
# Magnetic field and spin

Spin-orbit interaction  $\hat{H}_{SO} = \lambda \hat{L} \hat{S}$  with the base  $|n, l, s, m_l, m_s\rangle$

For s-states  $\hat{L} = 0 \Rightarrow \hat{L} \hat{S} = 0$

Total angular momentum operator  $\hat{J} = \hat{L} + \hat{S}$ , the base  $|j, m_j\rangle$

$$\bar{L}\bar{S} = \frac{1}{2}(\bar{J}^2 - \bar{L}^2 - \bar{S}^2) = L_z S_z + \frac{1}{2}(L_+ S_- + L_- S_+)$$



the base:  $|n, l, s, j, m_j\rangle$

shortly:  $|j, m_j\rangle$