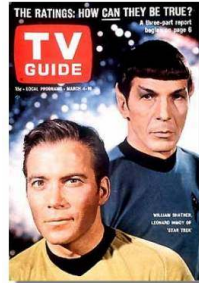


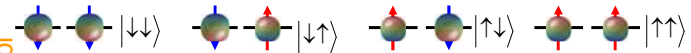
## Kwantowa kryptografia i teleportacja. Splątanie kwantowe

Jacek Szczętko, Wydział Fizyki UW

- a. Poplątane stany.
  - i. Eksperyment EPR.
  - ii. Eksperyment Bella
- b. Star-Trec, czyli teleportujcie mnie!
  - i. Co właściwie teleportujemy
  - ii. Ile kosztuje ubezpieczenie
- c. Kryptografia kwantowa
  - i. Czy są szyfry nie do złamania
  - ii. Klucze duże i małe
  - iii. Alice i Bob w świecie kwantów
  - iv. Ewa chce posłuchać



## Opis wielu cząstek kwantowych



### Stany Bella

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle) \quad |\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle) \quad |\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$$

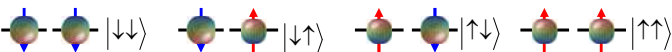
Ale stanów Bella nie da się przedstawić w postaci iloczynu dwóch funkcji

jednocząstkowych typu:  $|\beta\rangle = |\varphi_1\rangle|\varphi_2\rangle$  gdzie  $|\varphi_i\rangle = a_{i1}|\uparrow\rangle + a_{i2}|\downarrow\rangle$

Stany Bella są **SPLATANE**

spiny, polaryzacja fotonów, atom + foton, dwa atomy, atom w różnych stanach...

## Opis wielu cząstek kwantowych



### Stany Bella

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle) \quad |\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle) \quad |\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle) \quad |\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$$

### Observation of entanglement between a single trapped atom and a single photon

B. C. Blinov, D. L. Moehring, L.-M. Duan & C. Monroe  
NATURE | VOL 428 | 11 MARCH 2004 | www.nature.com/nature

PHYSICAL REVIEW A 69, 042316 (2004)

### Atom-photon entanglement generation and distribution

B. Sun, M. S. Chapman, and L. You  
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA  
(Received 21 August 2003; published 21 April 2004)

spiny, polaryzacja fotonów, atom + foton, dwa atomy, atom w różnych stanach...

## Zakaz klonowania

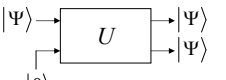


Jacek.Szczytko@fuw.edu.pl

## Zakaz klonowania

Klonowanie stanów kwantowych

$$|\Psi\rangle \rightarrow |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, \dots$$



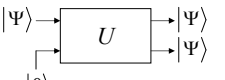
„czysta kartka”

Jacek.Szczytko@fuw.edu.pl

## Zakaz klonowania

Klonowanie stanów kwantowych

$$|\Psi\rangle \rightarrow |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, |\Psi\rangle, \dots$$



„czysta kartka”

Wtedy  $U|\Psi\rangle|0\rangle = |\Psi\rangle|\Psi\rangle$   
 $U|0\rangle|0\rangle = |0\rangle|0\rangle$   
 $U|1\rangle|0\rangle = |1\rangle|1\rangle$

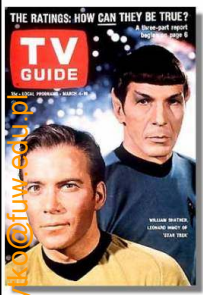
Ale dla  $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$  Dostajemy sprzeczność bo:

$$U|\Psi\rangle|0\rangle = U(\alpha|0\rangle + \beta|1\rangle)|0\rangle = \alpha U|0\rangle|0\rangle + \beta U|1\rangle|0\rangle = \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle \neq |\Psi\rangle|\Psi\rangle$$

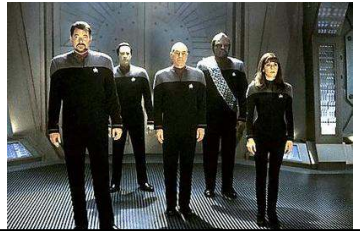
bo  $|\Psi\rangle|\Psi\rangle = \alpha^2|0\rangle|0\rangle + \alpha\beta|0\rangle|1\rangle + \beta\alpha|1\rangle|0\rangle + \beta^2|1\rangle|1\rangle$

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## Kwantowa teleportacja







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## Kwantowa teleportacja



**IBM CREW** Six researchers--Richard Jozsa, William K. Woollers, Charles H. Bennett (*back row, left to right*) Gilles Brassard, Claude Crepeau and Asher Peres (*front row*)--proposed quantum teleportation in 1993.

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# Kwantowa teleportacja

PHYSICAL REVIEW  
LETTERS

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VOLUME 70
29 MARCH 1993
NUMBER 13

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Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

Charles H. Bennett,<sup>(1)</sup> Gilles Brassard,<sup>(2)</sup> Claude Crépeau,<sup>(2),(3)</sup>  
Richard Jozsa,<sup>(2)</sup> Asher Peres,<sup>(4)</sup> and William K. Wootters<sup>(5)</sup>

<sup>(1)</sup>IBM Research Division, T.J. Watson Research Center, Yorktown Heights, New York 10598  
<sup>(2)</sup>Département IRO, Université de Montréal, C.P. 6128, Succursale "A", Montréal, Québec, Canada H3C 3J7  
<sup>(3)</sup>Laboratoire d'Informatique de l'École Normale Supérieure, 45 rue d'Ulm, 75230 Paris CEDEX 05, France<sup>(a)</sup>  
<sup>(4)</sup>Department of Physics, Technion-Israel Institute of Technology, 32000 Haifa, Israel  
<sup>(5)</sup>Department of Physics, Williams College, Williamstown, Massachusetts 01267

(Received 2 December 1992)

An unknown quantum state  $|\phi\rangle$  can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations. To do so the sender, "Alice," and the receiver, "Bob," must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state  $|\phi\rangle$  which Alice destroyed.

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# Kwantowa teleportacja

Dwucząstkowe stany splątane, baza Bella

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$   
 $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)$

$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$   
 $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$

Alice



Bob



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# Kwantowa teleportacja

$|\phi\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle$ 
 $|\Psi_{2,3}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_3\rangle - |\uparrow_2\downarrow_3\rangle)$

Alice



Bob



Jacek.Szczytko@fuw.edu.pl

# Kwantowa teleportacja

$|\phi\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle$ 
 $|\Psi_{2,3}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_3\rangle - |\uparrow_2\downarrow_3\rangle)$

Alice



Bob



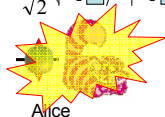
3

## Kwantowa teleportacja

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$$|\phi\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle$$

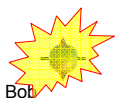
$$|\Psi_{2,3}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_3\rangle - |\uparrow_2\downarrow_3\rangle)$$



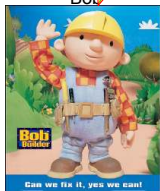
Alice



$$|\Psi_{2,3}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_3\rangle - |\uparrow_2\downarrow_3\rangle)$$



Bob

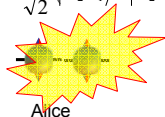


## Kwantowa teleportacja

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puff...

$$|\Psi_{2,1}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_1\rangle - |\uparrow_2\downarrow_1\rangle)$$



Alice



$$|\phi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} (a|\uparrow_3\rangle + b|\downarrow_3\rangle)$$



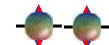
Bob



## Kwantowa teleportacja

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$$|\Psi_{2,1}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_1\rangle - |\uparrow_2\downarrow_1\rangle)$$



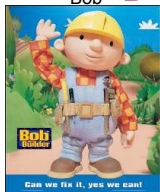
Alice



$$|\phi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} (a|\uparrow_3\rangle + b|\downarrow_3\rangle)$$



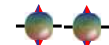
Bob



## Kwantowa teleportacja

Jacek.Szcztyko@fuw.edu.pl

$$|\Psi_{2,1}^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_2\uparrow_1\rangle - |\uparrow_2\downarrow_1\rangle)$$



Alice



$$|\phi\rangle = a|\uparrow_3\rangle + b|\downarrow_3\rangle$$



Bob



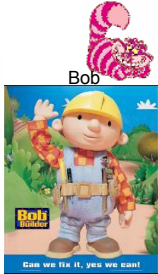
Jacek.Szczytko@fuw.edu.pl

## Kwantowa teleportacja

1. Przekazywana jest stan obiektu, a nie sam obiekt fizyczny.
2. Obiekt nie jest skopiowany (por. zasada Heisenberga, *zakaz klonowania*)
3. Informacja nie porusza się szybciej niż światło.
4. Trudności w pomiarze wszystkich czterech stanów Bella (wydajność).



Alice

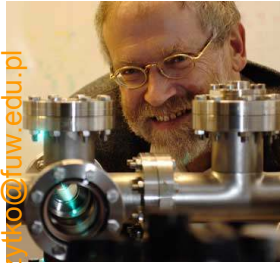


Bob



Jacek.Szczytko@fuw.edu.pl

## Kwantowa teleportacja



**Anton Zeilinger**  
Institute of Experimental Physics, University of Vienna,

Teleportacja stanów fotonowych  
Nature, **390**, 575 (1997)

### Experimental quantum teleportation

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger  
Institut für Experimentalphysik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria

Quantum teleportation—the transmission and reconstruction over arbitrary distances of the state of a quantum system—is demonstrated experimentally. During teleportation, an initial photon which carries the polarization that is to be transferred and one of a pair of entangled photons are subjected to a measurement such that the second photon of the entangled pair acquires the polarization of the initial photon. This latter photon can be arbitrarily far away from the initial one. Quantum teleportation will be a critical ingredient for quantum computation networks.

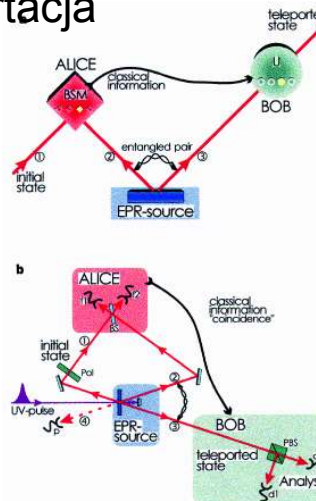
Jacek.Szczytko@fuw.edu.pl

## Kwantowa teleportacja



**Anton Zeilinger**  
Institute of Experimental Physics, University of Vienna,

teleportacja stanów fotonowych  
Nature, **390**, 575 (1997)



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## Kwantowa teleportacja



**Deterministic quantum teleportation with atoms**

M. Riebe<sup>1</sup>, H. Häffner<sup>1</sup>, C. F. Roos<sup>1</sup>, W. Hänsel<sup>1</sup>, J. Benhelm<sup>1</sup>, G. P. T. Lancaster<sup>1</sup>, T. W. Körber<sup>2</sup>, C. Becher<sup>1</sup>, F. Schmidt-Kaler<sup>1</sup>, D. F. V. James<sup>2</sup> & R. Blatt<sup>1,2</sup>

<sup>1</sup>Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria  
<sup>2</sup>Theoretical Division T-4, Los Alamos National Laboratory, Los Alamos NM 87545, USA  
<sup>3</sup>Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 25, A-6020 Innsbruck, Austria

Teleportacja stanów kwantowych jonów <sup>40</sup>Ca<sup>+</sup> w pułapce jonowej  
Nature, **429**, 734 (2004)

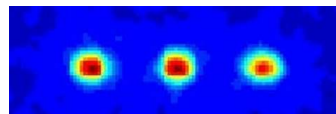
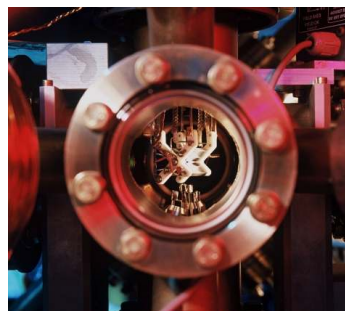
**Reiner Blatt**  
Institute of Experimental Physics, University of Innsbruck,



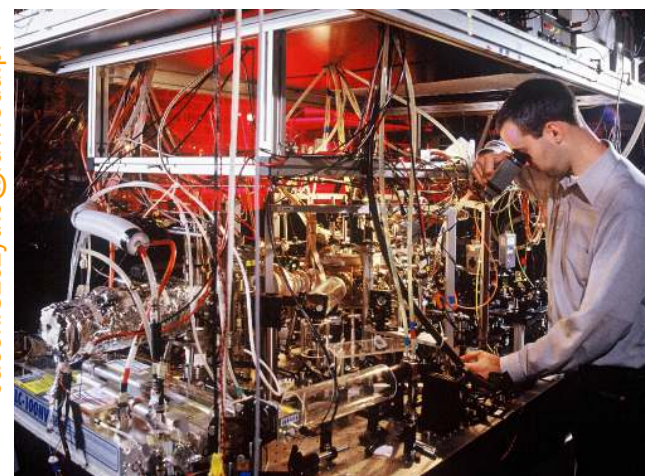
## Kwantowa teleportacja

Table 1 Pulse sequence of the teleportation protocol.

| Action                                            | Comment                                             |
|---------------------------------------------------|-----------------------------------------------------|
| 1 Light at 397 nm                                 | Doppler preparation                                 |
| 2 Light at 729 nm                                 | Sideband cooling                                    |
| 3 Light at 397 nm                                 | Optical pumping                                     |
| 4 $R_2^x(\pi/2, 3\pi/2)$                          | Entangle ion 3 with motional qubit                  |
| 5 $R_2^x(\pi, 3\pi/2)$                            | Prepare ion 2 for entanglement                      |
| 6 $R_2^x(\pi, \pi/2)$                             | Entangle ion 2 with ion 3                           |
| 7 Wait for 1 $\mu\text{s}$ – 10,000 $\mu\text{s}$ | Standby for teleportation                           |
| 8 $R_2^x(\pi, 0)$                                 | Hide target ion                                     |
| 9 $R_1^x(\theta, \phi)$                           | Prepare source ion 1 in state $\chi$                |
| 10 into Bell basis                                |                                                     |
| 11 $R_2^x(\pi, 3\pi/2)$                           | Get motional qubit from ion 2                       |
| 12 $R_1^x(\pi/2, \pi/2)$                          | Composite pulse for phasegate                       |
| 13 $R_1^x(\pi, 0)$                                | Composite pulse for phasegate                       |
| 14 $R_1^x(\pi/2, \pi/2)$                          | Composite pulse for phasegate                       |
| 15 $R_1^x(\pi, 0)$                                | Composite pulse for phasegate                       |
| 16 $R_1^x(\pi, \pi/2)$                            | Spin echo on ion 1                                  |
| 17 $R_2^x(\pi, \pi)$                              | Unhide ion 3 for spin echo                          |
| 18 $R_2^x(\pi, \pi/2)$                            | Spin echo on ion 3                                  |
| 19 $R_2^x(\pi, 0)$                                | Hide ion 3 again                                    |
| 20 $R_2^x(\pi, \pi/2)$                            | Write motional qubit back to ion 2                  |
| 21 $R_2^x(\pi/2, 3\pi/2)$                         | Part of rotation into Bell basis                    |
| 22 $R_2^x(\pi/2, \pi/2)$                          | Finalise rotation into Bell basis                   |
| 23 out                                            |                                                     |
| 24 $R_2^x(\pi, 0)$                                | Hide ion 2                                          |
| 25 PM detection for 250 $\mu\text{s}$             | Read out of ion 1 with photomultiplier              |
| 26 $R_2^x(\pi, 0)$                                | Hide ion 1                                          |
| 27 $R_2^x(\pi, \pi)$                              | Unhide ion 2                                        |
| 28 PM detection for 250 $\mu\text{s}$             | Read out of ion 2 with photomultiplier              |
| 29 $R_2^x(\pi, 0)$                                | Hide ion 2                                          |
| 30 Wait 300 $\mu\text{s}$                         | Let system rephase; part of spin echo               |
| 31 $R_2^x(\pi, \pi)$                              | Unhide ion 3                                        |
| 32 $R_2^x(\pi/2, 3\pi/2 + \phi)$                  | Change basis                                        |
| 33 Reconstruction                                 |                                                     |
| 34 $R_2^x(\pi, \phi)$                             | $i\phi_x$ conditioned on PM detection 1             |
| 35 $R_2^x(\pi, \pi/2 + \phi)$                     | $i\phi_y$ conditioned on PM detection 1             |
| 36 $R_2^x(\pi, \phi)$                             | $i\phi_z$ conditioned on PM detection 2             |
| 37 $R_2^x(\theta, \phi_x + \pi + \phi)$           | Inverse of preparation of $\chi$ with offset $\phi$ |
| 38 Light at 397 nm                                | Read out of ion 3 with camera                       |



## Maszyna do teleportacji



## Kwantowa teleportacja



David Wineland  
National Institute of Standards and Technology

### Deterministic quantum teleportation of atomic qubits

M. D. Barrett<sup>1</sup>, J. Chiaverini<sup>1</sup>, T. Schaetz<sup>1</sup>, J. Britton<sup>1</sup>, W. M. Itano<sup>1</sup>, J. D. Jost<sup>1</sup>, E. Knill<sup>2</sup>, C. Langer<sup>1</sup>, D. Leibfried<sup>1</sup>, R. Ozeri<sup>1</sup> & D. J. Wineland<sup>1</sup>

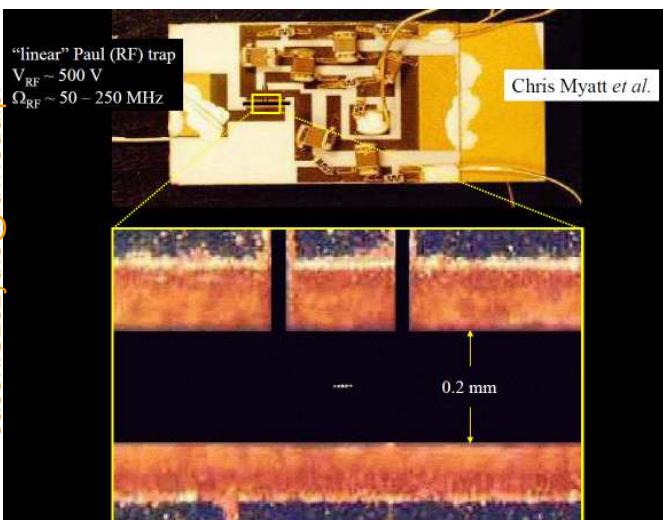
<sup>1</sup>Time and Frequency Division, NIST, Boulder, Colorado 80305, USA

<sup>2</sup>Mathematical and Computational Sciences Division, NIST, Boulder, Colorado 80305, USA

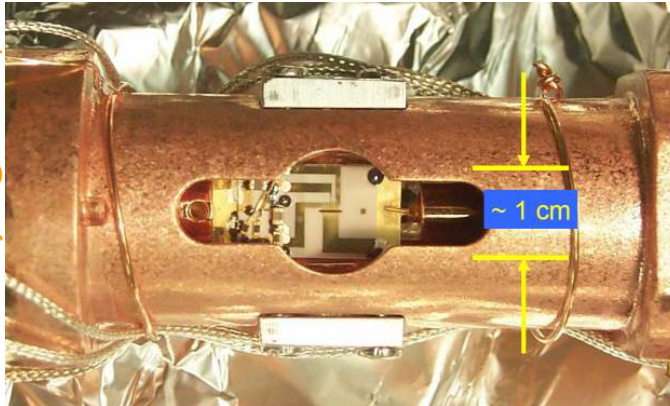
\* Present address: Department of Physics, University of Otago, PO Box 56, Dunedin, New Zealand

Teleportacja stanów kwantowych jonów  $^9\text{Be}^+$  w pułapce jonowej

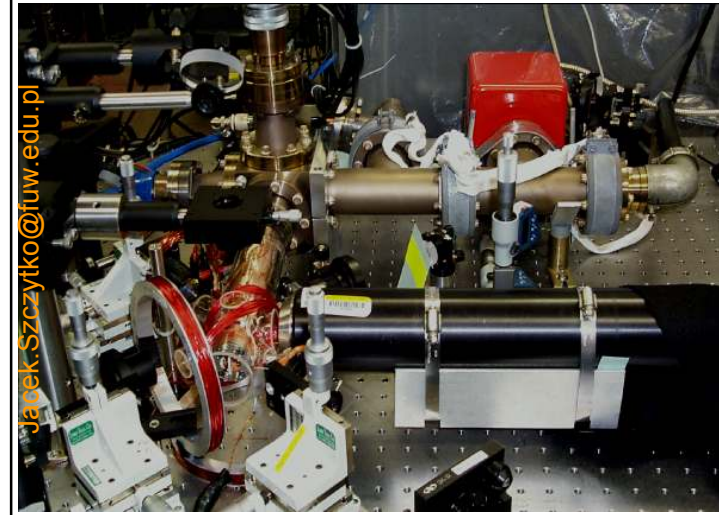
Nature, **429**, 737 (2004)



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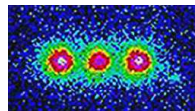


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## Kwantowa teleportacja

| 1 | 2 | 3 | 4 | 5   | 6  | 7 | 8 | Electrodes                |
|---|---|---|---|-----|----|---|---|---------------------------|
|   |   |   |   | 123 |    |   |   |                           |
|   |   |   |   | *** |    |   |   | 1. Preparation            |
|   |   |   |   |     |    |   |   | Spin echo                 |
|   |   |   |   | **  | *  |   |   | 2. Basis transformation   |
|   |   |   |   | *** |    |   |   | Spin echo                 |
|   |   |   |   | *   | ** |   |   | 3. Measurement 1          |
|   |   |   |   | *** |    |   |   | Spin echo                 |
|   |   |   |   | **  | *  |   |   | 4. Measurement 2          |
|   |   |   |   | **  | *  |   |   | Spin echo                 |
|   |   |   |   | **  | *  |   |   | 5. Conditional operations |

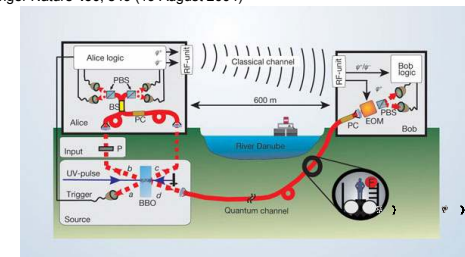


This colorized image shows the fluorescence from three trapped beryllium ions illuminated with an ultraviolet laser beam. Credit: NIST

**Figure 1** Schematic representation of the teleportation protocol. The ions are numbered 1, 2, 3 from left to right, as indicated at the top, and retain their order throughout. Positions, relative to the electrodes, are shown at each step in the protocol. The widths of the electrodes vary, with the width of the separation electrode (6) being the smallest at 100  $\mu\text{m}$ . The spacing between ions in the same trap is about 3  $\mu\text{m}$ , and laser-beam spot sizes (in traps 5 and 6) at the position of the ions are approximately 30  $\mu\text{m}$ . In step 1 we prepare the outer ions in an entangled (singlet) state and the middle ion in an arbitrary state (excited ion (1)). Steps 2–4 constitute a measurement in a Bell-basis for ions 1 and 2 (Alice's qubits), teleporting the state of ion 2 onto ion 3 (Bob's qubit), up to unitary operations that depend on the measurement outcomes. In step 5 we invoke these conditional operations, recovering the initial state. Interspersed are spin-echo pulses applied in trap 6 that protect the state from de-phasing due to fluctuating magnetic fields but do not affect the teleportation protocol.

## Kwantowa teleportacja

Rupert Ursin, Thomas Jennewein, Markus Aspelmeyer, Rainer Kaltenbaek, Michael Lindenthal, Philip Walther and Anton Zeilinger Nature **430**, 849 (19 August 2004)



The quantum channel (fibre F) rests in a sewage-pipe tunnel below the river in Vienna, while the classical microwave channel passes above it. A pulsed laser (wavelength, 394 nm; rate, 76 MHz) is used to pump a  $\beta$ -barium borate (BBO) crystal that generates the entangled photon pair  $c$  and  $d$  and photons  $a$  and  $b$  (wavelength, 788 nm) by spontaneous parametric down-conversion. The state of photon  $b$  after passage through polarizer P is the teleportation input;  $a$  serves as the trigger. Photons  $b$  and  $c$  are guided into a single-mode optical-fibre beam splitter (BS) connected to polarizing beam splitters (PBS) for Bell-state measurement. Polarization rotation in the fibres is corrected by polarization controllers (PC) before each run of measurements. The logic electronics identify the Bell state as either  $|\Psi^+\rangle_{bc}$  or  $|\Psi^-\rangle_{bc}$  and convey the result through the microwave channel (RF unit) to Bob's electro-optic modulator (EOM) to transform photon  $d$  into the input state of photon  $b$ .

## Quantum Teleportation: The Math

Three qubit joint state of Alice, Bob, and "Victor" who prepared  $|\psi\rangle$ :

$$|\chi\rangle = |\psi\rangle_V \otimes |\Phi^{(+)}\rangle_{AB} = (\alpha|0\rangle_V + \beta|1\rangle_V) \otimes (|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \\ = \frac{1}{2} [(|0\rangle_V \otimes |0\rangle_A + |1\rangle_V \otimes |1\rangle_A) \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + (|0\rangle_V \otimes |0\rangle_A - |1\rangle_V \otimes |1\rangle_A) \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) \\ + (|0\rangle_V \otimes |1\rangle_A + |1\rangle_V \otimes |0\rangle_A) \otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + (|0\rangle_V \otimes |1\rangle_A - |1\rangle_V \otimes |0\rangle_A) \otimes (\alpha|1\rangle_B - \beta|0\rangle_B)] \\ |\chi\rangle = \frac{1}{2} (|\Phi^{(+)}\rangle_{VA} \otimes |\psi\rangle_B + |\Phi^{(-)}\rangle_{VA} \otimes Z|\psi\rangle_B + |\Psi^{(+)}\rangle_{VA} \otimes X|\psi\rangle_B - |\Psi^{(-)}\rangle_{VA} \otimes Y|\psi\rangle_B)$$

A measurement by Alice in Bell-basis leaves Bob's qubit in one of four states:

$$\{|\psi\rangle_B, Z|\psi\rangle_B, X|\psi\rangle_B, Y|\psi\rangle_B\}$$

Alice uses the classical channel to tell Bob which Bell state she found.

Bob can then put his qubit in the unknown state, through application of a Pauli.



I. H. Deutsch, University of New Mexico  
Short Course in Quantum Information



## Kwantowa teleportacja

VOLUME 88, NUMBER 1

PHYSICAL REVIEW LETTERS

7 JANUARY 2002

### Experimental Nonlocality Proof of Quantum Teleportation and Entanglement Swapping

Thomas Jennewein, Gregor Weihs, Jian-Wei Pan, and Anton Zeilinger  
Institut für Experimentalphysik, Universität Wien Boltzmannsgasse 5, 1090 Wien, Austria  
(Received 15 August 2001; published 18 December 2001)

Quantum teleportation strikingly underlines the peculiar features of the quantum world. We present an experimental proof of its quantum nature, teleporting an entangled photon with such high quality that the nonlocal quantum correlations with its original partner photon are preserved. This procedure is also known as entanglement swapping. The nonlocality is confirmed by observing a violation of Bell's inequality by 4.5 standard deviations. Thus, by demonstrating quantum nonlocality for photons that never interacted, our results directly confirm the quantum nature of teleportation.

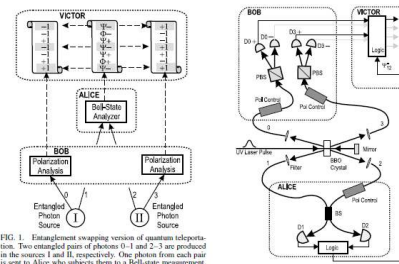


FIG. 1. Entanglement swapping version of quantum teleportation. Two entangled pairs of photons 0-1 and 2-3 are produced in the sources I and II, respectively. One photon from each pair is sent to Alice who subjects them to a Bell-state measurement, projecting them randomly into one of four possible entangled

FIG. 2. Setup of the experiment. The two entangled photons

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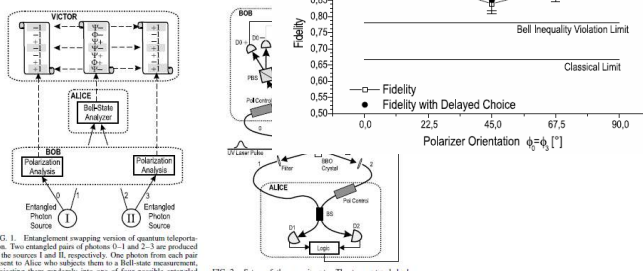


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FIG. 2. Setup of the experiment. The two entangled photons

## Kryptografia Kwantowa





## Kryptografia Kwantowa

- <http://zon8.physd.amu.edu.pl/~tanas/>



## Kryptografia Kwantowa

### 1.4 Proste szyfry

#### Szyfr Cezara

szyfr podstawieniowy monoalfabetyczny

ABCDEFGHIJKLMNOPQRSTUVWXYZ  
DEFGHIJKLMNOPRSTUVWXYZABC

tekst jawny → KRYPTOGRAFIA  
kryptogram → NUBTWSJUDILD

## Kryptografia Kwantowa

### Szyfr Vigenère'a

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A |
| C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B |
| D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |
| E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D |
| F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E |
| G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F |
| H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G |
| I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H |
| J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I |
| K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J |
| L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |
| M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M |
| O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
| P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q |
| S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R |
| T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
| U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U |
| W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |
| X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W |
| Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X |
| Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y |

klucz → SZYMPANSSZYM  
tekst → KRYPTOGRAFIA  
krypt. → CPWCIOUISEGM

## Kryptografia Kwantowa

### Szyfr Vernama (one-time pad)

tekst jawny → S Z Y F R  
binarnie → 01010011 01011010 01011001 01000110 01010010  
klucz → 01110010 01010101 11011100 10110011 00101011  
kryptogram → 00100001 00001111 10000101 11110101 01111001

- Klucz jest losowym ciągiem bitów.
- Kryptogram jest także losowym ciągiem bitów i jeśli nie znamy klucza to nie dowiemy się niczego o tekście jawnym.
- Jeśli klucz jest tak długi jak wiadomość i użyty tylko raz, to szyfr ten gwarantuje **bezpieczeństwo absolutne**.
- Współczesne metody kryptograficzne sprowadzają się do obliczeń w systemie binarnym, czyli operacji na bitach.

## Kryptografia i kryptoanaliza

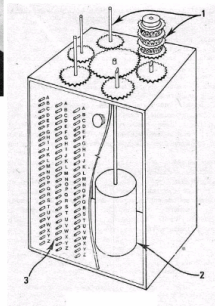


Jerzy Różycki

Marian Rejewski

Henryk Zygalski

Enigma 1918/1923



images from [http://www.armyradio.com/publish/Articles/The\\_Enigma\\_Code\\_Breach/](http://www.armyradio.com/publish/Articles/The_Enigma_Code_Breach/)

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## Szyfry nie do złamania



Klucze jednorazowe – nie do złamania!



Jacek.Szczytko@fuw.edu.pl

## Kryptografia



Alice



Bob

Eve (eavesdropper)



Jacek.Szczytko@fuw.edu.pl

## Kryptografia



Alice



Bob

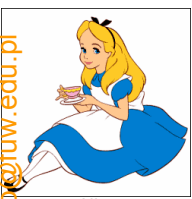
Eve



Jacek.Szczytko@fuw.edu.pl

Jacek.Szczytko@fuw.edu.pl

## Kryptografia



Alice



→





Bob

Aby mieć bezpieczny kanał łączności trzeba mieć bezpieczny kanał łączności...



Eve

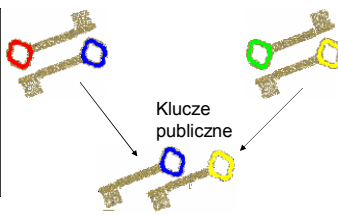


Jacek.Szczytko@fuw.edu.pl

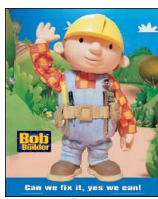
## Kryptografia klucza publicznego



Alice



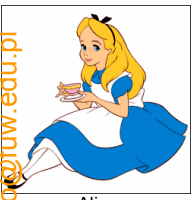
Klucze publiczne



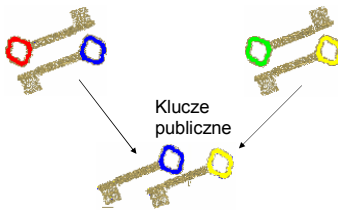
Bob

Jacek.Szczytko@fuw.edu.pl

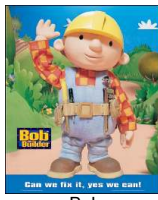
## Kryptografia klucza publicznego



Alice



Klucze publiczne



Bob

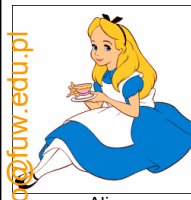


→

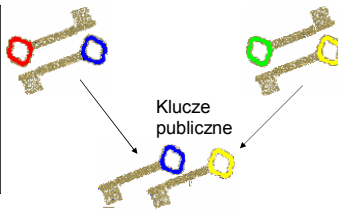


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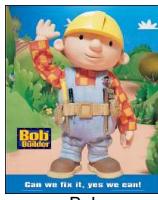
## Kryptografia klucza publicznego



Alice



Klucze publiczne



Bob



→



→



## Kryptografia klucza publicznego



- Bezpieczeństwo systemu kryptograficznego z kluczem publicznym jest oparte na istnieniu funkcji jednostronnych, dla których znalezienie wartości samej funkcji jest łatwe, zaś znalezienie argumentu funkcji, kiedy znany jest jej wynik, jest obliczeniowo trudne (jak trudne to zależy od aktualnego stanu wiedzy i rozwoju techniki)
- Najbardziej znany kryptosystem z kluczem publicznym, RSA, opiera się na trudności z rozkładem liczby na czynniki (faktoryzacja)
- Systemy takie nie gwarantują pełnego bezpieczeństwa. Nie można wykluczyć, że ktoś znajdzie efektywny algorytm faktoryzacji liczb. W istocie taki algorytm już istnieje. Jest to algorytm Shora! Wymaga on jednak komputera kwantowego!

<http://zon8.physd.amu.edu.pl/~tanasi>

## Kryptografia klucza publicznego



W 1994 r. RSA 129 został złamany na 1600 stacjach roboczych w ciągu 8 miesięcy



<http://zon8.physd.amu.edu.pl/~tanasi>

## Kryptografia kwantowa



Wielokrotna i bezpieczna procedura uzgadniania klucza jednorazowego

<http://wug.physics.uiuc.edu/courses/phys214/fall04/>

## Kryptografia kwantowa



Charles Bennett



Gilles Brassard

Wielokrotna i bezpieczna procedura uzgadniania klucza jednorazowego

QUANTUM CRYPTOGRAPHY: PUBLIC KEY DISTRIBUTION AND COIN TOSSENG  
Charles H. Bennett (IBM Research, Yorktown Heights NY 10598 USA)  
Gilles Brassard (dept. IRO, Univ. de Montreal, H3C 3J7 Canada)

International Conference on Computers, Systems & Signal Processing Bangalore, India December 10-12, 1984

When elementary quantum systems, such as polarized photons, are used to transmit digital information, the uncertainty principle gives rise to novel cryptographic phenomena unachievable with traditional transmission media, e.g. a communications channel on which it is impossible in principle to eavesdrop without a high probability of disturbing the transmission in such a way as to be detected. Such a quantum channel can be used in conjunction with ordinary insecure classical channels to distribute random key information between two users with the assurance that it remains unknown to anyone else, even when the users share no secret information initially. We also present a protocol for coin-tossing by exchange of quantum messages, which is secure against traditional kinds of cheating, even by an opponent with unlimited computing power, but ironically can be subverted by use of a still "classical" quantum phenomenon, the Einstein-Podolsky-Rosen paradox.

• principle impossible to counterfeit, and multiplexing two or three messages in such a way that reading one destroys the others. More recently (BB84), quantum coding has been used in conjunction with public key cryptographic techniques to yield several schemes for unforgeable subway tokens. Here we show that quantum coding by itself achieves one of the main advantages of public key cryptography by permitting secure distribution of random key information between parties who share no secret information initially, provided the parties have access, besides the quantum channel, to an ordinary channel susceptible to passive but not active eavesdropping. Even in the presence of active eavesdropping, the two parties can still distribute key securely if they share some secret information initially, provided the eavesdropping is not so active as to suppress communications completely. We also present a protocol for coin tossing by exchange of quantum messages. Except where otherwise noted, all rights reserved.



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## Kryptografia kwantowa

Rectilinear polarization mode

Diagonal polarization mode

Established bit value

|   |   |
|---|---|
| 0 | 1 |
|---|---|

Protokół BB84 (Bennett, Brassard, 1984)

Bolek publicznie informuje Alicję jakiej bazy używał, zaś Alicja informuje go czy była to baza właściwa czy nie.

Scientific American, January 2005

Jacek.Szcztyko@fuw.edu.pl

## Kryptografia kwantowa

**Uwagi:**

- Porównując bity wysłane przez Alicję z bitami zarejestrowanymi przez Boleka możemy podzielić bity zarejestrowane przez Boleka na trzy kategorie:
- bity pewne (średnio 50 %) — te dla których Bolek wybrał prawidłową bazę i które mogą być traktowane jako klucz kryptograficzny;
- bity prawidłowe pomimo złego wyboru bazy (średnio 25 %);
- bity nieprawidłowe (średnio 25 %).

Łącznie prawdopodobieństwo tego, że zarejestrowany bit będzie prawidłowy (taki sam jak bit wysłany) jest równe  $3/4$

Prawdopodobieństwo zarejestrowania bitu nieprawidłowego wynosi więc  $1/4$

Rectilinear polarization mode

Diagonal polarization mode

Established bit value

|   |   |
|---|---|
| 0 | 1 |
|---|---|

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## Kryptografia kwantowa

**Uwagi:**

- Jeśli Ewa podsłuchuje stosując strategię tzw. *nieprzezroczystego podsłuchu*, to wybiera losowo bazę prostą lub ukośną, dokonuje pomiaru polaryzacji w tej bazie i następnie przesyła do Boleka foton o takiej polaryzacji jaką zmierzyła.
- Dokonywane przez Ewę pomiary muszą wprowadzić błędy, które Alicja i Bolek mogą wykryć przy uzgadnianiu klucza.
- Takie błędy Alicja i Bolek mogą wykryć wybierając losowo pewną liczbę bitów klucza i porównując publicznym kanałem ich wartości. Te bity oczywiście następnie się wyrzuca.
- Jeśli liczba błędów przekracza założony poziom to uznaje się, że kanał był podsłuchiwany i procedurę uzgadniania klucza rozpoczyna się od nowa.

**Mechanika kwantowa nie dopuszcza możliwości pasywnego podsłuchu.**

**Bezpieczeństwo kwantowego systemu kryptograficznego gwarantowane jest przez prawa fizyki!**

|                              |   |   |   |   |   |   |   |   |   |
|------------------------------|---|---|---|---|---|---|---|---|---|
| Alice's bit sequence:        | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| Alice's filter scheme:       | + | + | + | + | + | + | + | + | + |
| Bob's detection scheme:      | + | + | + | + | + | + | + | + | + |
| Bob's bit measurements:      | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| Retained bit sequence (key): | 0 | — | 0 | 1 | — | — | 1 | 1 | — |

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## Produkty



# Kryptografia kwantowa

Artur Ekert

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## PHYSICAL REVIEW LETTERS

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VOLUME 67 5 AUGUST 1991 NUMBER 6

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### Quantum Cryptography Based on Bell's Theorem

Artur K. Ekert  
*Merton College and Physics Department, Oxford University, Oxford OX1 3PU, United Kingdom*  
 (Received 18 April 1991)

Practical application of the generalized Bell's theorem in the so-called key distribution process in cryptography is reported. The proposed scheme is based on the Bohm's version of the Einstein-Podolsky-Rosen gedanken experiment and Bell's theorem is used to test for eavesdropping.

# Kryptografia kwantowa

Artur Ekert

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VOLUME 67 NUMBER 6

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*Merton College and Physics Department, Oxford University, Oxford OX1 3PU, United Kingdom*

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Artur Ekert

# Kryptografia kwantowa

Artur Ekert

The diagram shows Alice and Bob performing measurements on entangled photon pairs. Alice's measurement results are listed as  $a_1$  (0°),  $a_2$  (45°), and  $a_3$  (90°). Bob's measurement results are listed as  $b_1$  (45°),  $b_2$  (90°), and  $b_3$  (135°). The diagram includes a 3D representation of the Bloch sphere and a Venn diagram showing the overlap of the two sets of results.

# Kryptografia kwantowa

Artur Ekert

$a_1$  0°  
 $a_2$  45°  
 $a_3$  90°

$b_1$  45°  
 $b_2$  90°  
 $b_3$  135°

Alice

Bob

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | x | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| t | 0 | 1 | t | 0 | 0 | t | 1 | 0 | 1 | x | t | 0 | 1 | t | t | 1 | t | t |

klucz: 0100101011...

# Kryptografia kwantowa

Artur Ekert

After the transmission has taken place, Alice and Bob can announce in public the orientations of the analyzers they have chosen for each particular measurement and divide the measurements into two separate groups: a first group for which they used different orientation of analyzers, and a second group for which they used the same orientation of their analyzers. They discard all measurements in which either or both of them failed to register a particle at all. Subsequently, Alice and Bob can reveal publicly the results they obtained but within the first group of measurements only. This allows them to establish the value of  $S$ , which, if the particles were not directly or indirectly "disturbed," should reproduce the result of Eq. (4). This assures the legitimate users that the results they obtained within the second group of measurements are anticorrelated and can be converted into a secret string of bits—the key. This secret key may be then used in a conventional cryptographic communication between Alice and Bob.

The eavesdropper cannot elicit any information from the particles while in transit from the source to the legitimate users, simply because there is no information encoded there. The information "comes into being" only after the legitimate users perform measurements and communicate in public afterwards. The eavesdropper may try to substitute his own prepared data for Alice and Bob to misguide them, but as he does not know which orientation of the analyzers will be chosen for a given pair of particles, there is no good strategy to escape from being detected.

$$S = \int \rho(\mathbf{n}_a, \mathbf{n}_b) d\mathbf{n}_a d\mathbf{n}_b [\sqrt{2} \mathbf{n}_a \cdot \mathbf{n}_b], \quad (6)$$

which implies

$$-\sqrt{2} \leq S \leq \sqrt{2}, \quad (7)$$

$$S = E(a_1, b_1) - E(a_1, b_2) + E(a_2, b_1) + E(a_2, b_2). \quad (3)$$

Again, quantum mechanics requires

$$S = -2\sqrt{2}. \quad (4)$$

# Kryptografia kwantowa

PHYSICAL REVIEW A

VOLUME 56, NUMBER 2

AUGUST 1997

Optimal eavesdropping in quantum cryptography. I. Information bound and optimal strategy

Christopher A. Fuchs,<sup>1</sup> Nicolas Gisin,<sup>2</sup> Robert B. Griffiths,<sup>3</sup> Chi-Sheng Niu,<sup>3</sup> and Asher Peres<sup>4,\*</sup>

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(Received 31 January 1997)

We consider the Bennett-Brassard cryptographic scheme, which uses two conjugate quantum bases. An eavesdropper who attempts to obtain information on qubits sent in one of the bases causes a disturbance to qubits sent in the other basis. We derive an upper bound to the accessible information in one basis, for a given error rate in the conjugate basis. Independently fixing the error rates in the conjugate bases, we show that both bounds can be attained simultaneously by an optimal eavesdropping probe. The probe interaction and its subsequent measurement are described explicitly. These results are combined to give an expression for the optimal information an eavesdropper can obtain for a given average disturbance when her interaction and measurements are performed signal by signal. Finally, the relation between quantum cryptography and violations of Bell's inequalities is discussed. [S1050-2947(97)01708-3]

# Kryptografia kwantowa

We take the framework for our problem directly from quantum cryptography. In order to take advantage of Alice's delayed information on the basis that was used, Eve's optimal strategy is the following: she lets a probe, initially in some standard state  $|\psi_0\rangle$ , interact unitarily with the qubit sent by Alice. (There is no loss of generality in this, because any physical nonunitary interaction is equivalent to a unitary one with a higher dimensional probe.) Eve's probe is then stored until Alice announces the basis that was used, and only after that is it measured by Eve.

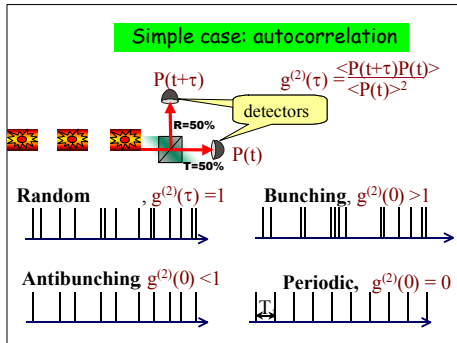
In a convenient notation, if Alice sends state  $|x\rangle$ , the result may be written as

$$|x\rangle \otimes |\psi_0\rangle \rightarrow |X\rangle, \quad (1)$$

where  $|X\rangle$  is an entangled state of the probe and the photon that Alice sent to Bob. Likewise, for the other signals that Alice may send, the results of Eve's intervention are entangled states,  $|Y\rangle$ ,  $|U\rangle$ , and  $|V\rangle$ . Since the interaction is unitary, it follows from

FIG. 2. Information vs disturbance for various eavesdropping methods.

## Badania na Hożej pojedyncze fotony na żądanie



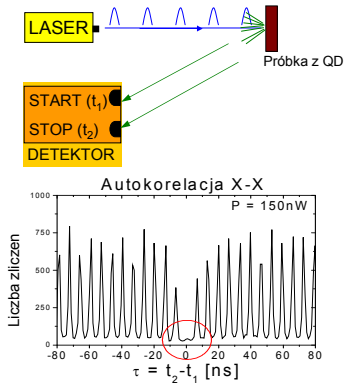
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## Problemy

Pojedyncze fotony na żądanie



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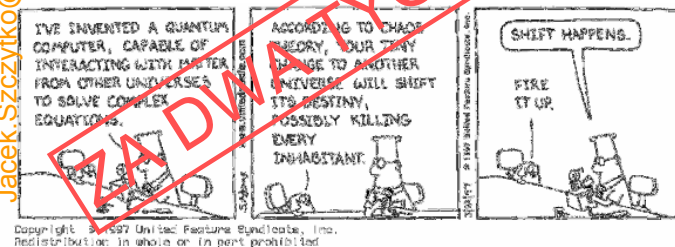
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## Quantum Computer I (QC)

1. Komputery kwantowe
  - a. Logika bramek
  - b. Kwantowe algorytmy
  - c. Jak zbudować taki komputer?







Dr Piotr Wasylczyk  
Nieliniowo, adaptacyjnie  
i femtosekundowo,  
czyli ekstremalnie w optyce.

