

Quantum Computers and Quantum Coherence

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If the states of spins in solids can be created, manipulated, and measured at the single-quantum level, an entirely new form of information processing, quantum computing, will be possible. We first give an overview of quantum information processing, showing that the famous Shor speedup of integer factoring is just one of a host of important applications for qubits, including cryptography, counterfeit protection, channel capacity enhancement, distributed computing, and others. We review our proposed spin-quantum dot architecture for a quantum computer, and we indicate a variety of first generation materials, optical, and electrical measurements which should be considered. We analyze the efficiency of a two-dot device as a transmitter of quantum information via the propagation of qubit carriers (i.e. electrons) in a Fermi sea.

I. INFORMATION PROCESSING AND QUANTUM MECHANICS

While we will spend much of this chapter considering fairly specifically the application of quantum magnetic systems to quantum computing, we want to first review more broadly the potential “quantum revolution” that is brewing in the area of information science. It is amusing for a physicist to note that quantum mechanics is now being taught as part of the standard curriculum in a growing number of graduate computer science programs! Why would computer scientists find it necessary to take up such an esoteric study from a different field? The problems which have interested them have nothing to do with the quantum world, and this is not being changed by this quantum revolution. Computer scientists have a wide range of tasks which they are interested in accomplishing successfully, safely, and/or efficiently [1]:

1. Given data X , compute $f(X)$ in the fewest number of steps. (*computational complexity* [2])
2. Given two parties holding data X and Y , compute $f(X, Y)$ with the least communication. (*communication complexity* [3])
3. Given two parties holding data X and Y , compute $f(X, Y)$ in such a way that the two learn no more about each other’s data than they know from the function value itself. (*discreet function evaluation* ([1], Chap. 5.8))
4. Transmit data X reliably from one party to a second as quickly as possible. (*channel capacity* [4])
5. Protect data X from duplication. (*counterfeit protection*)
6. Transmit data X from one party to a second in such a way that the data cannot be read by any third

party. (*key distribution/cryptography* ([1], Chap. 6))

7. Transmit data X from one party to a second in such a way that the receiver can be assured that the data was not corrupted during passage through the channel. (*authentication* [1])
8. Transmit data X from one party to a second in such a way that another party can later confirm that the second party did not alter X , and can confirm that it was produced by the first party. (*digital signature* [1])
9. Divide data X among n parties in such a way that no $n - 1$ of them can reconstruct data X , but all n working together can. (*secret sharing* [5])
10. Determine and execute optimal strategies in games. (*game theory; economics* [6])

In this information age, our society’s well being increasingly depends on being able to perform these and similar tasks well. Quantum mechanics is never mentioned in this list; nor should it be, since all of these tasks involve the possession and transmission of data in palpable, macroscopic form. “Quantum data” is not useful for members of our very macroscopic society; the inputs and outputs of these tasks must be in classical form. (One might question this assumption in some radically altered definition of “society”.)

But what we have increasingly realized is that the *tools* employed to accomplish these tasks can well be quantum mechanical. In addition to “classical” processing primitives involved in completing tasks (place a bit in memory, compute the AND of two bits, launch a bit into a communication channel), we can employ a host of quantum processing primitives: prepare a qubit (two-level quantum system) in a particular pure state; launch a qubit

(e.g., a photon or an electron, see Sec. III) into a communications channel; transform the state of two qubits according to the action of some two-body Hamiltonian.

The remarkable fact is that it is known how to achieve improvements in many (but by no means all) of the tasks mentioned above by employing these quantum primitives. We will review here briefly the “quantum state of the art” for our list of tasks:

1. *computational complexity*: Shor’s famous work [7] showed that some very important computations, for example prime factorization, have only polynomial complexity if quantum primitives are used, while this computation can (probably) not be done in polynomial time if only classical primitives are used. It is worth reviewing the general way in which the classical specification of the problem is converted into an application of quantum primitives: the data X (the number to be factored) is converted into a time-dependent two-body Hamiltonian function which is applied to a set of qubits prepared in a standard quantum state (e.g., all zeros). Then the answer $f(X)$ (the set of prime factors) is obtained by the results of a quantum measurement performed on each of the qubits. It may be necessary to repeat Shor’s procedure several times to obtain a factor.

It should be noted that there are some other computations for which it has been proved that *no* improvement in computational complexity is achieved by using quantum primitives [8]. For instance, the n^{th} iterate of a function provided as a look-up table takes n references to the table even if quantum primitives can be used [9]. Work continues to explore the cases in which quantum speed-ups are and are not possible [10].

2. *communication complexity*: In this work the advantages gained by communicating using qubits rather than bits have been explored [13]. There are some strong positive results in this area. Quantum communication is provably more efficient for the problem of two-party appointment scheduling: two persons have to compare their appointment books to choose a day to have lunch out of N possible days. For classical bit transmission $O(N)$ bits of communication are required in general. But it has been proved (it is an application of the “Grover” algorithm [12]) that no more than $O(\sqrt{N} \log N)$ quantum bits of transmission are needed to complete this task with high reliability [11]. There is a related task in which the quantum speedup is even more dramatic, in the area of “sampling complexity”: two parties must both pick a subset of cardinality \sqrt{N} from a common set of size N in such a way that their subsets are disjoint. Classically, $O(N)$ bits of communication are required to assure disjointness, but just $O(\log N)$ of quantum bit

transmission suffices [14]. Such dramatic provable speedups are apparently also possible even in a case where two parties share a string of random bits [15].

3. *discreet function evaluation*: This is an example of a category of task for which there is believed to be *no* quantum solution. This is true, at least, for the principal technique which computer scientists have used to analyze this task [1], which involves reducing it to a procedure called *bit commitment*, in which one party records a bit value of her choosing, locks the record in a safe and sends it to a second party (the “commit” phase); then at a later time of her choosing, she sends the key to the other party (the “opening” phase). Since safes can be x-rayed and locks picked, this protocol is not secure. It has been proved that bit commitment is never secure in a quantum world [16]; using entanglement, the sender can change the value of the bit between the commit phase and the opening phase.
4. *channel capacity*: Here the results are tantalizing, but not conclusive. The problem is this: given a classical bit channel and a quantum bit channel with the same levels of noise (the same probability that the bit will pass through the channel unaffected, roughly speaking), are fewer uses of the qubit channel needed to send a given classical message reliably than of the bit channel? No case has been found in which this “classical capacity of a quantum channel” exceeds the Shannon capacity for the classical channel, although the work of Fuchs *et al.* give indications that it may be possible [17]. Actually, there is one scenario in which the quantum capacity is definitely greater: if the sender and receiver have shared a prior supply of maximally entangled quantum states, which themselves carry no classical information, the quantum capacity can be boosted by the technique of superdense coding [18] by a factor of two or sometimes more (at least up to a factor of three for qubit transmission) [19].
5. *counterfeit protection*: There are really no strong classical techniques for protecting against counterfeiting. The first application of quantum primitives ever conceived, “quantum money” was devised in 1970 by S. Wiesner [20]. It is a beautifully straightforward application of the simple rules of quantum state preparation and measurement. The bank embeds qubits into its banknotes; each qubit is in a pure quantum state, but the states are drawn from a non-orthogonal ensemble (e.g., $|0\rangle$, $|1\rangle$, $|0\rangle + |1\rangle$, and $|0\rangle - |1\rangle$, or in spin language, in the eigenstates of either σ_z or σ_x). A record of the state preparation is kept at the bank, and the bill is sent into circulation. When the note returns to the bank, the bank can use its record to measure each qubit in a “non-demolition” [21] fashion, that is, in the

appropriate σ_z or σ_x basis so that the state is undisturbed and the measurement outcome is deterministic. If all measurements agree with the stored record, the bank can be assured that no attempt has been made by a counterfeiter to read the state of the qubits to duplicate them. This application has not received much attention lately, but perhaps its day will come with the further advance of quantum technology, when qubits can be stored (or error-corrected) over very long times.

6. *key distribution*: The most well-known success of quantum protocols is in “quantum cryptography [22].” The security of quantum transmission of random data (the key) begins with the same trick that is introduced in quantum money, sending one of a set of non-orthogonal quantum states that an eavesdropper cannot reliably distinguish, and that are in fact disturbed if the eavesdropper attempts to learn any information about them. The construction of a secure key from this primitive involves a lot more work, but Mayers has given a proof [23] that the a protocol naturally obtained from the one proposed by Bennett and Brassard in 1984 [22] is unconditionally secure. Another protocol in which state transmission is augmented by local quantum computation is considerably easier to prove secure [24].
7. *authentication*: Wegman and Carter [25] introduced a provably secure authentication technique that assumes that the sender and receiver possess a secret key; therefore, a secure key exchange using quantum primitives leads directly to a way of doing secure authentication. In today’s world there is another way to perform authentication: authentication is implied by digital signatures, which are routinely used in present-day cryptography, but —
8. *digital signatures*: The existence of quantum protocols has negated the ability to do digital signatures. First, no quantum protocol can apparently be introduced which can take the place of digital signatures used in public-key cryptography, in which a sender, by appending to the end of a message an encrypted version of that message, produces unalterable evidence that this message originated from him: anyone can later decrypt the “signature” using the sender’s public key and compare it with the putative message [1]. Second, the “proof” that this protocol is secure relies on the security of public-key cryptography, which is jeopardized by the ability to factor large numbers by quantum computation. Perhaps some entirely different quantum reasoning will again permit the accomplishment of this information processing task.
9. *secret sharing*: Only a little work has been done on this [26], but it appears that there will be a vari-

ety of ways of using multipartite states to split up a secret in such a way that it can only be reconstructed by the cooperative quantum operations of several parties. Buzek *et al.* have shown ways in which this problem can be approached using entangled states; it is perhaps more surprising that it is possible to use unentangled quantum states to perform this task. This arises from the recent discovery that there exist ensembles of multiparty orthogonal product states which can nevertheless not be distinguished by any local operations of those parties, even if they are allowed any amount of classical communication. Only a joint quantum measurement can distinguish them reliably. The detailed application of this discovery to a secure secret sharing protocol has only just begun.

10. *games*: This is a rather ill-defined area at the moment, but one with apparent promise. Meyer and Eisert *et al.* [27] have shown that if the players in a game can perform quantum mechanical manipulations in the game (e.g., moving a chess piece into a superposition of positions by a unitary operation), they can gain some advantages. It seems that some changes will have to take place in our society before some of these game results become applicable — can we have a quantum stock market? A quantum economy?

A final comment about this survey: while in some sense it covers everything that goes on in the research on quantum improvements of information processing tasks, in another way it misses a lot of what workers in this field really think about. Between the bottom level of quantum or classical primitives like data transmission and qubit measurement, and the top level of tasks to be accomplished, lies a whole realm of macros and subprocedures which use the primitives and provide tools for accomplishing the end tasks. We are very familiar with these in classical computing (fetch program instructions, invoke a floating-point multiplier, launch a packet onto an ethernet), but there is a whole host of quantum macros which have no classical analog and which are crucial for facilitating the quantum implementations of many tasks.

An important example of these is quantum error correction and fault-tolerant quantum computation [28], which put together the primitives of state preparation, measurement, and manipulation in such a way that the effective unitary evolution of a quantum computation is carried out reliably despite the intervention of noise (“quantum decoherence”). Another operation which one might consider as a quantum macro is the sharing of a *quantum* secret, recently discussed in [29]. Reliable qubit communication depends on other noise-suppression quantum macros; the most effective approach to this problem involves entanglement purification [30] (in which a large supply of partially entangled mixed quantum states is manipulated locally to produce a smaller supply of pure, maximally entangled quantum states). Another

crucial subprocedure in this noise-suppression macro is the celebrated “quantum teleportation [31].” Much of the recent “Star Trek” discussion of teleportation misses the point that it has a well-defined, scientifically valid role as an enabler of high-level quantum processing for anything involving the transmission of quantum information (e.g., distributed computing, key distribution).

So, quantum information processing isn’t just factoring! Quantum factoring alone is interesting and important; seeing the whole picture, though, indicates that we may be just at the beginning of something *really* big.

II. QUANTUM INFORMATION PROCESSING AND MAGNETIC PHYSICS

Specially-crafted magnetic materials and magnetoelectronic structures, we believe, are good candidates for providing some of the important primitive quantum tools for performing many of the tasks itemized above, as we will detail shortly. We will concentrate in this section on those applications which require the creation and manipulation of “fixed” qubits, which include the applications of quantum computing, counterfeit protection, and secret sharing, and pieces of the others, such as the encoding and decoding required in channel transmission. In Sec. III we will discuss a particular scenario based on mobile electrons [32] whose spins provide the “mobile” qubits needed in the other applications; some proposals are now being considered in which the coupling by solid-state optical cavities to photons [33] could provide the tools for the remainder of our tasks as well.

The magnetic structures that we envision are promising because the qubit is naturally defined (in terms of a localized single spin). This localized spin has the potential for being relatively well isolated from its environment – that is, for having low decoherence rates – and it can be manipulated by electrical, magnetic and/or spectroscopic tools and can be measured using advanced magnetometric or electronic techniques.

Of course, the magnetoelectronic structures that we propose are not the only possible approach to the realization of quantum information processing; efforts spanning many of the active areas of experimental quantum physics have led to successful demonstrations of quantum logic gates, and of operating systems for quantum cryptography, superdense coding, and quantum teleportation.

We can only give a brief mention of all the different quantum logic gate demonstrations that have been reported: In 1995, there was the demonstration of the two-qubit controlled-NOT reported using ion trap spectroscopy by the NIST group [34]. Since this demonstration, progress towards realizing the idea of the linear-ion trap quantum computer has been proceeding steadily; this group has recently demonstrated the deterministic creation of entanglement between two ions [35]. In

the area of cavity-quantum-electrodynamics, the vacuum cavity version of the solid state microcavity scheme mentioned above was first investigated in 1995 by the Cal Tech group [36], and many proposals have been made for how to use this device in a quantum communication network.

The processing of photons in fiber-optic experiments has also received a lot of attention. Full-scale quantum cryptography demonstrations have now been achieved in many different laboratories [37]. In addition, several other quantum information processing protocols have been realized in such systems: superdense coding has been achieved in systems where photon EPR pairs are created by parametric down conversion, and incomplete Bell measurements are performed using linear optical elements [38]. More recently, teleportation of photon polarization states has been achieved [39,40]. Now it has also become possible to teleport a “continuous” Hilbert space, the quadrature field coordinates of a coherent state of light [41].

Finally, it should be mentioned that there is another condensed matter implementation of quantum gates that has received a lot of experimental attention lately, one involving bulk NMR (nuclear magnetic resonance). Following on the original theoretical idea for using NMR for quantum gates [42], the idea was put into practical, realizable form in 1997 [43]. Since then, there has been a plethora of experimental investigations of 2 and 3 spin systems, including demonstrations of the Deutsch-Jozsa and Grover quantum algorithms [44] and of simple quantum error correction techniques [45]. There has even been a realization of intramolecular quantum teleportation [46].

A. Proposed Device Structure

Rather than giving a general discussion of the criteria which a magnetoelectronic device proposal must satisfy in order to be a good candidate for a quantum computer (which we have done previously; see Refs. [47–49]), we will simply proceed to describe the specific model that we have introduced [48–50]. From the discussion here it should be clear what are the critical requirements for this proposal to succeed.

Fig. 1 sketches the model that we have introduced in Ref. [48]. It is a quantum-dot array [51,52], produced in this version of the model by lateral confinement. The figure indicates a two-dimensional layer, for example a quantum well produced in a GaAs heterostructure, above which an array of electrodes is placed. As in the experiments of various groups [53–57,61], voltages on these electrodes can be used to deplete selectively regions of the two-dimensional electron gas below them, leaving isolated regions (the quantum dots shown dotted in the figure) in which electrons can be confined.

The qubit in this scheme is provided by the *electron*

spin of each quantum dot. In order that this qubit be well defined, the electron number must be controlled and constant throughout the operation of the device. This is assured by exploiting the well understood Coulomb blockade effect in these dots. We imagine that a transport (e.g., [55]) or capacitance [53] measurement is performed on every dot in the array separately, and the gate voltages (the gates are shown shaded in the figure) adjusted so that the energy of the N electron state is much lower than that of the $N - 1$ or $N + 1$ electron state. N will remain fixed throughout the quantum computation operations: this computer has no moving parts, not even the electrons move (at least not much). In order to use the electron spin as a quantum number, it is very likely essential that N be an odd number (if N is even, it would typically be the case that the total spin of the dot would be zero, so that no nearly-degenerate levels would be available to represent the qubit). If N is odd, the spin is at least $1/2$. In fact, $s = 1/2$ exactly is the ideal situation for representing a qubit. $s = 1/2$ is assured if $N = 1$, that is, if there is only one excess electron confined to the dot. For this reason partly, but mainly because of other considerations about the many-body physics of the dot, such as that discussed in Sec. III, we will consider only the $N = 1$ case. The $N > 1$ case may be usable for quantum computation, but it will require more analysis than we have performed up until now.

$N = 1$ is not easy to achieve experimentally. N in the range of a few tens has become relatively routine in the experiments cited, but in the very small- N regime it becomes difficult for electrons to tunnel in and out of the dot, and the quantum-dot potential can become disorder dominated. These are not severe difficulties in principle, but we acknowledge that it is a demanding requirement from the perspective of present-day experiments, and we are committed to studying the effect of using larger electron numbers on our proposed device operation.

B. Decoherence

Among the most crucial requirements for the implementation of quantum logic devices is a high degree of quantum coherence. Coherence is lost when a qubit interacts with other quantum degrees of freedom in its environment and becomes entangled with them. Predicting the coherence time of the electron spin states of the device described above is very difficult, as the possible couplings to all the other quantum degrees of freedom of the system must be considered. We are encouraged, however, by the general fact, observed in many experimental situations in condensed matter physics, that spin degrees of freedom have longer coherence times than charge degrees of freedom (ones for which the different electron states are associated with different orbital wavefunctions), simply due to the weaker couplings of spin states than orbital states to the environment.

This observation does not lead to any simple result about what the available decoherence times in our structure will be. Experiments of spin coherence times have been performed on somewhat related structures [58,59], with the result that a very wide range of decoherence times can be seen for the spins of electrons in semiconductor heterostructures and bulk doped semiconductors. In structures which are intentionally doped with magnetic ions (Mn), the coherence times are seen to be very small, on the order of picoseconds. But times ranging over six orders of magnitude, approaching microseconds in some structures, have now been seen depending on the details of the semiconductor structure. As we will discuss in the next section, microsecond decoherence times would be acceptable for beginning experiments on quantum gate operations, while times of milliseconds would be adequate for even large-scale quantum computing applications (because of the abilities offered by quantum error correction [28]).

We have considered in general the likely mechanisms of decoherence in structures such as Fig. 1, which should be useful in guiding designs of experiments which seek to lengthen the decoherence times. Our estimates [50] indicate that decoherence due to spin-orbit coupling should be negligible for conduction band electrons in GaAs (although not for holes); still, more detailed work needs to be done to quantify this effect. A potentially important mechanism for decoherence in these structures is the coupling to other spin states in the environment. As demonstrated in the Mn-doping experiments [58], this effect will be greatly influenced by the materials preparation of the devices, and can be a very strong pathway to decoherence. Thus, in our work [48] we have studied in detail models in which the qubits are coupled to a bath of other spins. The significance of the effect is entirely determined by the strengths of the coupling constants between the system and bath. The decohering effect of this bath can be enhanced during quantum gate operations [49], that is, when spins in neighboring quantum dots are coupled (see next section).

In addition to other electronic spins, there are unquestionably nuclear spins in the environment as well whose decohering effect must be considered. In GaAs in particular, 100% of the nuclei possess non-zero spin. We have studied the effect of these spins recently [50], and our calculations indicate that these spins can be a serious source of decoherence if the applied magnetic fields are low and the nuclear spins are in their thermal equilibrium state. However, it is relatively easy to modify these conditions, either by dynamically spin-polarizing the nuclear spins e.g. by known optical techniques, and/or by arranging that the operation of the devices is performed with a non-zero applied magnetic field. Actually, the presence of significantly spin-polarized nuclei may actually be very useful for performing gate operations on these qubits (see next section) [50].

Another “trivial” but practically important source of spin decoherence arises from uncertainties in the applied

Hamiltonians to be discussed in the next section. For example, in schemes in which the gate action involves the application of a uniform magnetic field, inhomogeneities in this field will result in inaccuracies in the gate operation. This decoherence effect is analogous to the broadening effect on absorption lines which is well known in traditional spin spectroscopies, where various “refocusing” and “spin-echo” techniques have been devised to ameliorate them. Such techniques may have to be developed and adapted to assure reliable quantum gate operation, but this problem has not been addressed systematically in any detail.

One might think that if fluctuating magnetic fields are a severe problem for quantum-dot quantum bits, then perhaps there would be some value in reconsidering the use of electron orbital states, which, after all, would be insensitive to such magnetic field effects. We are pessimistic on this account, not only because the decoherence times for orbital states are short for myriad other reasons, but also because there is reason to believe that some of the important decoherence mechanisms due to Fermi-sea effects will be non-Markovian. Markovian, or memoryless, decoherence is actually greatly desired over non-Markovian decoherence in quantum computation, as all the powerful techniques introduced in quantum error correction assume a memoryless error scenario [28]. No one has demonstrated that a qubit system with even very weak non-Markovian decoherence would be useful for quantum information processing.

C. Quantum gates

Another crucial requirement for quantum computing, and for many of the other quantum approaches to information processing tasks outlined in the first section, is that it must be possible to apply time-dependent one- and two-body Hamiltonians to the qubits according to the specifications of some program [60].

The structure of Fig. 1 can have many mechanisms for applying such “quantum gates” to the spin qubits. First, the structure has a set of gates which can control the position of the electron’s wavefunction within the two-dimensional electron gas, simply by varying the confining voltages on these gates. If two of these electrons in neighboring dots are pushed close together, the overlap of the orbital wavefunctions will, via the Pauli principle, produce an effective two-spin interaction between the two spin qubits. The Hamiltonian produced is that of an exchange interaction which is isotropic in spin space

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2. \quad (1)$$

Here the time dependence $J(t)$ is regulated by the time variation of the tunneling matrix element Γ of an electron from one dot to the other. According to perturbation theory, $J(t)$ is

$$J(t) \propto \frac{\Gamma(t)^2}{U}. \quad (2)$$

Here U is the Coulomb blockade energy, the charging energy required to add a second electron to one of the dots.

In Ref. [50] we give a more refined and detailed analysis of this switchable spin interaction, in particular we show that the long range part of the Coulomb interaction (if it is not screened) will produce an additional term in (2) of opposite sign that leads to a sign reversal of J for sufficiently large external magnetic fields as a result of competition between long-range Coulomb repulsion and magnetic wave function compression. By working at this magnetic field (where J vanishes) the exchange interaction can be pulsed on, even without changing the tunneling barrier between the dots, either by an application of a local magnetic field, or by exploiting a Stark electric field (which will also make the exchange interaction nonzero). See [50] for further information. We finally note that the exchange energy J can be understood as the level splitting induced by the formation of a molecular state between the two quantum dots [50]. The observation of such a molecular state in a double dot system containing several electrons has indeed been reported recently [61,62].

The exchange interaction of the form Eq. (1) is sufficient for the most general quantum computation, if it is supplemented by a suite of one-body time-dependent interactions (one-bit gates). This is discussed in [48–50], where it is shown that Eq. (1) will produce a quantum gate known as a “square-root of swap” (in which the exchange interaction is turned on for half the time required for it to produce a complete interchange (“swap”) of the quantum states of the two qubits). We show [48] that two square roots of swap, in conjunction with a set of one-qubit gates, will produce a quantum XOR (also known as a controlled-NOT) gate, which is known to be employable for any arbitrary quantum computation [63].

The speed at which these switchings are done will be an important parameter; the rule is, the faster the better, consistent with doing the prescribed manipulations with rather high accuracy (error correction theory says that the relative accuracy to be striven for is on the order of 10^{-4}). The fundamental physics says that the switching on and off of the tunneling could be done much faster than a nanosecond [50]—only at much, much shorter time scales will such fundamental limitations as adiabaticity enter the picture. It is necessary that the switching time be smaller than the decoherence time; again, error correction theory says that ultimately, it is desirable that the switching time be smaller than the decoherence time by about 10^{-4} . We think that 10^{-1} will be quite satisfactory for the initial round of measurements. We think that initially, the experimentalist should simply be guided by what is doable. Since high-frequency signals are difficult to transmit into quantum dot structures in the Coulomb blockade regime at 4K or so, we might suggest that one

should shoot for switching times in the neighborhood of 10^{-7} sec. A simple calculation indicates that only modest control-voltage excursions are needed to do square-root-of-swap in this time.

We note that the switching of the gates via an external control field $v(t)$ should be performed adiabatically [50], i.e. $|\dot{v}/v| \ll \delta\epsilon/\hbar$, where $\delta\epsilon$ is a characteristic energy scale of the problem. In the present case $\delta\epsilon$ should be taken to be on the order of the orbital energy-level separation. This adiabaticity requirement excludes e.g. switching pulses of rectangular shape, in which case many excitations into higher levels will occur. An adiabatic pulse shape of amplitude v_0 is e.g. given by $v(t) = v_0 \text{sech}(t/\Delta t)$, where $\Delta t = \tau_s/\alpha$ gives the width of the curve and α is chosen such that $v(t = \tau_s)/v_0$ becomes vanishingly small. In this case we have $|\dot{v}/v| = \frac{1}{\Delta t} |\tanh(t/\Delta t)| \leq 1/\Delta t = \alpha/\tau_s$, and thus for adiabaticity we need to choose τ_s such that $\alpha/\tau_s \ll \delta\epsilon/\hbar$. Note that the Fourier transform, $v(\omega) = \Delta t v_0 \pi \text{sech}(\pi\omega\Delta t)$, has the same shape as $v(t)$ but with a width $2/\pi\Delta t$, and we see that $v(\omega)$ decays exponentially in frequency ω , whereas it decays only as $1/\omega$ for a rectangular pulse. [We could, of course, also use a Gaussian pulse shape, however, in this case we would get $|\dot{v}/v| \propto t$ and some cutting of the long-time tails is required in order to satisfy adiabaticity for all times.] It is worth emphasizing, however, that for our quantum gate action the pulse shape is not relevant, the only parameter which counts is the integrated pulse shape, $\int_0^{\tau_s} dt P(v(t))$, where P stands for the exchange J or the magnetic field B which is switched. This stands in contrast to spectroscopic mechanisms based on resonance conditions where more details of the shape of the pulse are relevant. Also, even if adiabaticity is not well satisfied in our switching, not much will happen as long as spin-orbit coupling remains small since typically only charge degrees of freedom will be excited in a non-adiabatic process and not the spins representing the qubits.

The use of an inhomogeneous magnetic field (or an inhomogeneous g-factor) for gating mentioned above for two-bit gates is obligatory, in some form, for the accomplishment of the desired one-qubit gates. That is, every one-body Hamiltonian needed for quantum computing can be written in a standard Zeeman form

$$g\mu_B \mathbf{B}(t) \cdot \mathbf{S}. \quad (3)$$

It is necessary that the field $\mathbf{B}(t)$ (or the effective field) be applicable separately to each qubit (or at least that the effect on neighboring qubits be smaller and known), and that it can be applied along at least two different axes.

There are many ways that we can conceive of applying these local magnetic fields or local Zeeman interactions. If the switching time scale is to be the same as above (10^{-7} sec.), then field strengths of only a few Gauss are necessary, and this could be accomplished by a mechanism as simple as winding a small wire coil or by placing magnetic dots above/below each quantum dot, or by

placing the dots between a grid of current-carrying wires as in RAM devices [64]. Other methods of obtaining very localized fields, such as moving magnetic bubbles in a garnet film, using a magnetic-disk writing head, or a magnetic force microscope tip, can be considered.

Although strict localization of the applied field is not necessary, it does make life considerably easier, and there are several ideas which would make this field effectively much more localized. If the nuclear spins of the dot and the material surrounding the dot can be polarized as discussed above, then the electron spin (but not the orbital motion) experiences an effective internal magnetic field, the ‘‘Overhauser field’’, which can be on the order of several Tesla in GaAs [50]. If the Overhauser field is different in the dot and in the confining layers above and below it, then the field as seen by the confined electron can be varied by purely electric gating, that is, by pushing the electron more or less into the insulating barriers. In our original work [48] we introduced another variant of this idea, in which the confining materials possess a *real* magnetization due to a ferromagnetic moment. Such ferromagnetic insulating materials are not so common, but are not unheard of either (the garnets, the ferrites, and the Eu-chalcogenides are some examples); unfortunately, there is little experience in matching these materials epitaxially to the common dot materials such as GaAs, but first promising progress in this direction has been made recently, see [65]. We also would like to emphasize here that our set-up permits the performance of swaps of qubit states in such a way that we can easily move a spin state (not the electron spin itself) of a given quantum dot via a chain of adjacent quantum dots to a desired location in the network where we have localized magnetic fields available, act with the field on the qubit and then swap the qubit back to its original location. This is possible since the swapping operation does not involve single-qubit rotations and since we can swap two states even without knowing their particular state. But either the Overhauser field idea or the magnetic insulator idea can be extended to solve the very important problem of quantum measurement, to be discussed momentarily.

A brief word about error correction, which we have alluded to many times already: error correction provides a way of using redundancy and repeated quantum measurement during the course of computation, which detects and diagnoses the occurrence of decoherence, and undoes its effects. It uses exactly the same gates which we have just introduced, along with qubit measurements to be described shortly. The conventional analysis of quantum error correction [28] assumes that two-qubit gates can be performed between any two qubits. In our computational model, gate operations can only be performed between neighboring qubits. This is not a serious modification, the basic procedures of quantum error correction still work in this case [66]. The more crucial requirement for error correction to work is that two-qubit and one-qubit gates can be performed on many different qubits simultaneously as it is possible in our proposal. There

are other popular quantum register designs, for example the well-known linear ion trap model of Cirac and Zoller [67], for which error correction is not possible because gate operations cannot be done in parallel.

Finally, the concept of error correction promises to be important by itself. Indeed, in many areas of mesoscopic physics it would be highly desirable to maintain phase coherence *indefinitely*, a goal which we believe could be achieved with error correction schemes.

D. Quantum measurements

The final requirement which must be addressed for performing quantum information processing with the quantum-dot structure is the need to read out data reliably, which translates into the necessity of doing spin measurements at the single-spin level. It must be possible to address each individual spin in the structure (or at least some subset of the spins) and perform an “up/down” measurement on them. Solid-state magnetometry at the single-Bohr-magneton level has of course proved to be very difficult, as other contributions to this volume will discuss. We foresee, though, that using some of the capabilities of quantum computing, the very difficult single-spin measurement can be turned into a more manageable electrical (i.e., charge) measurement along the lines first proposed by us in Ref. [48].

We have recently reviewed in detail the possibilities in this area, we will just give an outline here, the interested reader is referred to [68]. The basic idea of turning the spin measurement into a charge measurement [48] (see also [69]) is this: we use the kind of magnetic (either ferromagnetic or nuclear-spin-polarized) barriers mentioned above as tunnel barriers, say in the form of a thin barrier separating two quantum dots or a quantum dot and a single-electron transistor. The tunneling barrier can be made strongly spin dependent (this is the well-known “spin-filter” effect); thus, at the time of measurement, the tunneling of a spin-up electron can be made very probable, while the tunneling of a spin-down electron remains very improbable. Thus, the job of measuring spin is converted into the job of measuring whether an electron has tunneled or not. But this is a feasible (and indeed, almost routine) electrometry measurement—many labs have demonstrated the feasibility of single-electron-charge magnetometry, either with single-electron transistors, quantum point contacts, and other mesoscopic electronic structures.

Another promising idea for single-spin measurement involves near-field optical probing of the spin state. We have not analyzed this approach in any detail, but it deserves future experimental and theoretical attention.

E. Test experiments

It is clear that the above concept, which we have developed over the last three years, has proved far too demanding to be undertaken all at once. It requires a combination of developments, in materials and device fabrication, in precision, high frequency electrical control, in hitherto unexplored, complex, nanoscale architectures, which are far beyond the scope of one generation of experimental investigation.

Therefore, it is very important to pull apart our quantum-dot quantum computer into small pieces, setting feasible shorter-term goals for the demonstration of particular capabilities. We only intend to give a brief idea here of the kind of near-term work which might be done; indeed, it seems that the possible ways of dividing our proposal into smaller, manageable chunks are almost infinite, and finding the most promising ones can only result from a detailed dialog between the theorist and experimentalist. But here is a selection of ideas which we now now might be promising for the next few years:

There is a clear need to demonstrate the controlled fabrication of spin quantum dots. As mentioned above, a desirable goal would be to routinely obtain dots with just *one* excess electron. More theory must be done to see whether using dots with an odd number of excess electrons would be acceptable. One-electron dots have been achieved [53], but not in geometries in which dots could potentially be coupled. Loading by transport in the Coulomb blockade regime would be the obvious way, but doping or optical techniques should also be considered.

If an array of such dots can be obtained, then characterization of the qubit energy levels, g-factors, and especially decoherence times would be the next thing to study. In fact, an initial version of this type of experiment has now been reported [59], which demonstrates that time-resolved optical probes of these systems are extremely promising for these kinds of initial characterizations. Further application of pulsed-spectroscopy techniques should yield further information about the controllability of such qubits (at least at the one-qubit gate level).

Another distinct line of investigation would involve demonstration of two-qubit gate capabilities. We have suggested [48–50] that gated double-dot structures that have been fabricated and studied in GaAs 2DEGs [55,56] could be the starting point of such studies; it will also be desirable to see if other types of dots, say in pillar structures or ones created by chemical nucleation, can be integrated into devices in which their coupling is subject to electrical or magnetic control. We envision experiments in which arrays of these dots can be subjected to identical preparations and probings. It may be that an experiment as straightforward as the measurement of the a.c. magnetic susceptibility of such a dot array as a function of a control voltage [48,49] will be sufficient to demonstrate the basic physics of quantum-mechanical

exchange coupling between neighboring spins.

The magnetoelectronic techniques that we have suggested for other gate operations and for single-spin quantum measurement involved additional and quite different experimental challenges. The basic materials issues of the integration of semiconducting and magnetic materials are not yet well enough developed to even propose a likely system to study at this time, although it is promising to note that there is now active research focussed on just this area, finding good matches between magnets and semiconductors which will show clean, reproducible interface properties. If, for example, it proves possible to grow EuS or EuO on GaAs, then an experiment can immediately be considered in which the basic spin filtering phenomenon of carriers in the semiconductor conduction band is looked for. This experiment would be very informative even in a traditional bulk tunneling geometry; there would be no need to even consider integrating these with quantum dot structures at first. Tunneling through Overhauser-polarized barrier materials may be less demanding from the materials science point of view, but will require integration of optical (for nuclear spin polarization) and electrical expertise. A later generation of experiment could consider integrating the spin-filter into a simple point-contact (say of the Ralls type) so that a combined spin-filter/Coulomb blockade effect could be demonstrated. This already takes us quite far into speculative territory.

We would like finally to briefly comment about questions that we have been asked about whether the many experiments on the *charge* degree of freedom in quantum dots could be directed towards the achievement of orbital-level qubits and quantum gates. While there may be a worthwhile approach in this direction, we are pessimistic about its ultimate chance of success compared with the spin approach, even though spin effects are at this time much less well developed in quantum-dot research. We say this based on the fact that orbital (i.e. charge) degrees of freedom of a dot will be much harder to make coherent than the spin of a dot, just based on the typically stronger coupling of charge (compared to magnetic moment) to the environment. A typical Fermi-sea charge environment also has a different, and possibly even worse, problem as already pointed out before: Fermionic baths are very non-Markovian, having power-law decays of correlations. Almost all the well-developed theory of quantum error correction applies only to Markovian baths [28], and it is very unclear whether any useful quantum computation can be done in the presence of a non-Markovian environment (however, see [70]). These considerations have been enough to justify, in our minds, a continued focus on the eventual possibilities of spin quantum dots only.

III. QUANTUM COMMUNICATION WITH ELECTRONS

In this section we would like to address the following question: is it possible to use mobile electrons, prepared in a definite (entangled) spin state, for the purpose of quantum communication? Such a question, for instance, is of central importance in a solid state quantum computer where one wishes to exchange quantum information between distant parts of a quantum network. The question is of course also of broader interest: if we could use electrons for creating entangled states, in particular so-called EPR pairs, and if we could move them around separately while preserving their spin entanglement, then we would be able to implement, for instance, tests of Bell's inequality; thereby, we could obtain tests of non-locality—one of the most striking concepts of quantum mechanics—for the first time with electrons. So far, all such tests have been done on photons [71], most recently by Gisin's group [72] who demonstrated in a remarkable experiment that photons propagating in optical fibers remain in an entangled state over more than 10 km's. It is quite amusing to note here that the Gedanken experiment which has been formulated by Einstein, Podolsky, and Rosen [73], and which underlies the Bell inequalities, makes use of point particles and not of massless particles such as photons. Thus, there can be no doubt that it would be highly desirable to extend tests of non-locality also to quantities which have a rest mass such as electrons in particular.

Now, as we have discussed before, one basic ingredient for quantum communication are entangled pairs of qubits which are shared by two parties. There are three separate requirements involved here which must be satisfied. First of all we need *mobile qubits* which can be transported from position A to position B. Second, we need a source of entanglement for such qubits which can be operated in a controllable way, and third, it must be possible to transport each of the qubits separately in a phase-coherent manner such that the entanglement between the two qubits of interest is not destroyed in the process of transporting them to their desired locations.

Now, our choice of representing the qubit in terms of the spin of a mobile electron satisfies the first requirement trivially (note that qubits defined as pseudospins are typically not mobile). The second requirement, to have a source of entanglement, can be satisfied by using the quantum gate mechanism based on coupled quantum dots [48–50] as we have described it in the preceding sections.

To assess the third requirement, transport of entangled qubits, we need to be more specific of how we actually envisage such transport. One realistic scenario is to attach leads to the quantum dots into which the electrons can be injected (e.g. by lowering the gate barriers between dot and lead). From an experimental point of view it is best to make leads and dots out of the same material.

For instance, if the dots are formed in a two-dimensional electron gas (2DEG) such as GaAs heterostructures it is not difficult to connect them to leads formed also in the 2DEG by electrostatic confinement or some etching techniques [61,55]. In a first step we inject an electron into quantum dot 1 and another one into quantum dot 2. In a second step, we perform a quantum gate operation to produce an entangled state out of the two electrons, say a singlet state, $|\psi_{kk'}\rangle = \frac{1}{\sqrt{2}}(|k, k'\rangle + |k', k\rangle)\chi_s$, where $\chi_s = (|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)/\sqrt{2}$ is the two spinor describing a spin-singlet state. The orbital part of the state, characterized by the quantum numbers k, k' , is symmetric whereas the spin singlet is antisymmetric. As a measure of correlations we consider transition amplitudes between an initial and a final state. We begin with the simplest case given by the wave function overlap of $|\psi_{kk'}\rangle$ with $|\psi_{qq'}\rangle$,

$$\langle\psi_{qq'}|\psi_{kk'}\rangle = \delta_{qk}\delta_{q'k'} + \delta_{qk'}\delta_{q'k}. \quad (4)$$

Thus, if e.g. $q = k$, and $q' = k'$, the overlap assumes its maximum value one, simply reflecting maximum correlation between the two states. If we prepare the two electrons in a triplet state instead of a singlet we will find a minus sign instead of the plus sign in Eq. (4). This means that this sign simply reflects the symmetry of the orbital part of the wave function, and thus the overlap (4) distinguishes only triplet from singlet states but not necessarily entangled from unentangled states. Indeed, the triplet states with $m_z = \pm 1$ are not entangled, whereas the triplet state with $m_z = 0$ as well as the singlet state are entangled. Since the (anti-)symmetry of the orbital part of the wave function leads to (anti-)bunching behavior in the noise spectrum [74], we can in principle distinguish singlet from triplet states. The triplet states themselves can be further distinguished by measuring the z -component of the total spin, S_z , which could be achieved e.g. by making use of spin filters in the leads and/or leads that are connected up to other quantum dots into which the electrons can tunnel and then be detected via SET measurements [32]. In this way it is possible (in principle) to distinguish all four spin states, in particular also to distinguish between entangled and unentangled states (provided we deal with these four particular states only—otherwise the expectation value of S_z does not distinguish between entangled and unentangled states in general).

Next we generalize this concept of the overlap to a dynamical situation as well as to the leads which contain many interacting electrons besides the two entangled electrons of interest. Again, we use a similar overlap as a measure of how much weight remains in the final state $|\psi_{qq'}, \psi_0, t\rangle$ when we start from some given initial state $|\psi_{kk'}, \psi_0\rangle$, where ψ_0 denotes the fermionic ground state of the electrons in the leads, which is simply given by a filled Fermi sea. For further discussion it is now convenient to make use of the standard second quantization formalism in terms of fermionic creation ($a_{k\sigma}^\dagger$) and anni-

hilation ($a_{k\sigma}$) operators, where $\sigma = \pm 1$ denotes spin \uparrow (\downarrow) in the S_z -basis. The (normalized) initial state, choosing a singlet, can then be written as

$$|\psi_{kk'}, \psi_0\rangle = \frac{1}{\sqrt{2}}(a_{k\uparrow}^\dagger a_{k'\downarrow}^\dagger - a_{k\downarrow}^\dagger a_{k'\uparrow}^\dagger) |\psi_0\rangle, \quad (5)$$

and similarly for the final state, again chosen to be a singlet state. The overlap (4) now becomes a singlet-singlet correlation function which we denote by $G^s(q', q, t; k, k')$, $t \geq 0$, and which is explicitly given by

$$G^s(q', q, t; k, k') = \frac{1}{2} \sum_{\sigma=\pm 1} [G(q', -\sigma; q, \sigma; t; k, \sigma; k', -\sigma) - G(q', -\sigma; q, \sigma; t; k, -\sigma; k', \sigma)], \quad (6)$$

where

$$G(q', -\sigma; q, \sigma; t; k, \sigma; k', -\sigma) = -\langle T a_{q'\sigma}(t) a_{q-\sigma}(t) a_{k-\sigma}^\dagger a_{k'\sigma}^\dagger \rangle \quad (7)$$

is a standard 2-particle Green's function, and $k = (\mathbf{k}, k_l)$, where $k_l = \pm 1$ refers to lead 1 (2). Here, T is the time-ordering operator and $\langle \dots \rangle$ the zero-temperature or ground state expectation value. We assume a time- and spin-independent Hamiltonian, $H = H_0 + \sum_{i<j} V_{ij}$, where H_0 describes the free motion of the N electrons, and V_{ij} is the bare Coulomb interaction between electrons i and j (extensions to more complicated situations including spin interactions will be considered elsewhere). This four-point correlation function is of the type $G(12; 1'2')$ and it provides a measure of how much overlap (or transition amplitude) is left after time t between an initial and final singlet state of two electrons which have been injected into a Fermi sea (leads) of $N - 2$ interacting electrons, and which propagate during time t in the leads before they are taken out again. We emphasize that after injection the two electrons of interest are, of course, no longer distinguishable from the electrons of the leads, and consequently the two electrons taken out of the leads will, in general, not be the same as the ones injected.

It is now a non-trivial many-body problem to find an explicit value for $G(12; 1'2')$. On the other hand, we can expect some simplification: without spin-dependent forces we know that the total spin must be conserved even if the two electrons strongly interact with the rest (and among themselves) via Coulomb interaction. It is thus not unreasonable to expect that we still find some spin correlations, in particular entanglement, between initial and final states. But how much is it? And why and how do we lose some of the correlations, etc.? These questions are of fundamental interest, and we can find answers to them by evaluating $G(12; 1'2')$ explicitly with the help of standard many-body techniques [75,32]. Omitting most of the details [32] here we briefly state the main results. First we note that the four-point

Green's function considerably simplifies for the realistic situation where there is no Coulomb interaction between the electrons in lead 1 and the electrons in lead 2. As a result the 2-particle vertex part vanishes and we get $G(12;1'2') = G(11')G(22') - G(12')G(21')$, i.e. the Hartree-Fock approximation is exact and the problem is reduced to the evaluation of single-particle Green's functions $G_1(\mathbf{k}, t)$, $G_2(\mathbf{k}', t)$ pertaining to lead 1 and 2, resp. (these leads are still interacting many-body systems though). In particular, we now find

$$G^{s/t}(q', q, t; k, k') = -\{G_1(\mathbf{q}, t)G_2(\mathbf{q}', t)\delta_{qk}\delta_{q'k'} \pm G_1(\mathbf{q}', t)G_2(\mathbf{q}, t)\delta_{qk'}\delta_{q'k}\}, \quad (8)$$

where the upper (lower) sign refers to the spin singlet (triplet), and where we have chosen $k_l = 1$. For the special case $t = 0$, $N = 2$, and no interactions, we have $G_j = -i$, and thus G^s reduces to the rhs of Eq. (4). For the general case, we evaluate the (time-ordered) single-particle Green's functions G_j close to the Fermi surface and get the standard result [75]

$$G_j(\mathbf{q}, t) \approx -iz_q\Theta(\epsilon_q - \epsilon_F)e^{-i\epsilon_q t - \Gamma_q t}, \quad (9)$$

where $\epsilon_q = q^2/2m$ is the quasiparticle energy (of our additional electron), ϵ_F is the Fermi energy, and $1/\Gamma_q$ is the quasiparticle lifetime. In a 2DEG, $\Gamma_q \propto (\epsilon_q - \epsilon_F)^2 \log(\epsilon_q - \epsilon_F)$ [76] within the random phase approximation (RPA), which accounts for screening and which is obtained by summing all polarization diagrams [75]. Thus, the lifetime becomes infinite when the energy of the added electron approaches ϵ_F . Eq. (9) is valid for $0 \leq t \lesssim 1/\Gamma_q$, in which case the incoherent part of the Green's function is negligible. Now, we come to the most important quantity in the present context, the renormalization factor or quasiparticle weight, $z_F = z_{q_F}$, evaluated at the Fermi surface; it is defined by

$$z_F = \frac{1}{1 - \frac{\partial}{\partial \omega} \text{Re} \Sigma(q_F, \omega = 0)}, \quad (10)$$

where $\Sigma(q, \omega)$ is the irreducible self-energy occurring in the Dyson equation. The quasiparticle weight, $0 \leq z_q \leq 1$, describes the weight of the bare electron in the quasiparticle state \mathbf{q} , i.e. when we add an electron with energy $\epsilon_q \geq \epsilon_F$ to the system, some weight (given by $1 - z_q$) of the original state \mathbf{q} will be distributed among all the electrons due to the Coulomb interaction. This rearrangement of the Fermi system due to interactions happens very quickly, at a speed given approximately by the plasmon velocity, which exceeds the Fermi velocity (typically 10^5 m/s in GaAs). Restricting ourselves now to momenta close to the Fermi surface and to identical leads (i.e. $G_1 = G_2$) we then have

$$|G^{s/t}(q', q, t; k, k')| = z_F^2 |\delta_{qk}\delta_{q'k'} \pm \delta_{qk'}\delta_{q'k}| \quad (11)$$

for all times satisfying $0 < t \lesssim 1/\Gamma_q$. Thus we see that it is the quasiparticle weight squared, z_F^2 , which is the measure of our spin correlation function G^s we were looking

for. It is thus interesting to evaluate z_F explicitly. This is indeed possible, again within RPA, and we find after some calculation [32]

$$z_F = 1 - r_s \left(\frac{1}{2} + \frac{1}{\pi} \right), \quad (12)$$

in leading order of the interaction parameter $r_s = 1/q_F a_B$, where $a_B = \epsilon_0 \hbar^2 / m e^2$ is the Bohr radius. In particular, in a GaAs 2DEG we have $a_B = 10.3$ nm, and $r_s = 0.614$, and thus we obtain from (12) the value $z_F = 0.665$. We note that a more accurate numerical evaluation of the exact RPA self-energy yields $z_F = 0.691155$ [32], again for GaAs. [For 3D metallic leads with say $r_s = 2$ (e.g. $r_s^{Cu} = 2.67$) the loss of correlation is somewhat less strong, since then the quasiparticle weight becomes $z_F = 0.77$ [77].]

In summary, we see that the spin correlation is reduced by a factor of about two (from its maximum value one) as soon as we inject the two electrons (entangled or not) into separate leads consisting of *interacting* Fermi liquids in their ground state. These findings are quite encouraging in view of experimental investigations, as they demonstrate that the spin correlations of a pair of electrons in a Fermi liquid will indeed be preserved in time (albeit with a reduced amplitude) as long as we can neglect spin-dependent forces such as spin-orbit interaction and spin flips induced by spin impurities or nuclear spins etc. Given the high purity of present-day GaAs 2DEG's and the possibility of suppressing the dephasing effects of nuclear spins by dynamical spin polarization [50], it looks promising to use mobile electrons in nanostructures as a means for quantum communication. Similar investigations [32] of such spin correlations are under way for non-equilibrium transport situations, as well as for leads containing impurities or consisting of superconducting or non-Fermi liquid materials, etc.

In conclusion, we believe that various aspects of quantum communication have a high chance of being realized in the not-too-distant future. As we have seen, all that is needed is one single quantum gate which is attached to leads and which can be used as a source of entanglement for mobile qubits along the lines proposed here. Although the realization of such a device is still an experimental challenge at present we are optimistic that it is within technological reach.

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[1] We are indebted to an excellent monograph, G. Brassard,

- Modern Cryptology, A Tutorial* (Springer-Verlag, Lecture Notes in Computer Science, eds. G. Goos and J. Hartmanis, Vol. 325, New York, 1988), for knowledge about much of the subject matter of this section. Our discussion of information processing with quantum-mechanical tools is an update of Brassard's discussion in the light of the many new advances in quantum information processing in the last ten years. See also A. J. Menezes, P. C. van Oorschot, and S. A. Vanstone, *Handbook of Applied Cryptography* (CRC Press, 1996).
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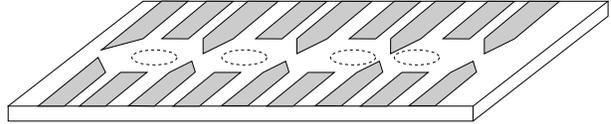


FIG. 1. A schematic of the quantum-dot array quantum computer. Single electrons are confined in a two-dimensional electron gas, and to dot regions in between the electrodes. Electrodes are shown shaded, dots are shown as dashed circles. The electrode potentials can be varied so as to push pairs of electrons into contact (see the third and fourth dots), which results in the execution of a two-bit quantum gate. One-bit gates are accomplished by the action of inhomogeneous magnetic fields (or effective fields). Readout is accomplished by tunneling the electrons through a spin-selective barrier. The magnetic elements of the device are not shown.