Physics of Condensed Matter I

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1100-4INZ`PC

Solid state 2

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Bravais lattice Regularna a = b = cIn three-dimensional space, $\alpha = \beta = \gamma = 90^{\circ}$ there are 14 Bravais lattices. Simple Face-centered Body-centered cubic cubic cubic They form 7 lattice systems $a = b \neq c$ Tetragonalna $\alpha = \beta = 90^{\circ}$ $a = b \neq c$ $\gamma = 120^{\circ}$ $\alpha = \beta = \gamma = 90^{\circ}$ Simple Body-centered Hexagonal tetragonal tetragonal Heksagonalna Rombowa $a \neq b \neq c$ $\alpha = \beta = \gamma = 90^{\circ}$ Face-centered Simple Body-centered **Base-centered** orthorhombic orthorhombic orthorhombic orthorhombic $a \neq b \neq c$ Jednoskośna Romboedryczna $\alpha \neq \beta \neq \gamma$ $a \neq b \neq c$ a = b = c $\alpha = \gamma = 90^{\circ}$ $\alpha = \beta = \gamma < 120^\circ \neq 90^\circ$ $\beta \neq 90^{\circ}$ Triclinic Base-centered Simple Trójskośna monoclinic Rhombohedral Monoclinic 2016-01-25

Planes in the crystal

A family of lattice planes are written (hkl), and denote the family of planes that intercepts the three points:

$$\frac{\vec{a}_1}{h}, \frac{\vec{a}_2}{k}, \frac{\vec{a}_3}{l}$$

If one of the indices is zero, it means that the planes do not intersect that axis (1/0 = infinity)

Also: the family of planes orthogonal to: $h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3$ Where $\vec{g}_1, \vec{g}_2, \vec{g}_3$ are reciprocal lattice vectors

(*hkl*) plane {*hkl*} set of planes [*hkl*] diections (*hkl*) set of directions







Planes in the crystal













Planes in the crystal

The crystalline structure is studied by means of the diffraction of photons, neutrons, electrons or other light particles



Crystals

1912 - Max von Laue noted that the wavelength of X-rays are comparable to the distances between atoms in the crystal. This suggestion was quickly confirmed by Walter Friedrich and Paul Knipping



Max von Laue 1879 - 1960

Model of the crystal: set of discrete parallel planes separated by a constant parameter *d*

 $2dsin\theta = n\lambda$

William Lawrence Bragg (son) and William Henry Bragg (father), 1913

e.g . λ =1,54 Å, d = 4 Å, crystals with cubic symmetry, the first reflex θ = 11°





Brehmsstrahlung – promieniowanie hamowania



Crystals



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Laue method

- The crystal is illuminated with white light.
- As a result of scattering the waves of different wavelengths are distributed in different directions. We get different points for different colors (wavelengths).
- the pattern of the spots has a symmetry of the crystal along the direction of the incident wave



Debaye-Scherer method







Peter Joseph Debye 1884 – 1966 Paul Scherrer 1890 - 1969

Typowy debajogram

Debaye-Scherer method

The powder of the crystals with chaotic orientation in space is measured. It is illuminated by the monochromatic wave. X-rays scattered by the differently oriented crystals creates arcs corresponding to the planes on which the X-ray wave was scattered.





Typowy debajogram

Atomic form factor (czynnik atomowy)



Both salts have the same crystal structure, but different diffraction, why?

Atomic form factor (czynnik atomowy)



different diffraction, why?

Atomic form factor (czynnik atomowy)



- K⁺ and Cl⁻ have the same number of electrons. They scatter similarly X-rays.
- For certain directions the destructive interference occurs (total extinction)
- Na⁺ and Cl⁻ waves are scattered by atoms with different electrons, no total extinction.
- Thus there is atomic form factor



Atomic form factor (czynnik atomowy)



Rys. 8.2. Dyfrakcja fali elektromagnetycznej przez chmurę gęstości ładunku elektronów w atomie

Elsatic scattering of X-rays by electron cloud: i.e. local electron concnetration $\rho(\vec{\xi})$

$$\left|\vec{k}\right| = \left|\vec{k}'\right| = k$$



Rys. 8.3. Oznaczenia używane w obliczeniach

$$\Delta_{1} = \xi \cos \alpha = \xi \frac{\vec{k}\vec{\xi}}{k\xi} = \frac{\vec{k}\vec{\xi}}{k}$$
$$\Delta_{2} = \xi \cos \alpha' = \xi \frac{\vec{k}'\vec{\xi}}{k\xi} = \frac{\vec{k}'\vec{\xi}}{k}$$
$$\Delta = (\Delta_{2} - \Delta_{1}) = \frac{(\vec{k}' - \vec{k})\vec{\xi}}{k} = \frac{\Delta \vec{k} \vec{\xi}}{k}$$
$$\varphi = \frac{2\pi\Delta}{\lambda} = k\Delta = \Delta \vec{k} \vec{\xi}$$

Atomic form factor (czynnik atomowy)

$$\Delta = (\Delta_2 - \Delta_1) = \frac{(\vec{k'} - \vec{k})\vec{\xi}}{k} = \frac{\Delta \vec{k} \cdot \vec{\xi}}{k}$$
$$\varphi = \frac{2\pi\Delta}{\lambda} = k\Delta = \Delta \vec{k} \cdot \vec{\xi}$$
Charge density in $\vec{\xi} = 0$
$$\Psi(0) = \frac{A}{r} \exp[i(\vec{k}\vec{r} - \omega t)]\rho_e(\vec{\xi} = 0)$$

$$\Psi(\vec{\xi}) = \frac{A}{r} \exp[i(\vec{k}\vec{r} - \omega t - \Delta \vec{k}\vec{\xi})]\rho_e(\vec{\xi})$$



Rys. 8.3. Oznaczenia używane w obliczeniach

Scattered wave:

$$\Psi(\vec{\xi})d\vec{\xi} = \frac{A}{r}\exp[i(\vec{k}\vec{r} - \omega t)]\rho_e(\vec{\xi})\exp[-i\Delta\vec{k}\vec{\xi}]d\vec{\xi}$$
Density of charge
$$Atomic \text{ form factor} \quad f = -\frac{1}{e}\int \rho_e(\vec{\xi})\exp[-i\Delta\vec{k}\vec{\xi}]d\vec{\xi}$$

Atomic form factor (czynnik atomowy)

For instance spherical distribution of electrons

$$f = -\frac{1}{e} \int \rho_e(\vec{\xi}) \exp\left[-i\Delta \vec{k}\vec{\xi}\right] d\vec{\xi} = -\frac{1}{e} 2\pi \int \rho_e(\vec{\xi}) \exp\left[-\Delta \vec{k}\vec{\xi}\right] \xi^2 d(\cos\theta) d\xi$$

$$= -\frac{2\pi}{e} \int \xi^2 \rho_e(\vec{\xi}) \frac{\exp[\Delta k\xi] - \exp[-\Delta k\xi]}{ik\xi} d\xi = -\frac{4\pi}{e} \int \xi^2 \rho_e(\vec{\xi}) \frac{\sin \Delta k\xi}{k\xi} d\xi$$

For small angles of scattering $\Delta k \xi \rightarrow 0$ and f = -Z

The atomic scattering factor f is the ratio of the amplitude of the radiation scattered by the actual distribution of electrons in the atom to the amplitude of the radiation scattered by one electron.

Atomic form factor
$$f = -\frac{1}{e} \int \rho_e(\vec{\xi}) \exp[-i\Delta \vec{k}\vec{\xi}] d^3\xi$$

Atomic form factor (czynnik atomowy)



For small angles of scattering ef = Q (total charge)

1A (1)]	Key:														7A	8A
ן י י	2A (2)	Metals Nonmetals											4A (14)	5A (15)	6A (16)	H	He
Li	Ве	Metalloids										в	С	N	0	F	Ne
Na	Mg	3B (3)	4B (4)	5B (5)	6B (6)	7B (7)	(8)	- 8B - (9)	(10)	1B (11)	2B (12)	AI	Si	Р	s	CI	Ar
к	Ca	Sc	Ti	v	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Мо	Тс	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Те	T	Xe
Cs	Ва	La	Hf	Та	w	Re	Os	lr	Pt	Au	Hg	ті	Pb	Bi	Ро	At	Rn
Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	112	113	114	115	110		
		/															
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb	Lu		
		Th	Ра	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		
			15 1	_				1			0		1				

$$f = -\frac{1}{e} \int \rho_e(\vec{\xi}) \exp\left[-i\Delta \vec{k}\vec{\xi}\right] d^3\xi$$

Wave scattered on one of the atoms (j)

$$\Psi = A \; e^{i\left(\vec{k}'\vec{r} - \omega t\right)} f_j$$

Wave scattered on all atoms (in direction \vec{k}'):



$$\vec{R}_{0j}$$
Basis $\vec{R}_{nj} = \vec{R}_{0j} + \vec{T}$





$$\Psi = A \ e^{i(\vec{k}'\vec{r}-\omega t)} f_j$$

Wave scattered on all atoms:
$$\Psi = A \ \sum_n \sum_j e^{i(\vec{k}'\vec{r}-\omega t)} f_j e^{-i\Delta \vec{k} \vec{R}_{nj}}$$

Atoms in basis



$$\Psi = A \ e^{i(\vec{k}'\vec{r} - \omega t)} f_j$$

Wave scattered on all atoms:
$$\Psi = A \sum_{n} \sum_{j} e^{i(\vec{k}'\vec{r} - \omega t)} f_j \ e^{-i\Delta \vec{k} \vec{R}_{nj}}$$

Atoms in basis
Period of the lattice



$$\Psi = A \ e^{i(\vec{k}'\vec{r} - \omega t)} f_j$$

Wave scattered on all atoms:
$$\Psi = A \sum_{n} \sum_{j} e^{i(\vec{k}'\vec{r} - \omega t)} f_j \ e^{-i\Delta \vec{k}(\vec{R}_{0j} + \vec{T})}$$

Atoms in basis
Period of the lattice



$$\Psi = A e^{i(\vec{k}'\vec{r}-\omega t)} f_j$$

Wave scattered on all atoms:
$$\Psi = A \sum_{n} \sum_{j} e^{i(\vec{k}'\vec{r}-\omega t)} f_j e^{-i\Delta \vec{k}(\vec{R}_{0j}+\vec{T})} =$$
$$= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_{n} \sum_{j} f_j e^{-i\Delta \vec{k}(\vec{R}_{0j})} e^{-i\Delta \vec{k}(\vec{T})} =$$

$$\begin{split} \Psi &= A \ e^{i(\vec{k}'\vec{r}-\omega t)} f_j \\ \text{Wave scattered on all atoms:} \\ \Psi &= A \ \sum_n \sum_j e^{i(\vec{k}'\vec{r}-\omega t)} f_j \ e^{-i\Delta \vec{k}(\vec{R}_{0j}+\vec{T})} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_n \sum_j f_j \ e^{-i\Delta \vec{k}(\vec{R}_{0j})} e^{-i\Delta \vec{k}(\vec{T})} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_n \sum_j f_j \ e^{-i\Delta \vec{k}(\vec{R}_{0j})} \sum_n e^{-i\Delta \vec{k}(n_1 \vec{t}_1 + n_2 \vec{t}_2 + n_3 \vec{t}_3)} = \end{split}$$

$$\begin{split} \Psi &= A e^{i(\vec{k}'\vec{r}-\omega t)} f_j \\ \text{Wave scattered on all atoms:} \\ \Psi &= A \sum_n \sum_j e^{i(\vec{k}'\vec{r}-\omega t)} f_j e^{-i\Delta \vec{k}(\vec{R}_0 j + \vec{T})} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_n \sum_j f_j e^{-i\Delta \vec{k}(\vec{R}_0 j)} e^{-i\Delta \vec{k}(\vec{T})} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_j f_j e^{-i\Delta \vec{k}(\vec{R}_0 j)} \sum_n e^{-i\Delta \vec{k}(n_1 \vec{t}_1 + n_2 \vec{t}_2 + n_3 \vec{t}_3)} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \sum_j f_j e^{-i\Delta \vec{k}(\vec{R}_0 j)} \sum_n e^{-i\Delta \vec{k}(n_1 \vec{t}_1 + n_2 \vec{t}_2 + n_3 \vec{t}_3)} = \\ &= A e^{i(\vec{k}'\vec{r}-\omega t)} \left[\sum_j f_j e^{-i\Delta \vec{k}(\vec{R}_0 j)} \right] \left[\sum_{n_1} e^{-i\Delta \vec{k}(n_1 \vec{t}_1)} \right] \left[\sum_{n_2} e^{-i\Delta \vec{k}(n_2 \vec{t}_2)} \right] \left[\sum_{n_3} e^{-i\Delta \vec{k}(n_3 \vec{t}_3)} \right] \end{split}$$

Maximal value of the intensity

$$\left[\sum_{n_1} e^{-i\Delta \vec{k}(n_1 \vec{t}_1)}\right] \left[\sum_{n_2} e^{-i\Delta \vec{k}(n_2 \vec{t}_2)}\right] \left[\sum_{n_3} e^{-i\Delta \vec{k}(n_3 \vec{t}_3)}\right]$$

When?



Maximal value of the intensity

$$\left[\sum_{n_1} e^{-i\Delta \vec{k}(n_1 \vec{t}_1)}\right] \left[\sum_{n_2} e^{-i\Delta \vec{k}(n_2 \vec{t}_2)}\right] \left[\sum_{n_3} e^{-i\Delta \vec{k}(n_3 \vec{t}_3)}\right]$$

When $e^{-i\Delta \vec{k}(n_1 \vec{t}_1)} = 1$

 $\Delta \vec{k} \vec{t}_1 = 2\pi h$ $\Delta \vec{k} \vec{t}_2 = 2\pi k$ Laue conditions $\Delta \vec{k} \vec{t}_3 = 2\pi l$

$$\Delta \vec{k}(n_3 \vec{t}_3) \bigg]$$

$$\vec{T} = n_1 \vec{t}_1 + n_2 \vec{t}_2 + n_3 \vec{t}_3$$

Maximal value of the intensity

$$\left[\sum_{n_1} e^{-i\Delta \vec{k}(n_1 \vec{t}_1)}\right] \left[\sum_{n_2} e^{-i\Delta \vec{k}(n_2 \vec{t}_2)}\right] \left[\sum_{n_3} e^{-i\Delta \vec{k}(n_3 \vec{t}_3)}\right]$$

When $e^{-i\Delta \vec{k}(n_1 \vec{t}_1)} = 1$

 $\Delta \vec{k} \vec{t}_1 = 2\pi h$

 $\Delta \vec{k} \vec{t}_2 = 2\pi k$

 $\Delta \vec{k} \vec{t}_3 = 2\pi l$

Laue conditions

$$\Delta \vec{k} \equiv G = h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3 \qquad \vec{g}_i\vec{t}_j = 2\pi\delta_{ij}$$

$$|\vec{g}_i| = \frac{2\pi}{a_i} \qquad \vec{g}_i = 2\pi \ \frac{\vec{t}_j \times \vec{t}_k}{\vec{t}_i (\vec{t}_j \times \vec{t}_k)}$$

reciprocal lattice

 $\vec{T} = n_1 \vec{t}_1 + n_2 \vec{t}_2 + n_3 \vec{t}_3$

Structure factor
$$S_G$$
 $S_G = \int_{cell} dV \rho(\vec{R}) e^{-i\vec{G}\vec{R}}$



Structure factor

$$F(h,k,l) = \sum_{j} f_{j} e^{-i2\pi(n_{1}h+n_{2}k+n_{3}l)}$$

The crystal of Li and TlBr (bcc lattice - body centered cubic) – find the possible values of the structure factor

$$r_1 = (0,0,0)$$
 $r_2 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$



$$\begin{split} F_{Li}(h,k,l) &= \sum_{j} f_{j} \; e^{-i2\pi(n_{1}h+n_{2}k+n_{3}l)} = f_{Li}e^{-i2\pi(0+0+0)} + f_{Li}e^{-i2\pi\left(\frac{1}{2}h+\frac{1}{2}k+\frac{1}{2}l\right)} & \text{odd} \\ F_{Li}(h,k,l) &= f_{Li}\left(1+e^{-i\pi(h+k+l)}\right) & \text{odd} \\ F_{TlBr}(h,k,l) &= \sum_{j} f_{j} \; e^{-i2\pi(n_{1}h+n_{2}k+n_{3}l)} = f_{Tl}e^{-i2\pi(0+0+0)} + f_{Br}e^{-i2\pi\left(\frac{1}{2}h+\frac{1}{2}k+\frac{1}{2}l\right)} & \text{odd} \\ F_{TlBr}(h,k,l) &= f_{Tl} + f_{Br}e^{-i\pi(h+k+l)} & \text{odd} \end{split}$$

even

Neutrons

Neutrons - generated in the reactor are slowed down by collisions with the moderator (graphite) to v = 4 km/s, which corresponds to the energy E = 0.08 eV and that energy that corresponds to $\lambda = 1$ Å

The neutrons interact with: nuclei (one can determine the density of the probability of finding nuclei), determine the phonon dispersion curves, the magnetic moments of nuclei.



$$E = \frac{\hbar^2}{2M\lambda^2}$$

$$M = 1,675 \times 10^{-24} \text{ g}$$

$$\lambda(A) = \frac{0,28}{\sqrt{E(eV)}}$$

1 Å for *E*=0,08 eV



J. Ginter

Electrons

Electrons have an electric charge and interact strongly with matter, penetrate it very shallow. The phenomenon of diffraction of electrons allows for structural studies of surfaces and very thin layers E =





T. Stacewicz & A. Witowski

Electrons



Electrons

Rafał Dunin-Borkowski

Magnetic domains in a thin cobalt

film The colors in the image show the different directions of the magnetic field in a layer of polycrystalline cobalt that has a thickness of only 20 nm. The field of view is approximately 200 microns



http://www.rafaldb.com/pictures-micrographs/index.html

Electrons

Rafał Dunin-Borkowski

Magnetic nanotubes. The nanotubes were fabricated in the University of Cambridge Engineering department by Yasuhiko Hayashi, who grew them using a Cobalt-Palladium catalyst. This alloy remains present in the ends of the nanotubes, and is magnetic. The nanotubes you see here have a 70-100 nm diameter.



http://www.rafaldb.com/pictures-micrographs/index.html

Electrons

Rafał Dunin-Borkowski

This image won First Prize in the "Science Close-Up" category in the Daily Telegraph Visions of Science competition. The image shows a multi-walled carbon nanotube, approximately 190 nm in diameter, containing a 35-nm-diameter iron crystal encapsulated inside it. Electron holography has been used to obtain a map of the magnetic field surrounding the iron particle, at a spatial resolution of approximately 5 nm.



http://www.rafaldb.com/pictures-micrographs/index.html

Electrons

Rafał Dunin-Borkowski

The image shows the magnetic field lines in a single **magnetosome chains** in a **bacterial cell**. The fine white lines are the magnetic field lines in the cell, which were measured using offaxis electron holography.



http://www.rafaldb.com/pictures-micrographs/index.html