Entanglement-based effects in two-photon propagation

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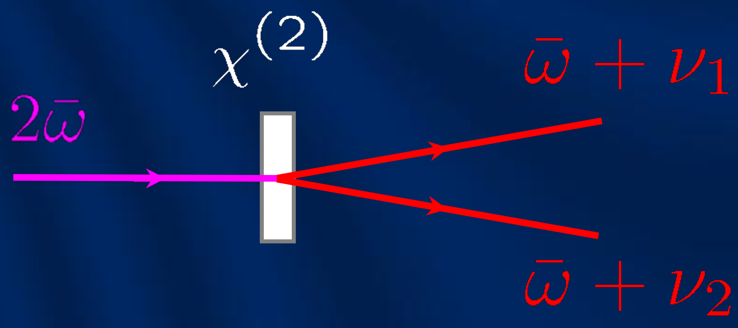
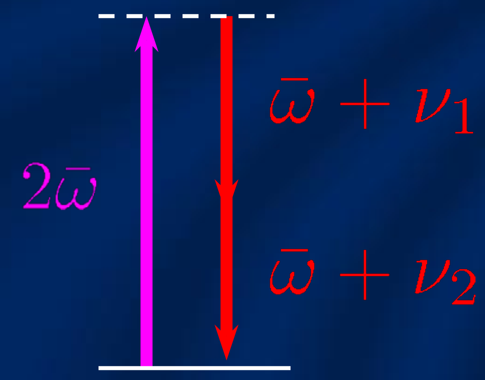
Parametric down-conversion





Energy conservation: $\nu_1 = -\nu_2$

Very approximate two-photon wave function assuming

- broadband phase matching
- cw pump

$$\tilde{\psi}(\nu_1, \nu_2) \propto \delta(\nu_1 + \nu_2)$$

Temporal domain

When:

- The spectrum is not too broad compared to the central frequency
- Observations are coarse-grained in time with respect to the optical oscillation period

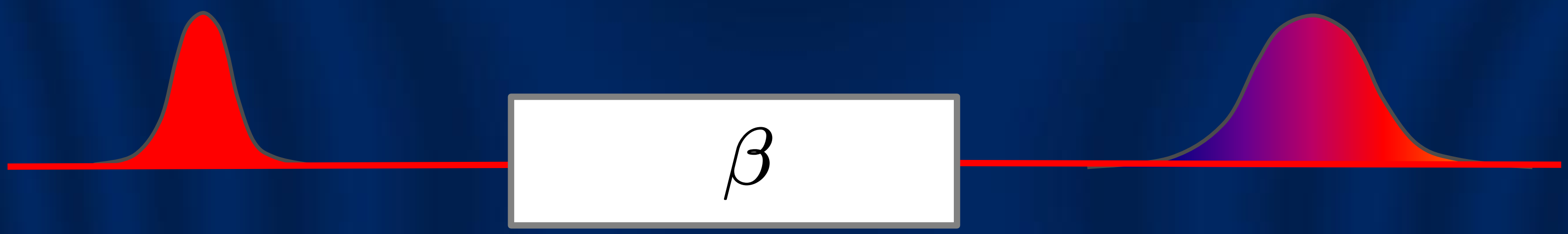
$$\psi(t_1, t_2) \propto \int \mathrm{d}\nu_1 \int \mathrm{d}\nu_2 \, \tilde{\psi}(\nu_1, \nu_2) \mathrm{e}^{-\mathrm{i}\nu_1 t_1 - \mathrm{i}\nu_2 t_2}$$
$$\propto \delta(t_1 - t_2)$$

Spectral anticorrelations $\Leftrightarrow$ Temporal correlations

(as long as the two-photon state is pure)

Group velocity dispersion

Light pulse centered at $\bar{\omega}$:



Transformation of a classical spectral amplitude:

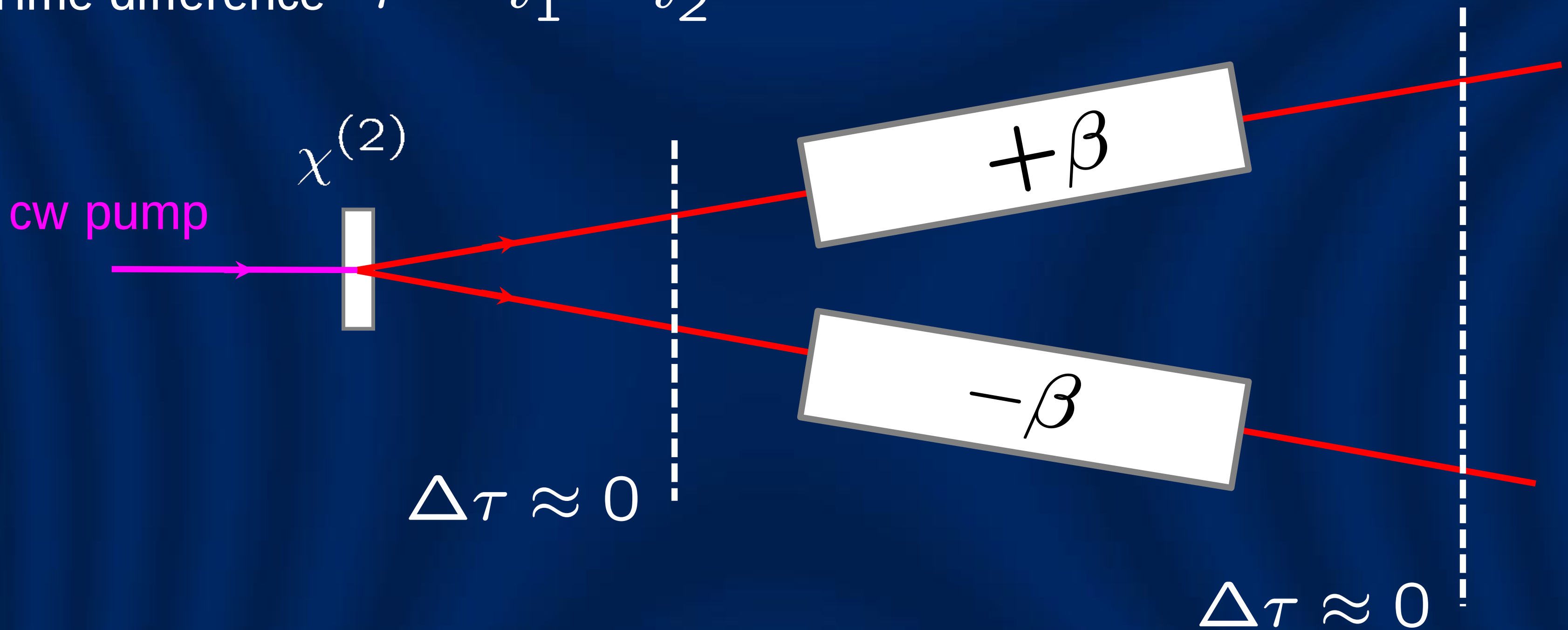
$$\alpha(\bar{\omega}+\nu) \rightarrow \mathrm{e}^{\mathrm{i}\beta\nu^2}\alpha(\bar{\omega}+\nu)$$

(plus time shift introduced by the group velocity)

Nonlocal dispersion cancellation

J. Franson, Phys. Rev. A **45**, 3126 (1992)

Time difference $\tau = t_1 - t_2$



$$\tilde{\psi}(\nu_1, \nu_2) \propto \delta(\nu_1 + \nu_2)$$

At the output:

$$\tilde{\psi}'(\nu_1, \nu_2) \propto \mathrm{e}^{\mathrm{i}\beta\nu_1^2}\mathrm{e}^{-\mathrm{i}\beta\nu_2^2}\delta(\nu_1 + \nu_2) = \delta(\nu_1 + \nu_2)$$

Classical fields

Electric field:

$$E(t) = \mathrm{e}^{-\mathrm{i}\bar{\omega}t}\mathcal{E}(t) + \mathrm{e}^{-\mathrm{i}\bar{\omega}t}\mathcal{E}^*(t)$$

Spectral amplitude:

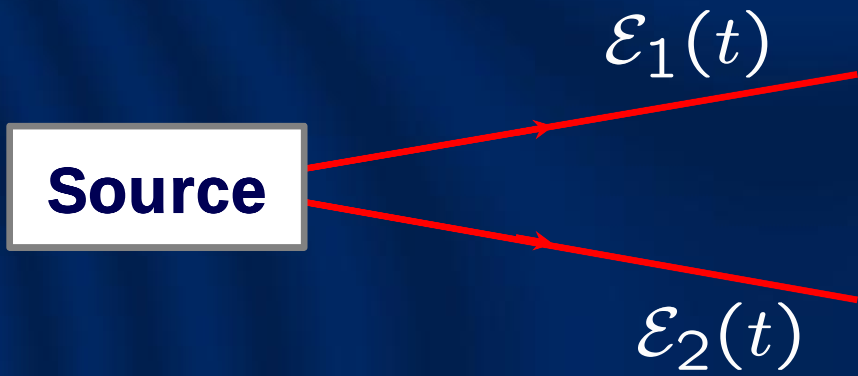
$$\tilde{\mathcal{E}}(\nu) = \int \mathrm{d}t\,\mathcal{E}(t)\mathrm{e}^{-\mathrm{i}\nu t}$$

Photocount probability:

$$p(t)\Delta t \propto |\mathcal{E}(t)|^2 \Delta t$$

Correlated classical fields

V. Torres-Company *et al*., New. J. Phys. **11**, 063041 (2009);
J. H. Shapiro, Phys. Rev. A **81**, 023824 (2010); J. D. Franson, *ibid*., 023825 (2010)



- Stochastic
- Time stationary
- Gaussian
- Zero mean

Non-vanishing second moments:

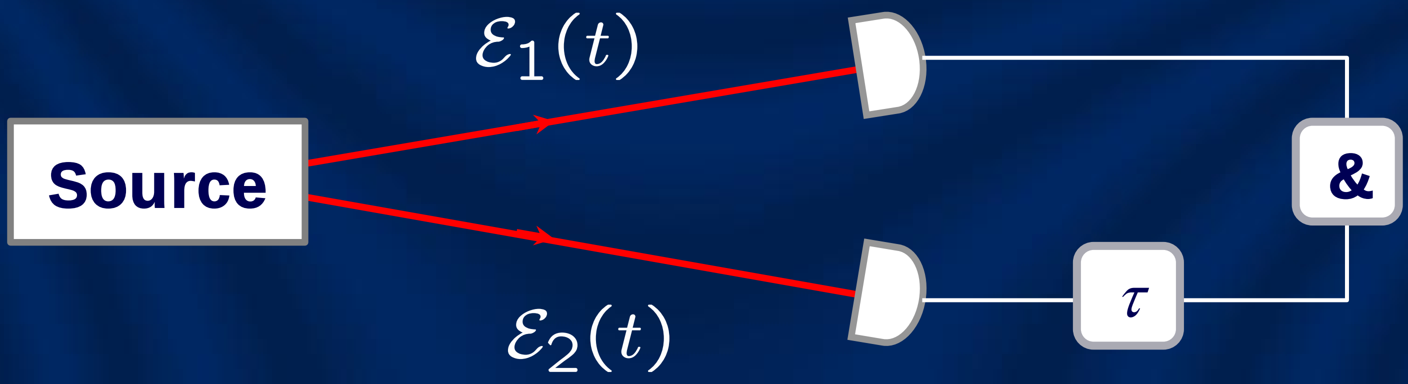
Phase-independent autocorrelation function:

$$S_j(\tau) = \left\langle \mathcal{E}_j^*(t+\tau)\mathcal{E}_j(t) \right\rangle, \qquad j = 1, 2$$

Phase-dependent cross-correlation function:

$$S_X(\tau) = \left\langle \mathcal{E}_1(t+\tau)\mathcal{E}_2(t) \right\rangle$$

Coincidences



Probability of a coincidence:

$$p(t+\tau, t) \propto \left\langle |\mathcal{E}_1(t+\tau)|^2 |\mathcal{E}_2(t)|^2 \right\rangle$$

$$= \left\langle |\mathcal{E}_1(t+\tau)|^2 \right\rangle \left\langle |\mathcal{E}_2(t)|^2 \right\rangle + \left| \left\langle \mathcal{E}_1(t+\tau) \mathcal{E}_2(t) \right\rangle \right|^2$$

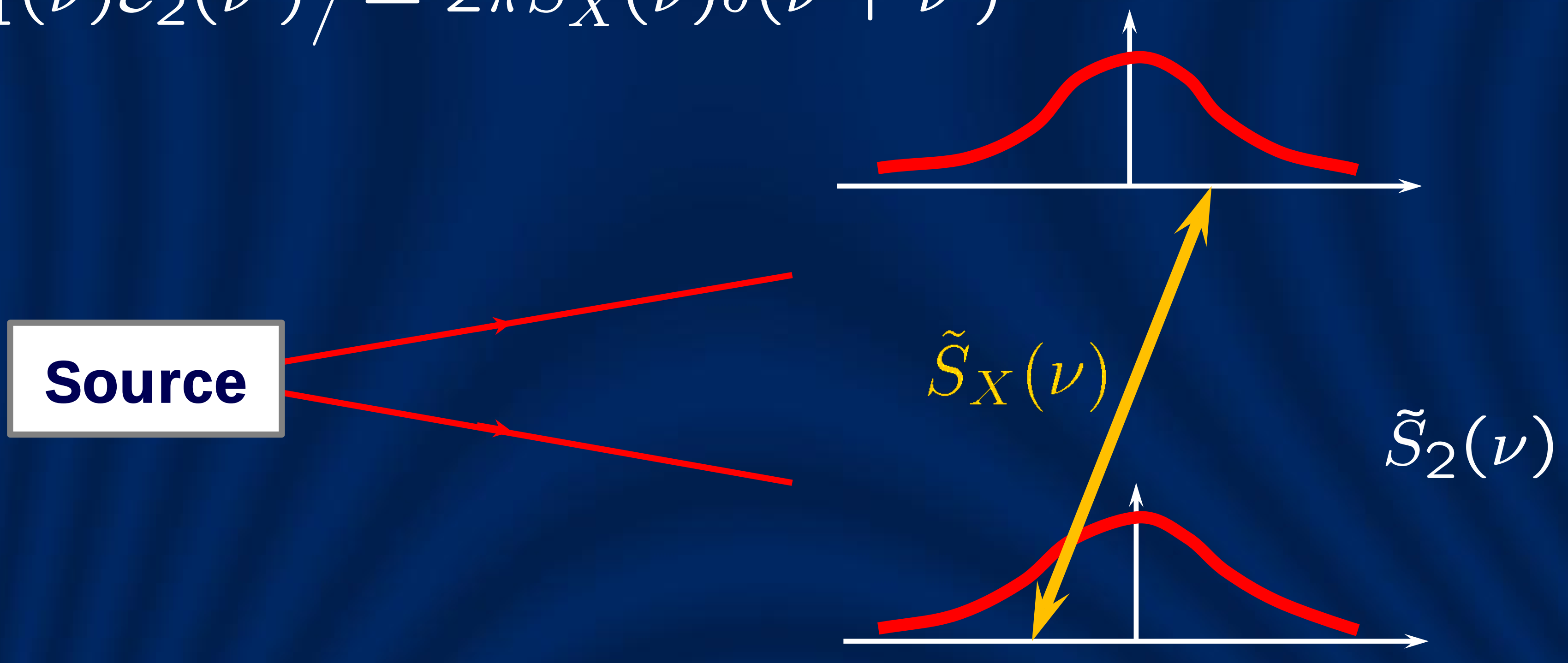
$$= S_1(\tau) S_2(0) + |S_X(\tau)|^2$$

Spectral correlations

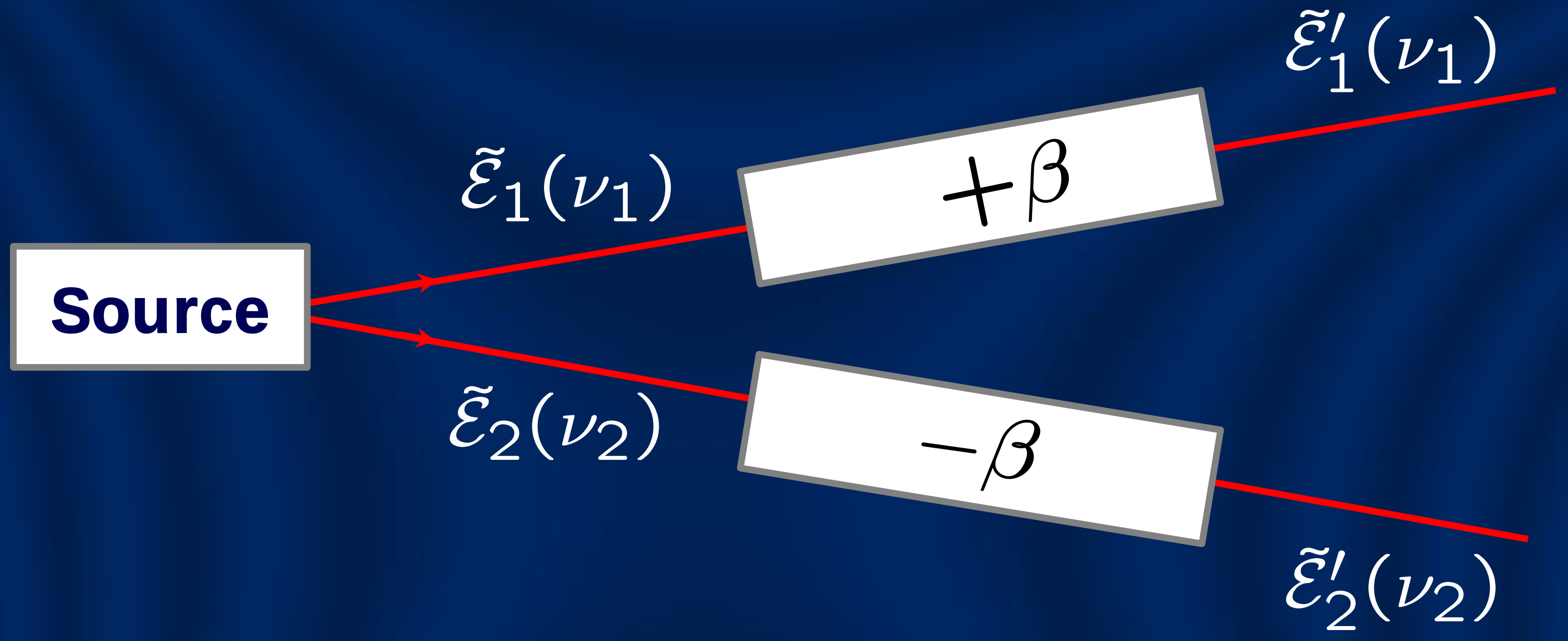
$$\tilde{S}_\square(\nu) = \int \mathsf{d}\tau\, \mathsf{e}^{-\mathsf{i}\nu\tau} S_\square(\tau)$$

$$\left\langle \tilde{\mathcal{E}}_j^*(\nu)\tilde{\mathcal{E}}_j(\nu')\right\rangle = 2\pi \tilde{S}_j(\nu)\delta(\nu - \nu'), \qquad j = 1, 2$$

$$\left\langle \tilde{\mathcal{E}}_1(\nu)\tilde{\mathcal{E}}_2(\nu')\right\rangle = 2\pi \tilde{S}_X(\nu)\delta(\nu + \nu')$$



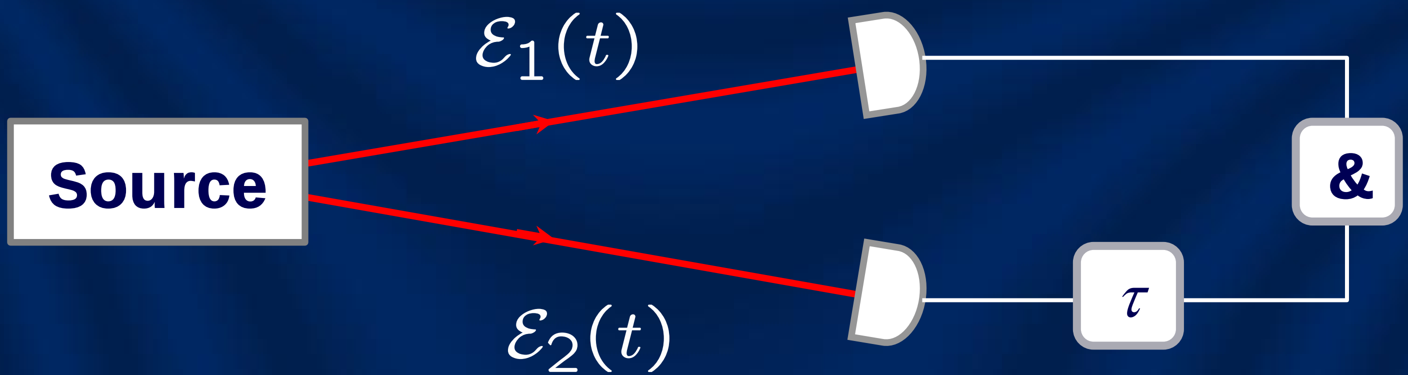
Dispersion



$$\left\langle \tilde{\mathcal{E}}'_1(\nu_1)\tilde{\mathcal{E}}'_2(\nu_2) \right\rangle = \mathrm{e}^{\mathrm{i}\beta(\nu_1^2-\nu_2^2)} 2\pi \tilde{S}_X(\nu_1)\delta(\nu_1+\nu_2)$$
$$= \left\langle \tilde{\mathcal{E}}_1(\nu_1)\tilde{\mathcal{E}}_2(\nu_2) \right\rangle$$

hence temporal correlations are preserved!

Background



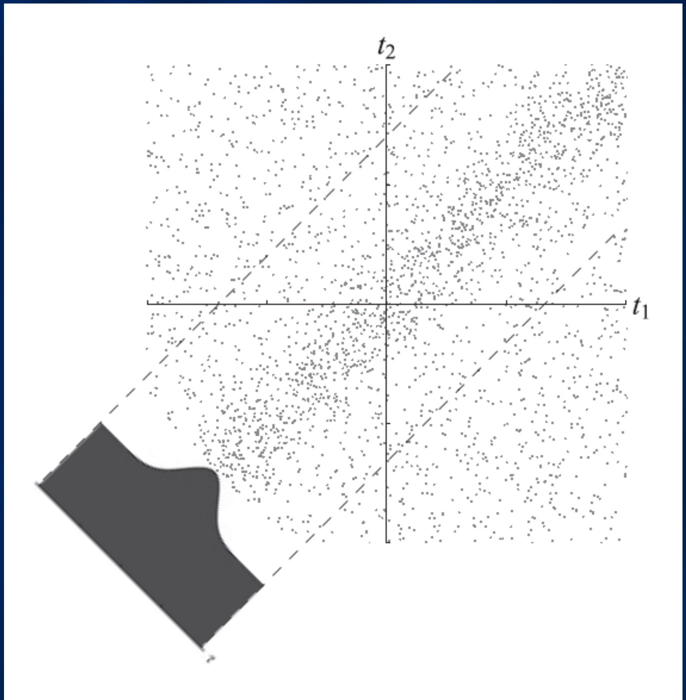
$$p(t+\tau, t) \propto S_1(\tau)S_2(0) + |S_X(\tau)|^2$$

Schwarz inequality:

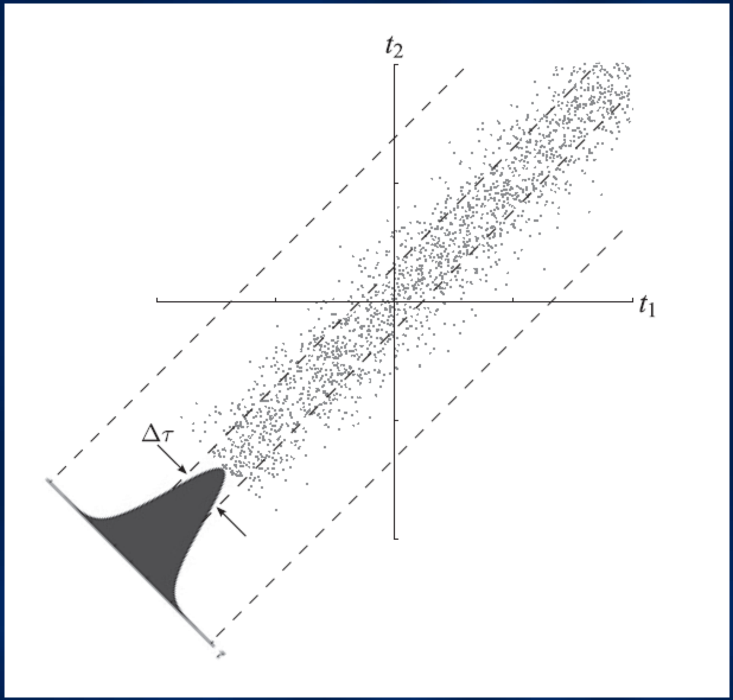
$$|S_X(\tau)|^2 = \left|\left\langle \mathcal{E}_1(t+\tau)\mathcal{E}_2(t)\right\rangle\right|^2$$
$$\leq \left\langle |\mathcal{E}_1(t+\tau)|^2\right\rangle\left\langle |\mathcal{E}_2(t)|^2\right\rangle = S_1(\tau)S_2(0)$$

Classical vs quantum correlations

Classical fields

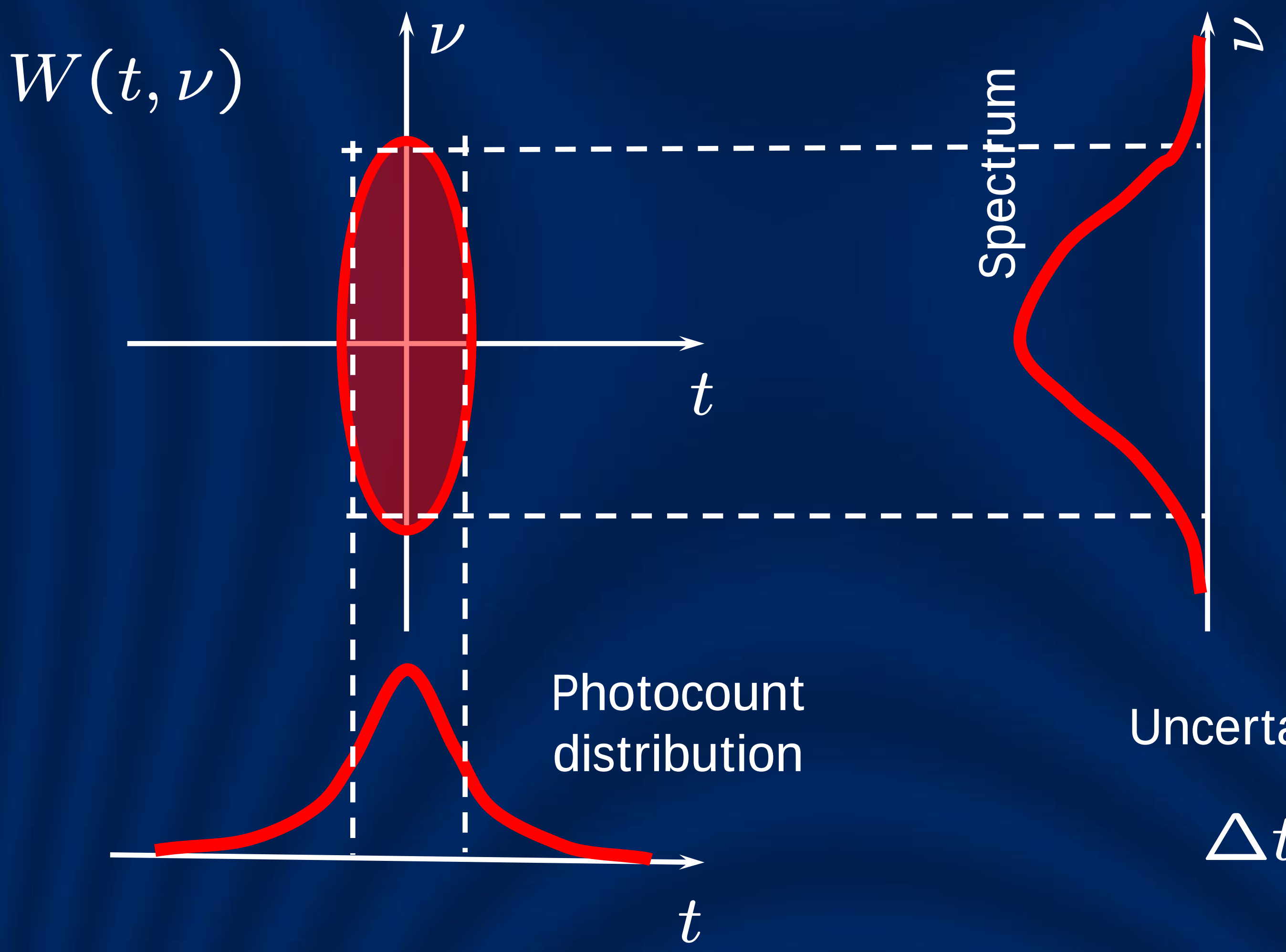


Down-converted photons



Chronocyclic Wigner function

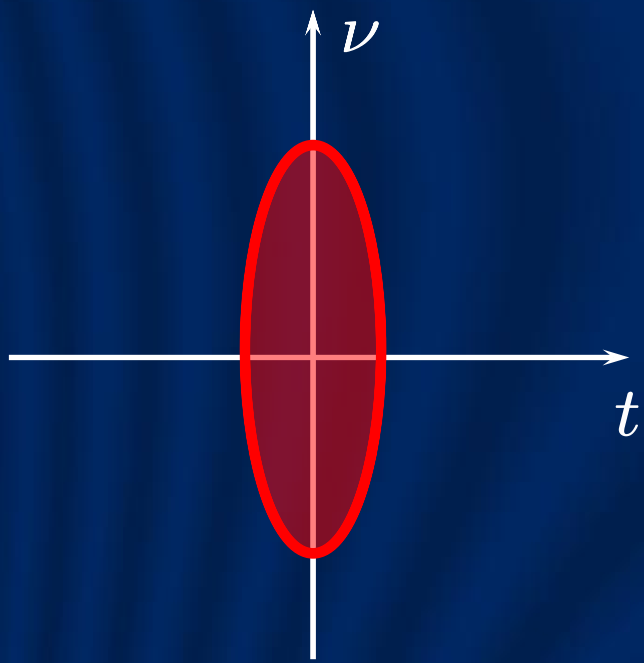
$$W(t,\nu) = \frac{1}{2\pi}\int \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\nu\tau}\left\langle \hat{\mathcal{E}}^{\dagger}(t-\tfrac{\tau}{2})\hat{\mathcal{E}}(t+\tfrac{\tau}{2})\right\rangle$$



Uncertainty relation

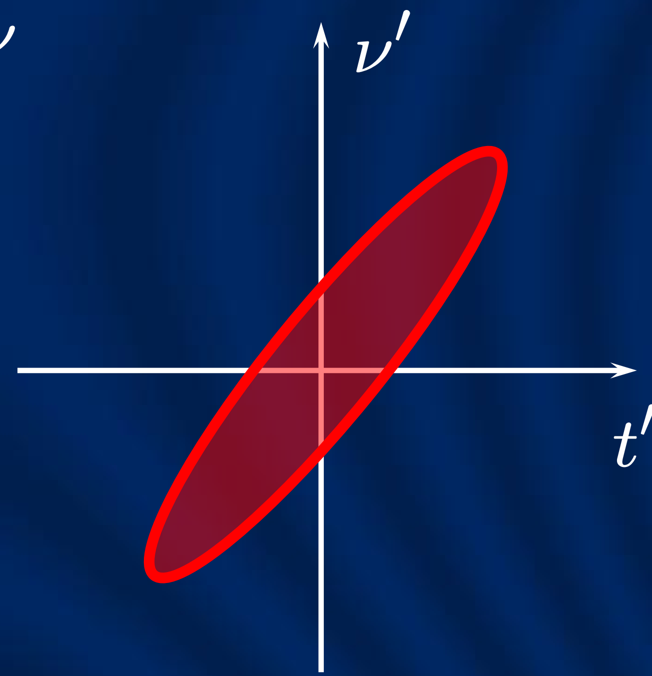
$$\Delta t\, \Delta\nu \geq \frac{1}{2}$$

Dispersion

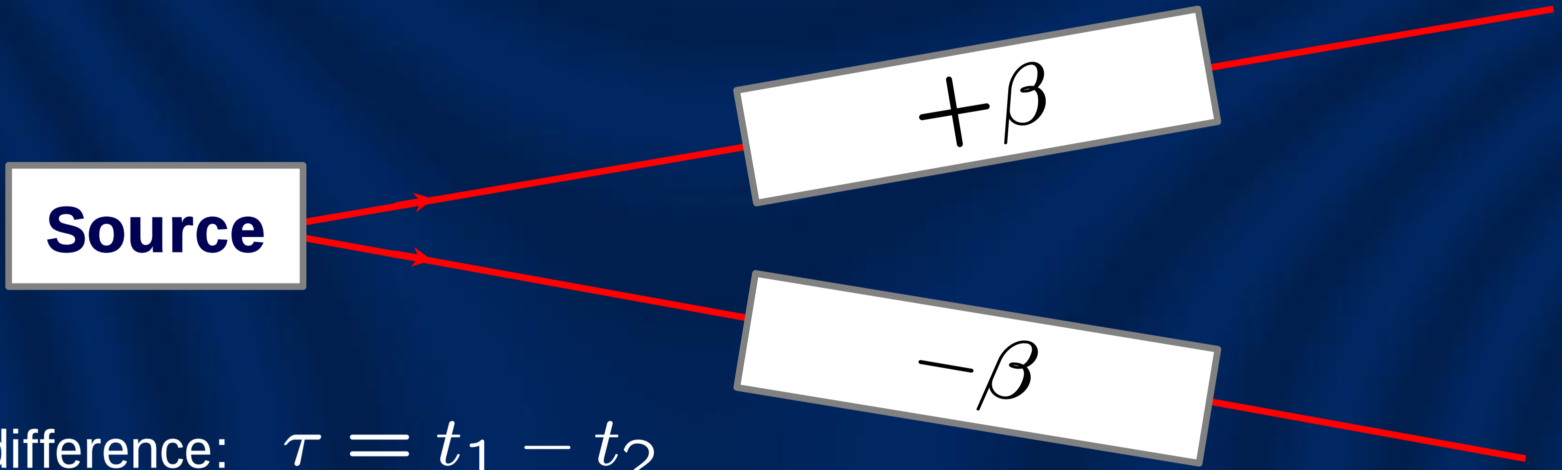


$$t' = t + 2\beta\nu$$

$$\nu' = \nu$$



Variances



Time difference: $\tau = t_1 - t_2$

Sum frequency: $\Omega = \nu_1 + \nu_2$

Propagation:

$$\langle(\Delta\tau')^2\rangle = \langle(\Delta\tau)^2\rangle + 4\beta\langle\Delta\tau\Delta\Omega\rangle + 4\beta^2\langle(\Delta\Omega)^2\rangle$$

After symmetrization 1 $\leftrightarrow$ 2:

$$\langle(\Delta\tau')^2\rangle_{\mathrm{sym}} = \langle(\Delta\tau)^2\rangle + 4\beta^2\langle(\Delta\Omega)^2\rangle$$

Separability criterion

S. M. Tan, Phys. Rev. A **60**, 2752 (1999)
S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005)

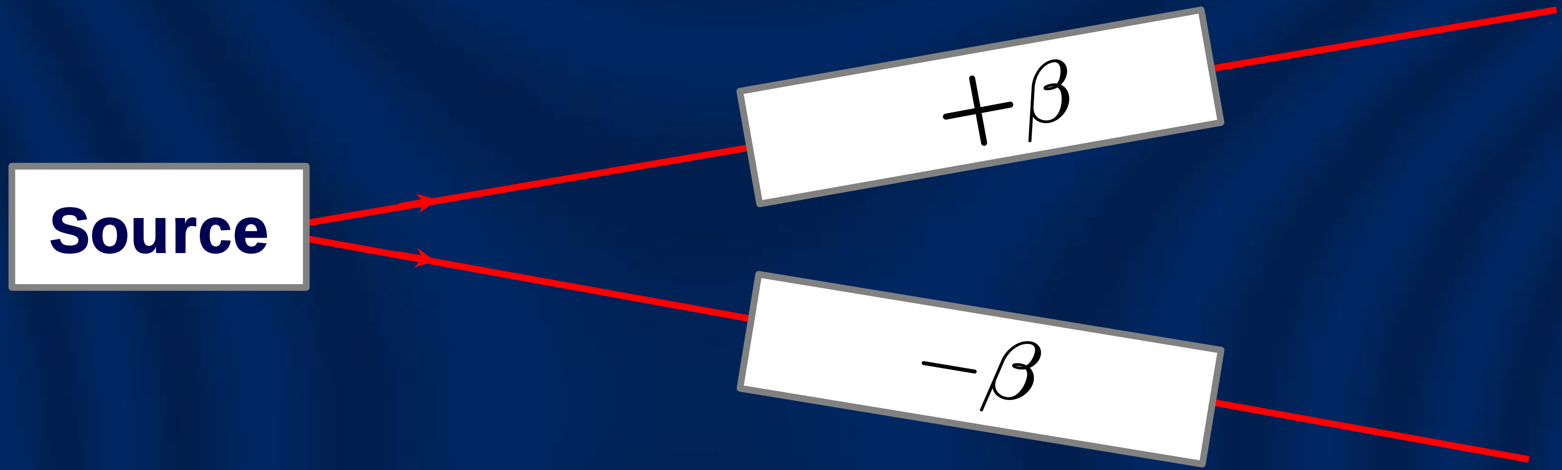
If two beams are uncorrelated, i.e. $\hat{\varrho} = \hat{\varrho}_1 \otimes \hat{\varrho}_2$

$$\langle(\Delta\tau)^2\rangle\langle(\Delta\Omega)^2\rangle \geq 1$$

This holds also for any separable state, when $\hat{\varrho} = \sum_i p_i \, \hat{\varrho}_1^{(i)} \otimes \hat{\varrho}_2^{(i)}$

Minimum broadening

T. Wasak, P. Szańkowski, W. Wasilewski, and K. Banaszek, Phys. Rev. A **82**, 052120 (2010)

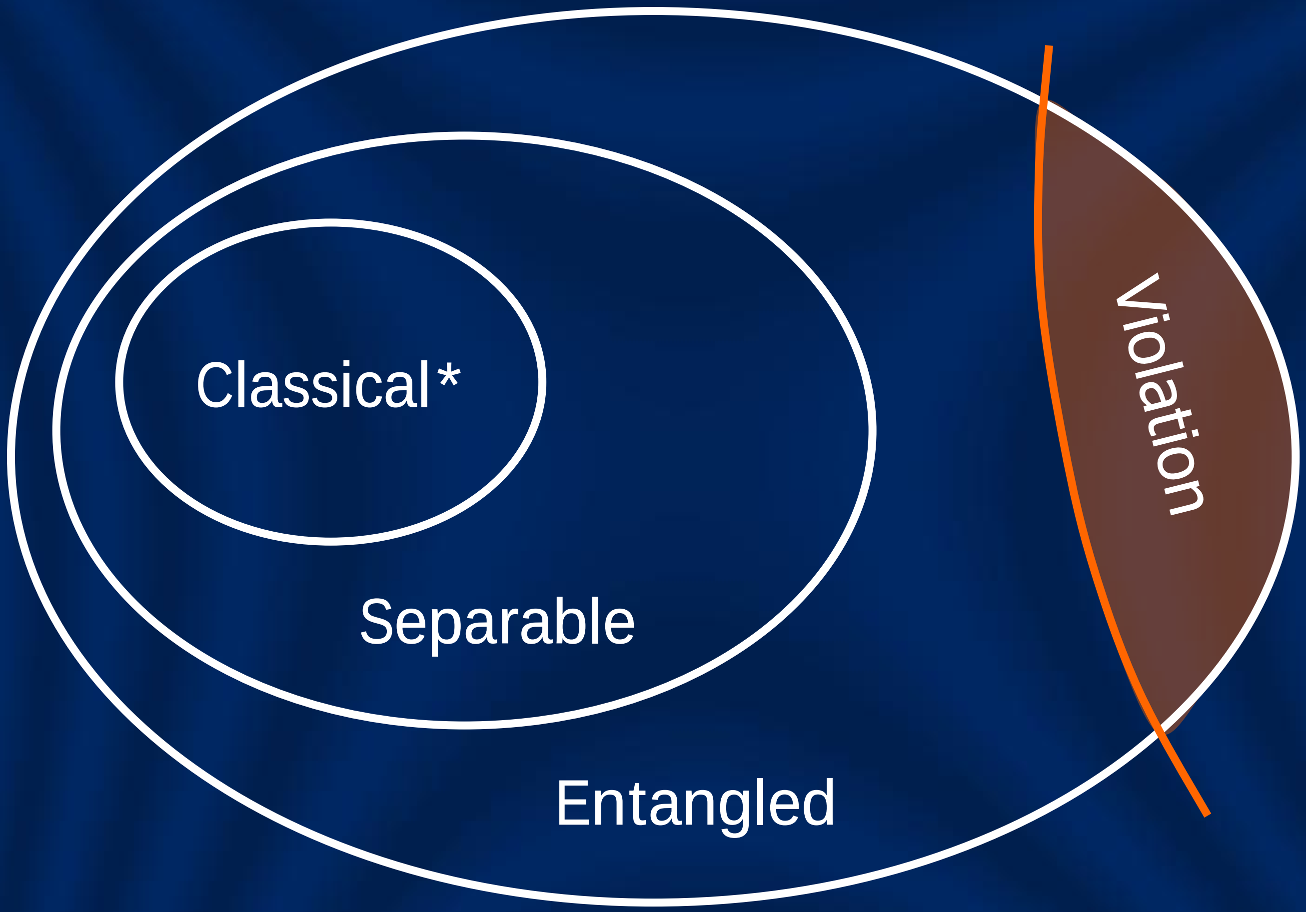


For a separable state
(in particular for any mixture of coherent states):

$$\langle (\Delta\tau')^2 \rangle_{\text{sym}} \geq \langle (\Delta\tau)^2 \rangle + \frac{4\beta^2}{\langle (\Delta\tau)^2 \rangle}$$

This inequality can be violated *only* with non-classical light!

Applicability



Classical* = coherent states and their statistical mixtures

Experimental considerations

Variance broadening due to detector jitter:

$$\langle(\Delta\tau')^2\rangle_{\mathsf{obs}} = \langle(\Delta\tau)^2\rangle + \langle(\Delta\tau)^2\rangle_{\mathsf{jitter}}$$

1) Violating the inequality

$$\langle(\Delta\tau')^2\rangle_{\mathsf{sym\ obs}} \geq \langle(\Delta\tau)^2\rangle_{\mathsf{obs}} + \frac{4\beta^2}{\langle(\Delta\tau)^2\rangle_{\mathsf{obs}}}$$

implies incompatibility with the original criterion!

2) Pump bandwidth: $\langle(\Delta\Omega)^2\rangle \ll \frac{1}{\langle(\Delta\tau)^2\rangle_{\mathsf{jitter}}}$

3) Non-negligible dispersion contribution when $2\beta \gtrsim \langle(\Delta\tau)^2\rangle_{\mathsf{jitter}}$

Conclusions

- Nonlocal dispersion cancellation *is* an intrinsically quantum phenomenon...
- ...if a suitable criterion is tested.
- Experimental verification should be feasible...
- ...but it puts stringent requirements on temporal resolution and pump spectral bandwidth.
- Complete analogy: spatial correlations and diffraction

Quantum correlations

$$\mathrm{Tr}[\hat{\varrho}\hat{\tilde{\mathcal{E}}}_j^\dagger(\nu)\hat{\tilde{\mathcal{E}}}_j(\nu')] = 2\pi\tilde{S}_j(\nu)\delta(\nu-\nu')$$

$$\mathrm{Tr}[\hat{\varrho}\hat{\tilde{\mathcal{E}}}_1(\nu)\hat{\tilde{\mathcal{E}}}_2(\nu')] = 2\pi\tilde{S}_X(\nu)\delta(\nu+\nu')$$

Commutation relations imply

$$|\tilde{S}_X(\nu)|^2 \leq [1+\tilde{S}_1(\nu)]\tilde{S}_2(-\nu)$$

while classically

$$|\tilde{S}_X(\nu)|^2 \leq \tilde{S}_1(\nu)\tilde{S}_2(-\nu)$$