Criteria for realistic single photon sources in quantum repeater applications

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1. Abstract

A possible solution to overcome the problem of inefficiency of direct quantum communication on long distances can be using quantum repeater protocols. In this work we derive criteria characterizing usefulness of realistic single photon sources in two recently proposed quantum repeater schemes.

2. General scheme for photon subtraction

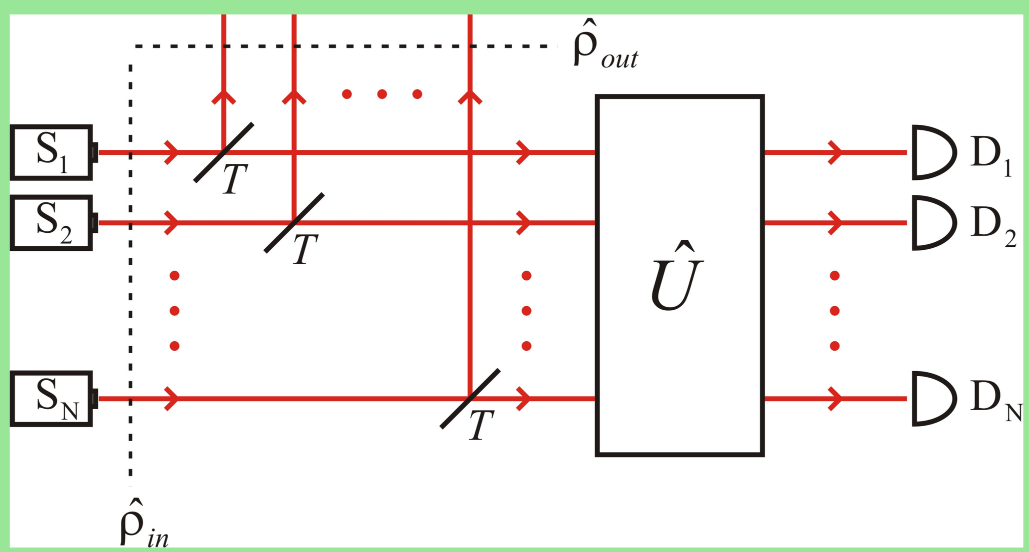

Figure 1: *A general scheme for photon subtraction.*

Transformation introduced by the scheme in the asymptotic case of $T \to 0$ (unnormalized):

$$\hat{\rho}_{out} = \frac{T^{i_1+\dots+i_N}}{i_1!\dots i_N!}\hat{M}\hat{\rho}_{in}\hat{M}^{\dagger}.$$

where

$$\hat{M} = \left[\prod_{j=1}^{N}\left(\hat{U}^{-1}\hat{a}_j\hat{U}\right)^{i_j}\right],$$

T – transmition of the beam-splitters

η – detection efficiency of the detectors

$\hat{a}_j$ – annihilation operator acting on mode j

i_j – number of clicks registered in the detector $\mathrm{D_j}$

3. Generation of entanglement between quantum memories

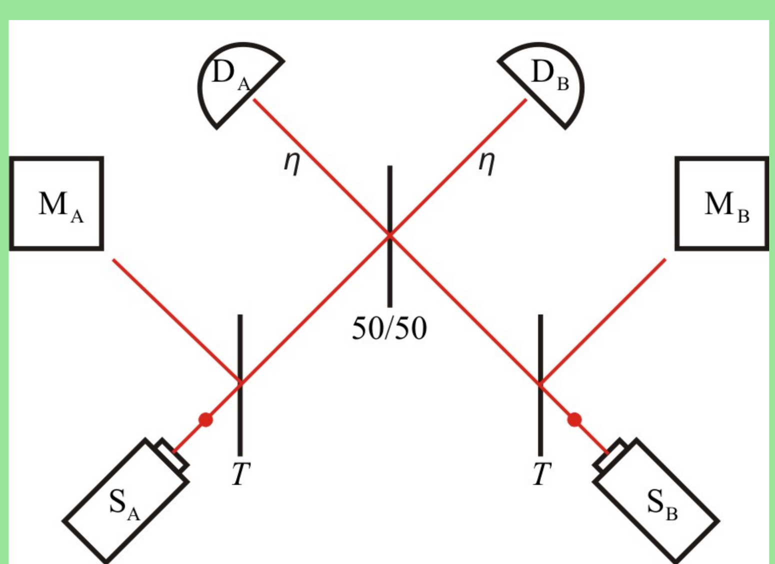

Figure 2: *The scheme for the generation of entanglement between two quantum memories (denoted by $\mathrm{M_A}$ and $\mathrm{M_B}$) proposed first in [1]*

State written in the memories conditioned on a click registered in the detector $\mathrm{D_A}$ ($\mathrm{D_B}$) in the case of $T \to 0$ (unnormalized):

$$\hat{\rho}_1 = (\hat{a}_A \pm \hat{a}_B)\,\hat{\rho}_1'\left(\hat{a}_A^{\dagger} \pm \hat{a}_B^{\dagger}\right)$$

where

$$\hat{\rho}_1' = \sum_{m,n=0}^{\infty} p_m p_n \left|m;n\right\rangle\left\langle m;n\right|,$$

p_i - probablility of emitting i photons by realistic single photon source in a given pulse

Condition for entanglement of the state $\hat{\rho}_1$ after its projection on the subspaces of 0 or 1 photon in each of the memories (from PPT separability criterion):

$$p_1^2\left(\frac{p_1^2}{4} - 2p_0p_2\right) - AT - BT^2 - CT^3 - DT^4 > 0$$

p_i - probablility of emitting i photons by realistic single photon source in a given pulse

A, B, C, D – non-negative coefficients

The lower bound for entanglement (for $T \to 0$):

$$p_1^2 - 8p_0p_2 > 0$$

Side note: for statistical mixtures of coherent states a nonequivalent condition holds

$$p_1^2 \leq 2p_0p_2$$

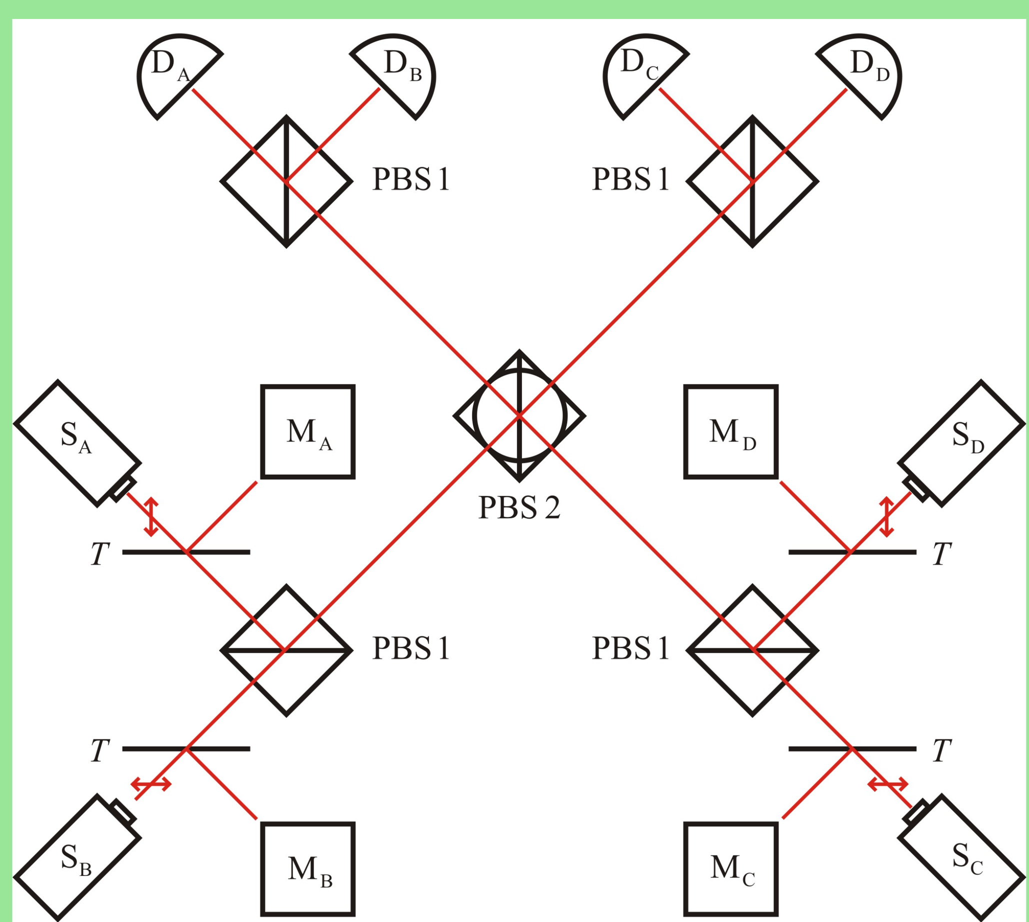

Figure 3: *A modification of the scheme for the generation of entanglement between four quantum memories proposed first in [2]*

State written in the memories conditioned on simultaneous clicks registered in the detectors $\mathrm{D_A}$ and $\mathrm{D_D}$ in the case of $T \to 0$ (unnormalized):

$$\hat{\rho}_2 = \hat{X}\hat{\rho}_2'\hat{X}^{\dagger}$$

where

$$\hat{X} = (\hat{a}_B + \hat{a}_C)^2 - (\hat{a}_A - \hat{a}_D)^2$$

and $\hat{\rho}_2'$ is a four-mode state analogous to $\hat{\rho}_1'$.

The lower bound for entanglement of the state $\hat{\rho}_2$ after its projection on the subspaces of 1 photon in the memory $\mathrm{M_A}$ or $\mathrm{M_B}$ and 1 photon in the memory $\mathrm{M_C}$ or $\mathrm{M_D}$:

$$p_1^3 - 5p_0p_1p_2 - 3p_0^2p_3 > 0$$

4. Violation of Bell's inequalities

The schemes presented on Fig. 2 and Fig. 3 can also be used for testing Bell's inequalities. By performing numercial calculations we manage to find another criterions for the statistics of realistic single photon sources needed to be fulfilled in order to show the violation of these inequalities.

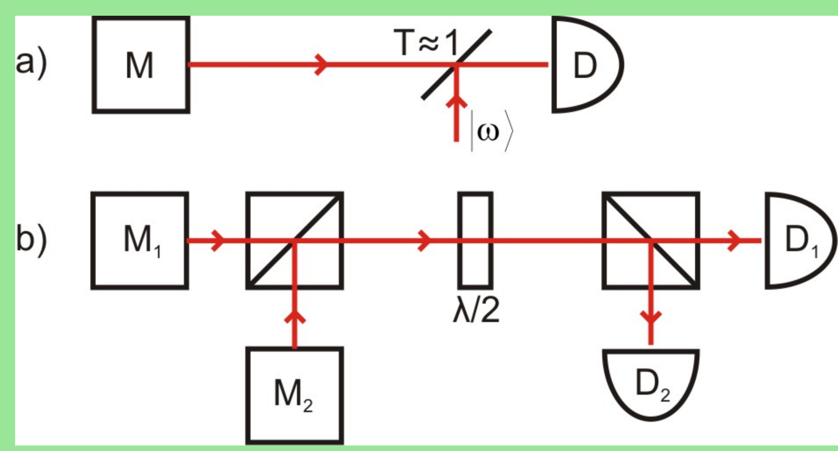

Figure 4: *Setups for testing Clauser-Horne inequality (a) and CHSH inequality (b) by using schemes for the generation of entanglement between quantum memories presented on Fig. 2 and Fig. 3 respectively.*

5. Comparison between different criterions

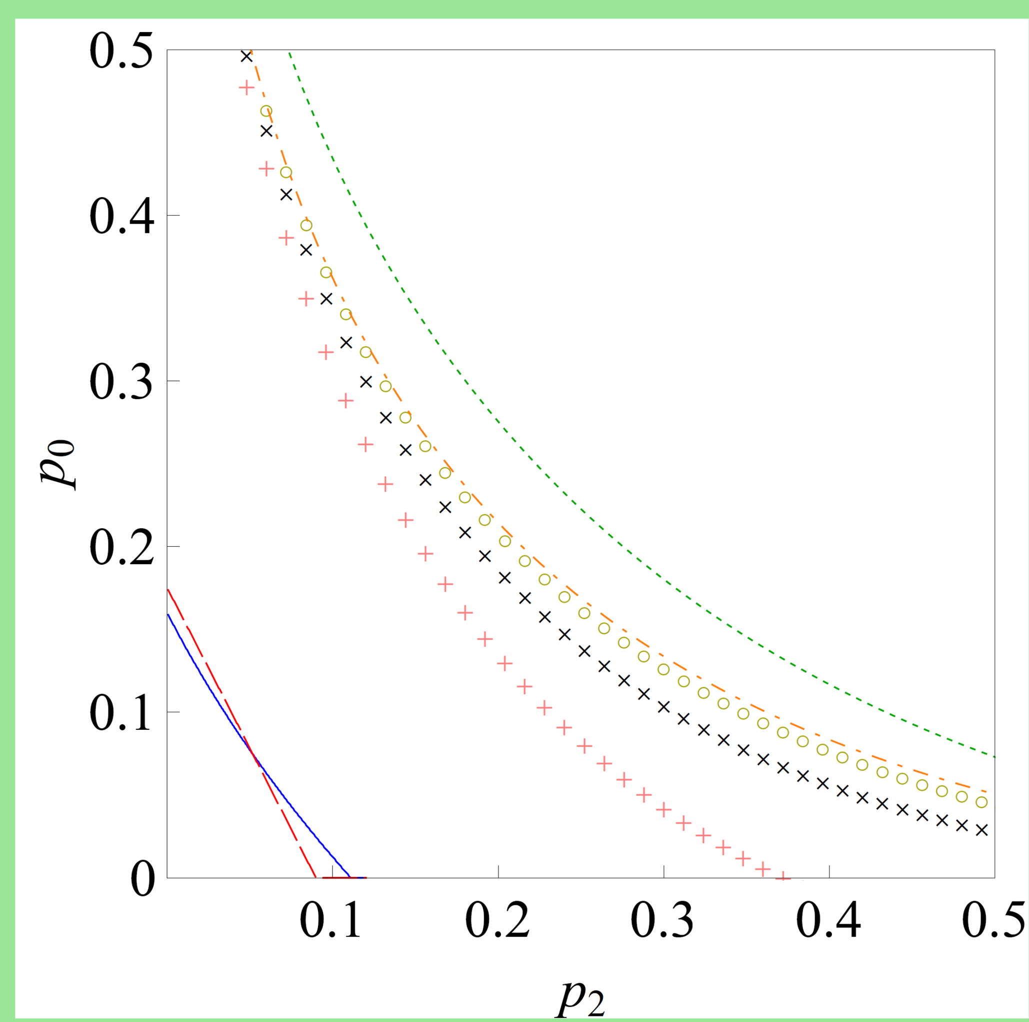

Figure 5: *Conditions for the statistics of a realistic single photon source which need to be fulfilled in order for the state created in the quantum memories to be entangled in the cases of using the schemes presented in [1] (for $\eta = 1$, $T = 10^{-5}$ – orange dash-dotted line, $\eta = 1$, $T = 0.1$ – yellow circles, $\eta = 1$, $T = 0.5$ – black x's, $\eta = 0.1$, $T = 0.5$ – pink pluses) and in [2] (for $\eta = 1$, $T = 10^{-5}$ – dotted green line) plus requirements needed to be satisfied by the source for the violation of Cluaser-Horne inequality (in the case of [1] – dashed red line) and CHSH inequality (in the case of [2] – solid blue line). Here we assume that $p_0 + p_1 + p_2 = 1$.*

References

[1] N. Sangouard *et al.* Phys. Rev. A **76,** 050301 (2007)

[2] Z.-B. Chen *et al.* Phys. Rev. A **76,** 022329 (2007)

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