Neutrino Masses, Mixing and Oscillations: Current Status and Future Prospects

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
Kavli IPMU, University of Tokyo, Japan

Neutrino Day
Faculty of Physics, University of Warsaw
January 14, 2014

Plan of the Lecture

1. Introduction.

2. Massive Neutrinos, Neutrino Mixing and Oscillations: Overview.

3. Three Neutrino Mixing. Massive Majorana versus Massive Dirac Neutrinos I. Dirac and Majorana CP Violation.

4. Neutrino Oscillations in Vacuum: Theory and Experimental Evidences.

5. Matter Effects in Neutrino Oscillations: Theory.
Neutrino Oscillations in the Earth.
CP Violation in Neutrino Oscillations.
Flavour Conversions of Solar Neutrinos.

6. Three Neutrino Mixing: the Angle θ_{13} and Indications for Dirac CP Violation.

7. Open Questions in the Physics of Massive Neutrinos.

8. Understanding the Pattern of Neutrino Mixing.

9. The Absolute Scale of Neutrino Masses.

10. The Nature of Massive Neutrinos.
Massive Majorana versus Massive Dirac Neutrinos II.
Origins of Dirac and Majorana Massive Neutrinos.
The Seesaw Mechanisms of Neutrino Mass Generation.

11. Determining the Nature of Massive Neutrinos.

12. Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m^2_{31})$ and CP Violation (?).

13. Conclusions.

Will not cover: Leptogenesis Scenario of Generation of the Baryon Asymmetry of the Universe. Dirac and Majorana Leptonic CP-Violation and Leptogenesis.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active $\nu' s$ and $\tilde{\nu}' s$
- The notion of "type" ("flavour") - dynamical;

ν_e: $\nu_e + n \rightarrow e^- + p$; ν_μ: $\pi^+ \rightarrow \mu^+ + \nu_\mu$; etc.

- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}$, $\tilde{\nu}_l \neq \tilde{\nu}_{l'}$, $l \neq l' = e, \mu, \tau$; $\nu_l \neq \tilde{\nu}_{l'}$, $l, l' = e, \mu, \tau$.

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu_l' | \nu_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}_l' | \tilde{\nu}_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}_l' | \nu_l \rangle = 0$.

• Data (relativistic ν's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard Theory: ν_l, $\tilde{\nu}_l$ - $\nu_{lL}(x)$;

$\nu_{lL}(x)$ form doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau \, .$$

• No (compelling) evidence for existence of (relativistic) ν's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)

If ν_R, $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l, $\tilde{\nu}_l$: ν_R, $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν's can play crucial role
i) in the generation of $m(\nu) \neq 0$,
ii) in understanding why $m(\nu) \ll m_l, m_q$,
iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = const.$, $l = e, \mu, \tau$;
$L \equiv L_e + L_\mu + L_\tau = const.$

There have been remarkable discoveries in neutrino physics in the last ~ 15 years.

Compellings Evidence for ν–Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of μ–like events**

Dominant $\nu_\mu \rightarrow \nu_\tau$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_μ from accelerators): $\nu_\mu \rightarrow \nu_e$

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj}\,|\nu_j>\,, \quad \nu_j:\ m_j \neq 0\,; \quad l = e,\mu,\tau\,; \quad n \geq 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\,\nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\ m_j \neq 0\,; \quad l = e,\mu,\tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x)\gamma_\alpha \,(1-\gamma_5)\, \nu_{l\mathrm{L}}(x)\, W^\alpha(x) + \text{h.c.}\,,$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\ m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, ...$) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,...} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly", data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly");

ii) $M_{4,5,...} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
$M_{4,5,...} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

We can also have, in principle:

$m_4 \sim 1$ eV ($\nu_{e(\mu)} \rightarrow \nu_S$), $m_5 \sim 5$ keV (DM), $M_6 \sim (10 - 10^3)$ GeV (seesaw).

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathrm{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \ \text{impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\text{el}} = 0,\ L_l = 0,\ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

Special Properties of the Currents of $\chi(x)$−Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0)\,;$$

Has imortant implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0\,)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \; d_\chi = 0 \; (d_\Psi \neq 0, \; if \; CP \; is \; not \; conserved)$$

$\chi(x)$ cannot couple to a real photon (field) .

$\chi(x)$ couples to a virtual photon through an anapole moment :

$$(g_{\alpha\beta}\, q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5 \; F_a(q^2)\,.$$

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48\ (2.44) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425\ (0.437)$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234\ (0.0239)$, NH (IH).

F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241$ (0.0244), NH (IH).

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48$ (2.44) $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425$ (0.437), NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0239), NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$, $1\sigma(\sin^2\theta_{23}) = 14\%$;
- $1\sigma(\sin^2\theta_{13}) = 10\%$,
- $3\sigma(\Delta m^2_{21})$: $(6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.259 - 0.359)$;
- $3\sigma(|\Delta m^2_{31(23)}|)$: $2.19(2.17) - 2.62(2.61) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23})$: $0.331(0.335) - 0.637(0.663)$;
- $3\sigma(\sin^2\theta_{13})$: $0.0169(0.0171) - 0.0313(0.0315)$.

Fogli *et al.*, arXiv:1205.5254v3

- **Dirac phase** δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{\rm CP}^{(l,l')} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^*\, U_{\mu 1}^*\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- **Majorana phases** α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left| < m > \right|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Solar Neutrinos ν_e, $E \sim$1 MeV: B. Pontecorvo 1946

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

R. Davis et al., 1967 - 1996: 615 t C_2Cl_4; 0.5 Ar atoms/day, exposure 60 days.

$$\nu + e^- \rightarrow \nu + e^-$$

Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);

$$\nu_e + D \rightarrow e^- + p + p \; , \qquad \text{SNO}$$

$$\nu_l + D \rightarrow \nu_l + n + p \, , \; l = e, \mu, \tau \qquad \text{SNO}$$

Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

Atmospheric Neutrinos ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$, $E \sim 1$ GeV (0.20 - 100 Gev)

$$\nu_\mu + N \to \mu^- + X, \quad \tilde{\nu}_\mu + N \to \mu^+ + X$$

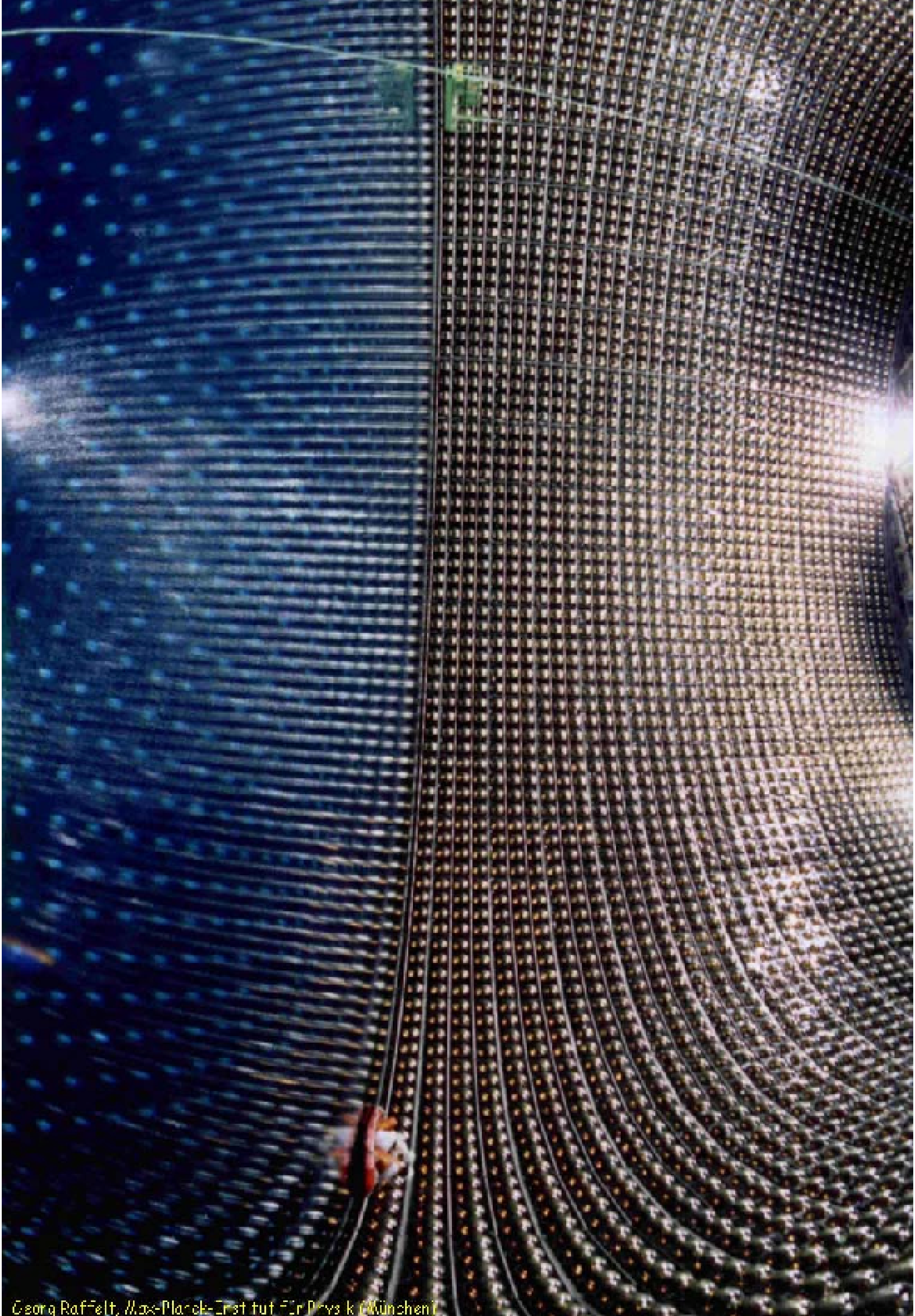
$$\nu_e + N \to e^- + X, \quad \tilde{\nu}_e + N \to e^+ + X$$

K2K, MINOS, T2K, ν_μ ($\bar{\nu}_\mu$), $E \sim 1$ GeV

$$\nu_\mu + N \to \mu^- + X \quad (\nu_e + N \to e^- + X)$$

Reactor $\bar{\nu}_e$: CHOOZ,KamLAND, Double Chooz, RENO, Daya Bay ($E \cong 2 - 8$ MeV)

$$\bar{\nu}_e + p \to e^+ + n$$


Georg Raffelt, Max-Planck-Institut für Physik (München)

Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Sendai
Tokyo
Kawasaki
Yokohama
Gifu
Nagoya
Kyoto
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e> \; = |\nu_1> \cos\theta + |\nu_2> \sin\theta,$$

$$|\nu_{\mu(\tau)}> \; = -|\nu_1> \sin\theta + |\nu_2> \cos\theta; \quad \nu_{1,2}: \quad m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e>_t \; = e^{-iE_1 t}|\nu_1> \cos\theta + e^{-iE_2 t}|\nu_2> \sin\theta, \; E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \to \nu_\mu; t) = <\nu_\mu|\nu_e>_t = \frac{1}{2}\sin 2\theta \; (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta \; (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \to \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \to \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L,\ E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}},\ L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta\ (1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}),\ L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$L_{osc}^{vac} \cong 2.5\ m\ \frac{E[MeV]}{\Delta m^2[eV^2]}$

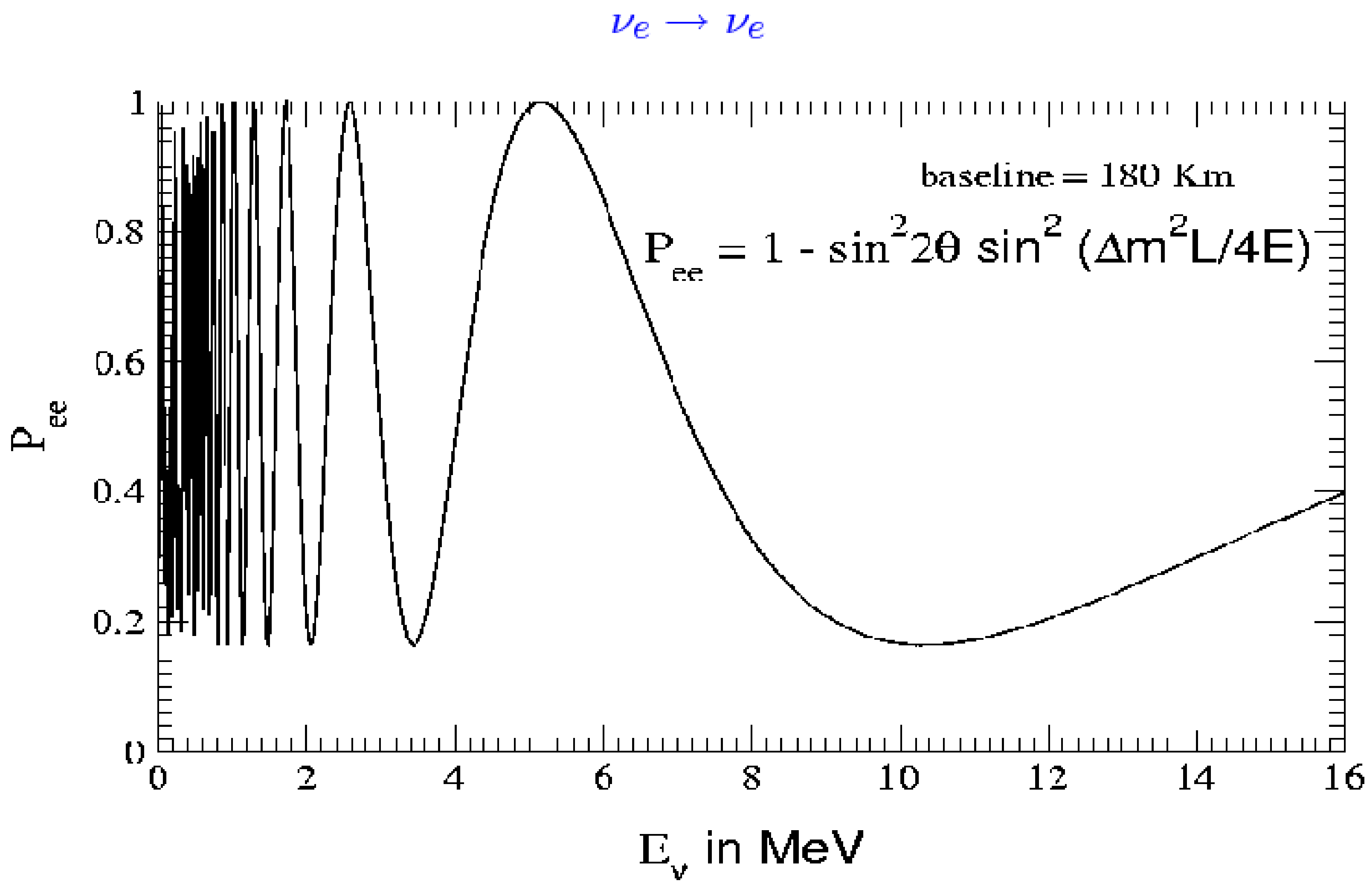
$E \cong 3$ MeV, $\Delta m^2[eV^2] \cong 8 \times 10^{-5}$: $L_{osc}^{vac} \cong 100$ km

$E \cong 1$ GeV, $\Delta m^2[eV^2] \cong 2.5 \times 10^{-3}$: $L_{osc}^{vac} \cong 1000$ km

Effects of oscillations observable if

$\sin^2 2\theta - sufficiently\ large, \quad L \gtrsim L_{osc}^{vac}$

Two basic parameters: $\sin^2 2\theta$, Δm^2
SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \to \nu_\tau$
KamLAND: $\bar{\nu}_e \to \bar{\nu}_e$; $\bar{\nu}_e \to (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$


$\nu_e \longrightarrow \nu_e$
baseline = 180 Km
$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L/4E)$
P_{ee}
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
E_ν in MeV

Source	Type of ν	$\bar{E}$[MeV]	L[km]	min(Δm^2)[eV^2]
Reactor	$\tilde{\nu}_e$	$\sim$ **1**	**1**	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	$\sim$**1**	**100**	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	**1**	~ 1
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	**1000**	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Correspond to: CHOOZ, Double Chooz, RENO, Daya Bay ($L \sim 1$ km), KamLAND ($L \sim 100$ km); $\tilde{\nu}_e$ disappearnce; $E = (1.8 \div 8.0)$ MeV;

to accelerator experiments - past ($L \sim 1$ km); past, current: K2K ($L \sim 250$ km), MINOS ($L \sim 730$ km), ν_μ disappearnce; OPERA ($L \sim 730$ km), $\nu_\mu \to \nu_\tau$;

T2K ($L \sim 295$ km), future NOνA ($L \sim 800$ km), ν_μ disappearnce, $\nu_\mu \to \nu_e$; $E \sim 1$ GeV;

SK experiment studying atmospheric ν_μ, $\tilde{\nu}_\mu$, ν_e, $\tilde{\nu}_e$ ($E \cong 0.1 \div 100$ GeV), and solar ν_e ($E \cong 5 \div 14$ MeV) oscillations, and to the solar ν experiments ($E \cong 0.29 \div 14$ MeV).

$$A(\nu_l \to \nu_{l'}) = \Sigma_j \, U_{l'j} \, D_j \, U^\dagger_{jl} \,, \quad l, l' = e, \mu, \tau \,,$$

$$D_j = e^{-i\tilde{p}_j\,(x_f - x_0)} = e^{-i(E_j T - p_j L)} \,, \quad p_j \equiv |\mathbf{p}_j| \,.$$

$$\delta\varphi_{jk} \cong \frac{m_j^2 - m_k^2}{2p} L = 2\pi \frac{L}{L^v_{jk}} \text{sgn}(m_j^2 - m_k^2) \,, \;\; p = (p_j + p_k)/2,$$

$$L^v_{jk} = 4\pi \frac{p}{|\Delta m^2_{jk}|} \cong 2.5 \text{ m} \; \frac{p[MeV]}{|\Delta m^2_{jk}|[eV^2]}$$

is the neutrino oscillation length associated with Δm^2_{jk}.

- One can safely neglect the dependence of p_j and p_k on the masses m_j and m_k and consider p to be the zero neutrino mass momentum, $p = E$.

- The phase $\delta\varphi_{jk}$ is Lorentz invariant.

$$\sigma_{m^2} = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$$

Condition for producing coherently $\nu_1, \nu_2, ...$:

$$\sigma_{m^2} > |\Delta m^2_{jk}|$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos ν_j. It accounts only for the movement of the center of the wave packet describing ν_j. In the wave packet treatment of the problem, the interference between the states of ν_j and ν_k is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of ν_j and ν_k at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads: $\sigma_{xP}, \sigma_{xD} < L^v_{jk}/(2\pi)$, $\sigma_{xP(D)}$ being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets detetermined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing ν_j and ν_k should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities $v_j \neq v_k$, satisfies $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$. If the interval of time T is not measured, T in the preceding condition must be replaced by the distance L between the neutrino source and the detector.

Examples

- Spatial localisation condition

ΔL - dimensions of the ν- source (and/or detector):

$$2\pi\Delta L/L^{v}_{jk} \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

$$2\pi(L/L^{v}_{jk})(\Delta E/E) \lesssim 1.$$

If $2\pi\Delta L/L^{v}_{jk} \gg 1$, and/or $2\pi(L/L^{v}_{jk})(\Delta E/E) \gg 1$,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2\, |U_{lj}|^2$$

Two-Neutrino Oscillations in Vacuum

SK ((100-12742) km), K2K (250 km); CNGS (OPERA), MINOS (730 km); T2K (295 km); dominant $\nu_\mu \rightarrow \nu_\tau$;

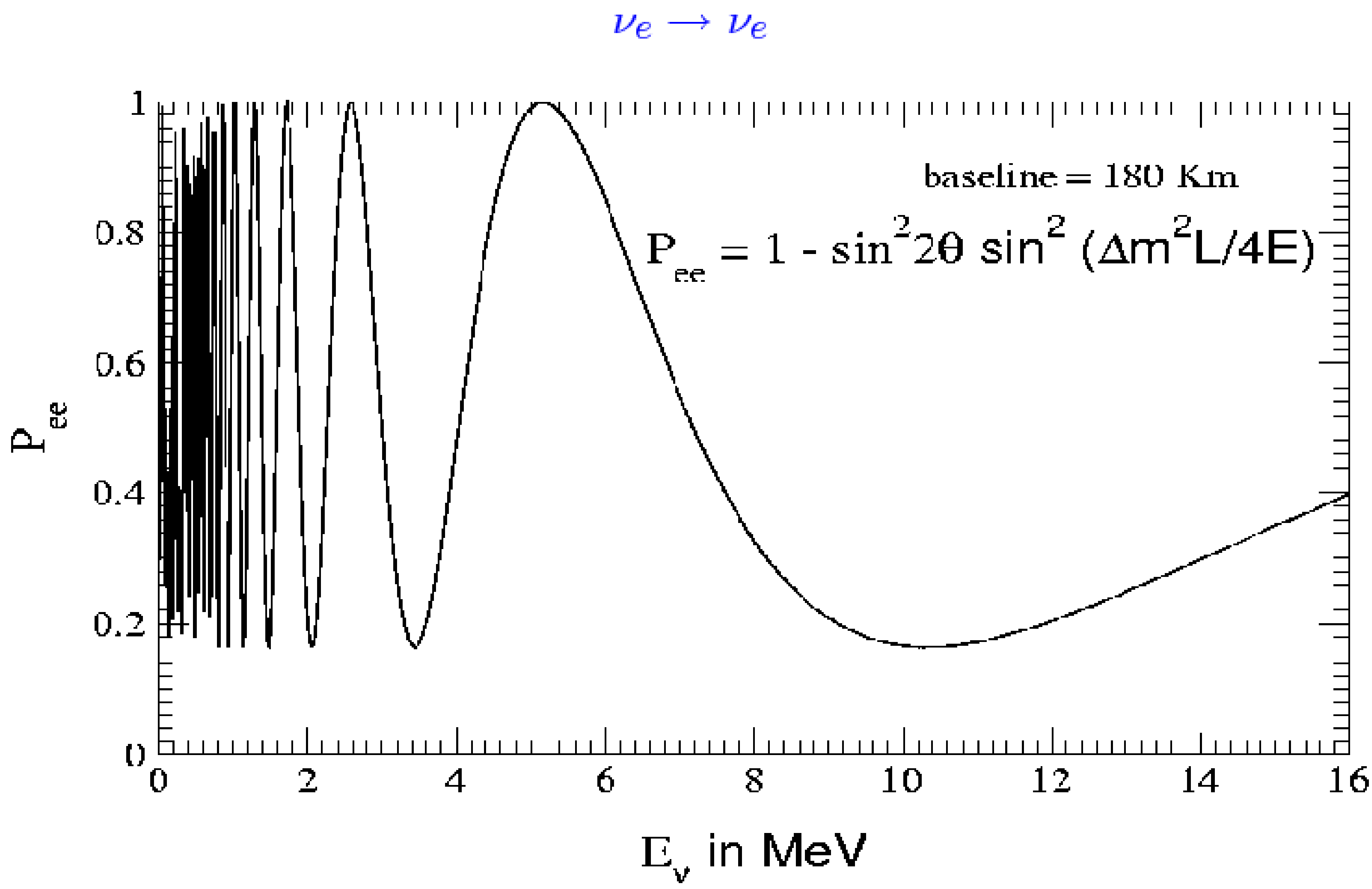
$$P(\nu_\mu \rightarrow \nu_\tau; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau; L) \cong \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m^2_{31} \, L}{4E};$$
$$P(\nu_\mu \rightarrow \nu_\mu; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu; L) = 1 - P(\nu_\mu \rightarrow \nu_\tau; L).$$

KamLAND ($\sim$ 180km): $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2}\sin^2 2\theta_{12} \, (1 - \cos \frac{\Delta m^2_{21} \, L}{2E}).$$

CHOOZ, Double Chooz, Daya Bay, RENO ($\sim$ 1 km): $\bar{\nu}_e \rightarrow \bar{\nu}_e$

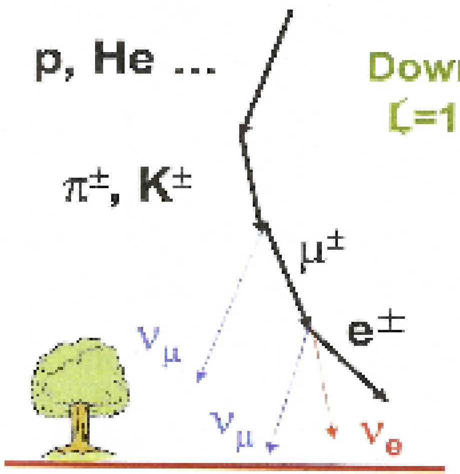
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2}\sin^2 2\theta_{13} \, (1 - \cos \frac{\Delta m^2_{31} \, L}{2E}).$$



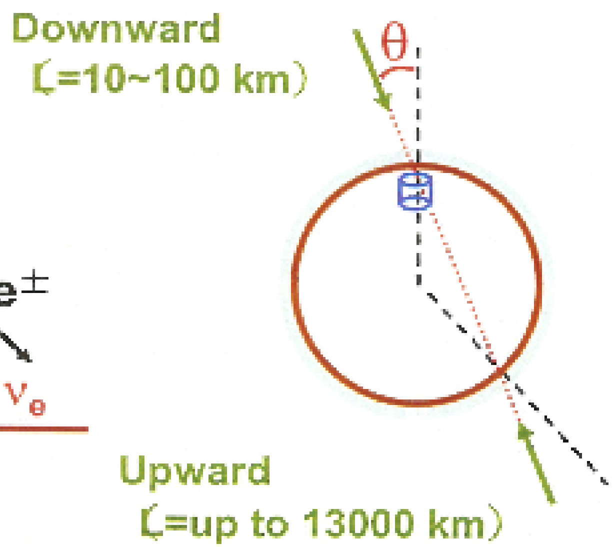
νe → νe
baseline = 180 Km
Pee = 1 - sin²2θ sin² (Δm²L/4E)
Pee
Eν in MeV

Observing the Oscillations of Neutrinos

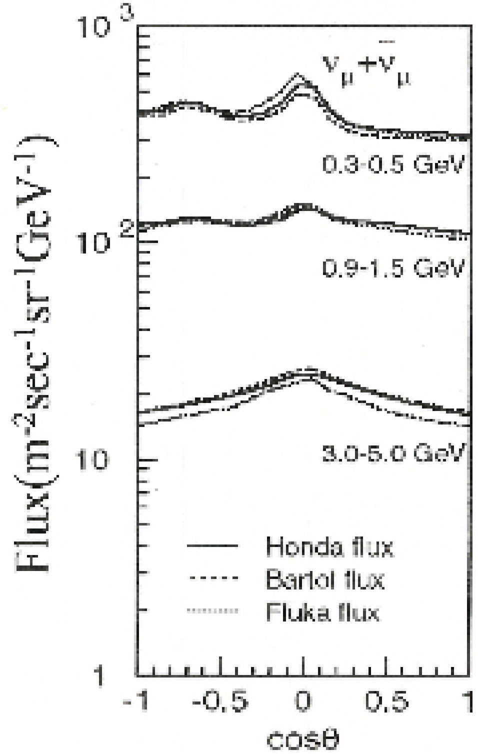
Atmospheric neutrinos



Zenith angle



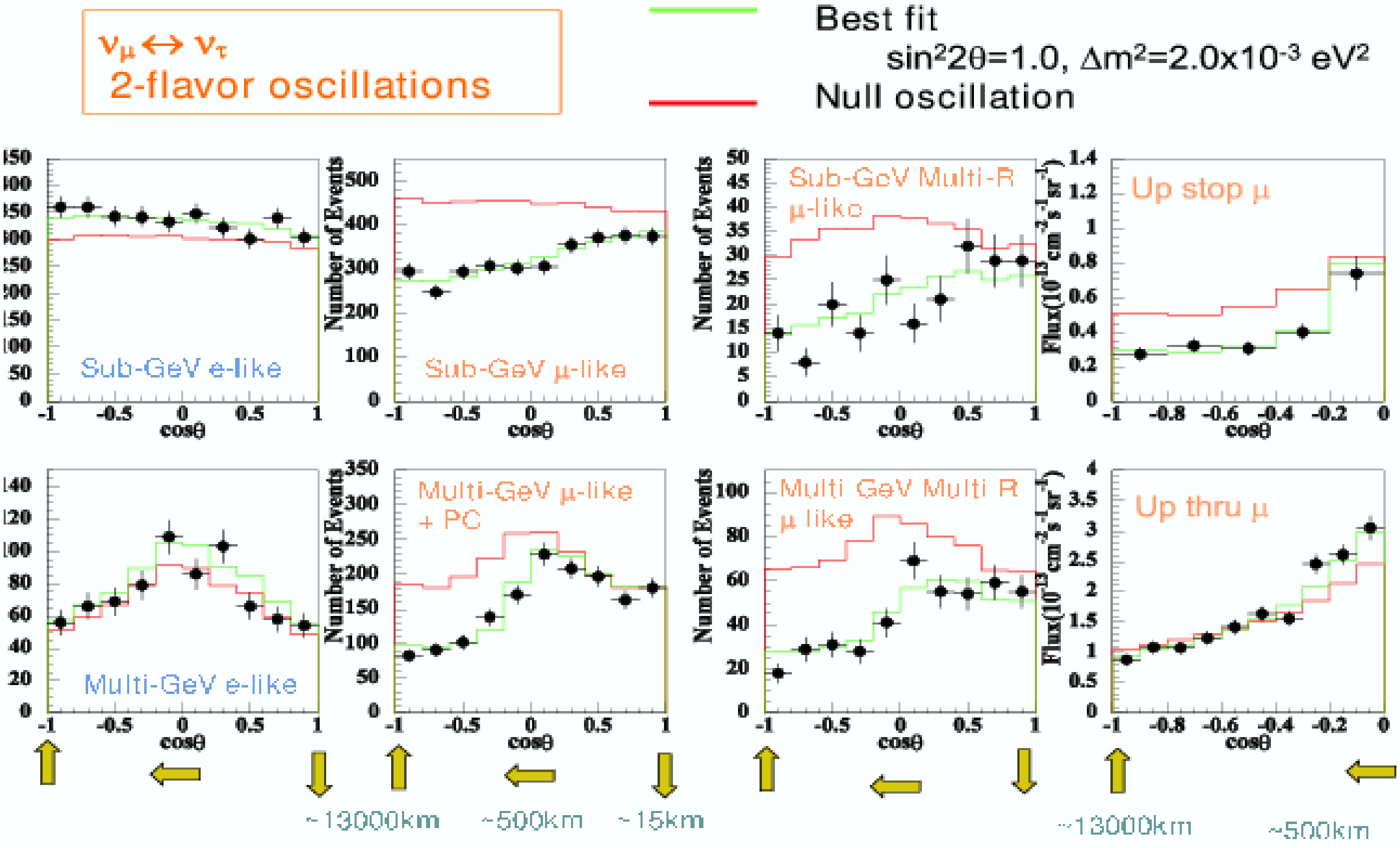
Zenith angle dist. of Atmospheric ν flux



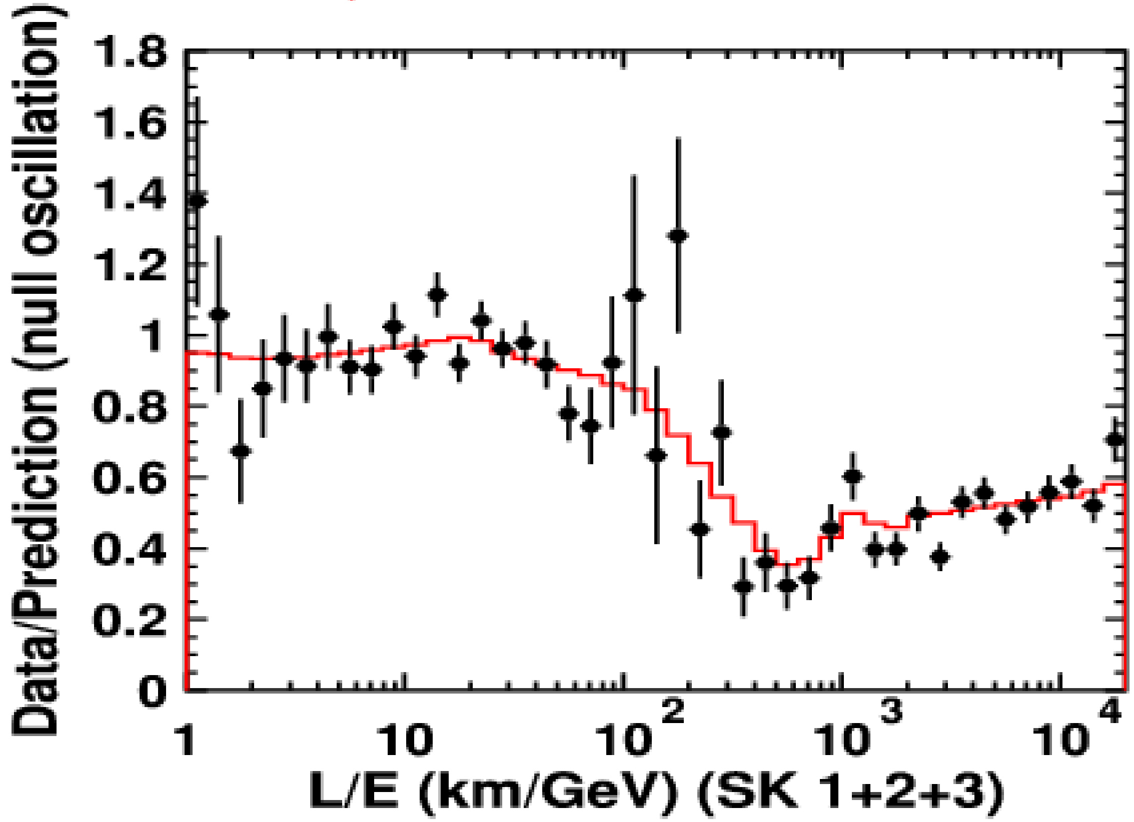
Eν > a few GeV

Up/Down Symmetry

Zenith angle distributions



SK: L/E Dependence, $\mu-$Like Events

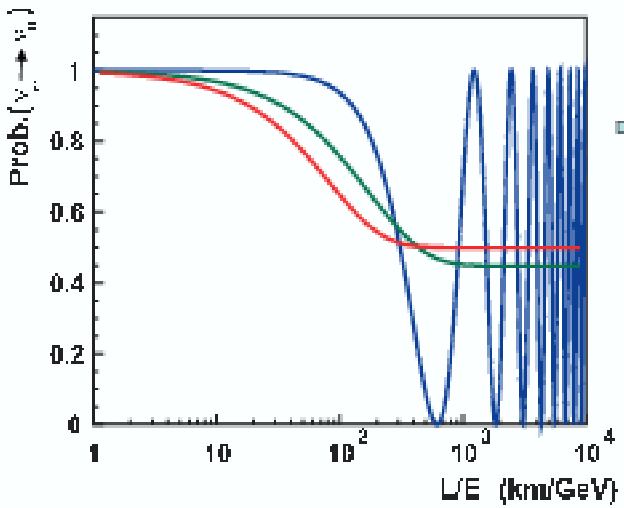


L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2\theta + \sin^2\theta \times \exp(-\frac{m}{2\tau}\frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2}\sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



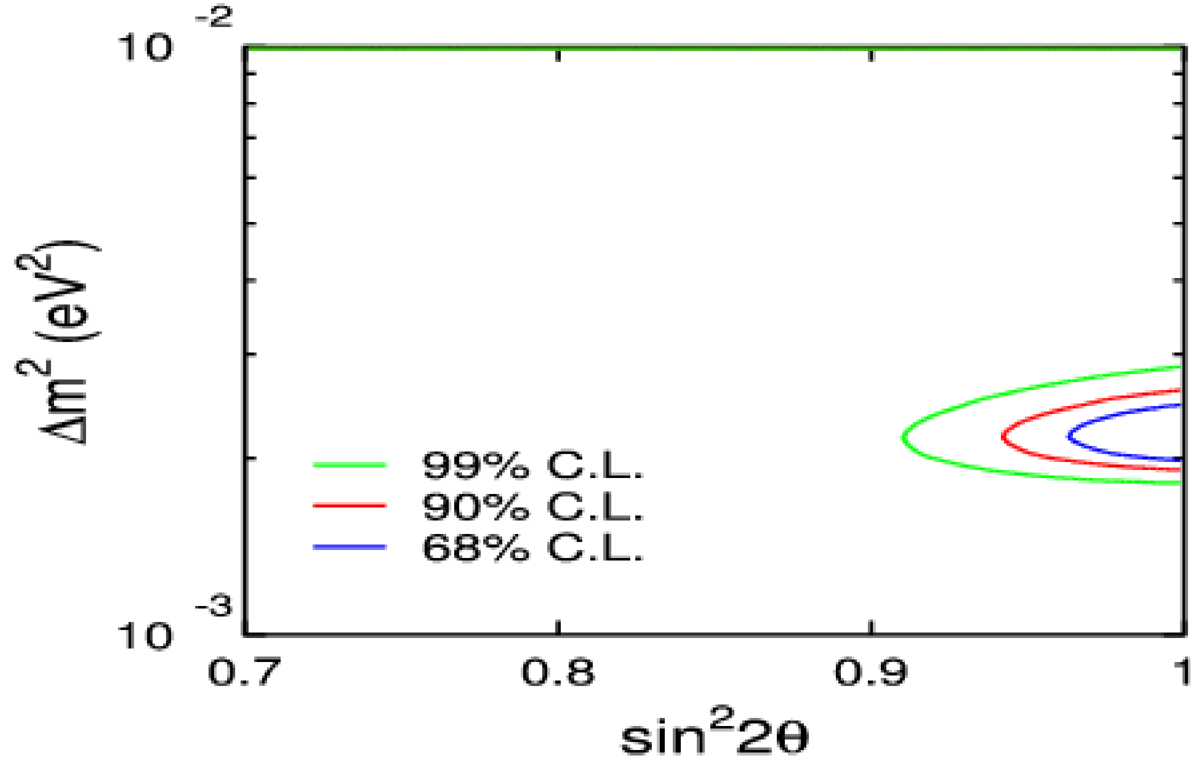
Use events with high resolution in L/E

→ The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

SK: Atmospheric ν Data



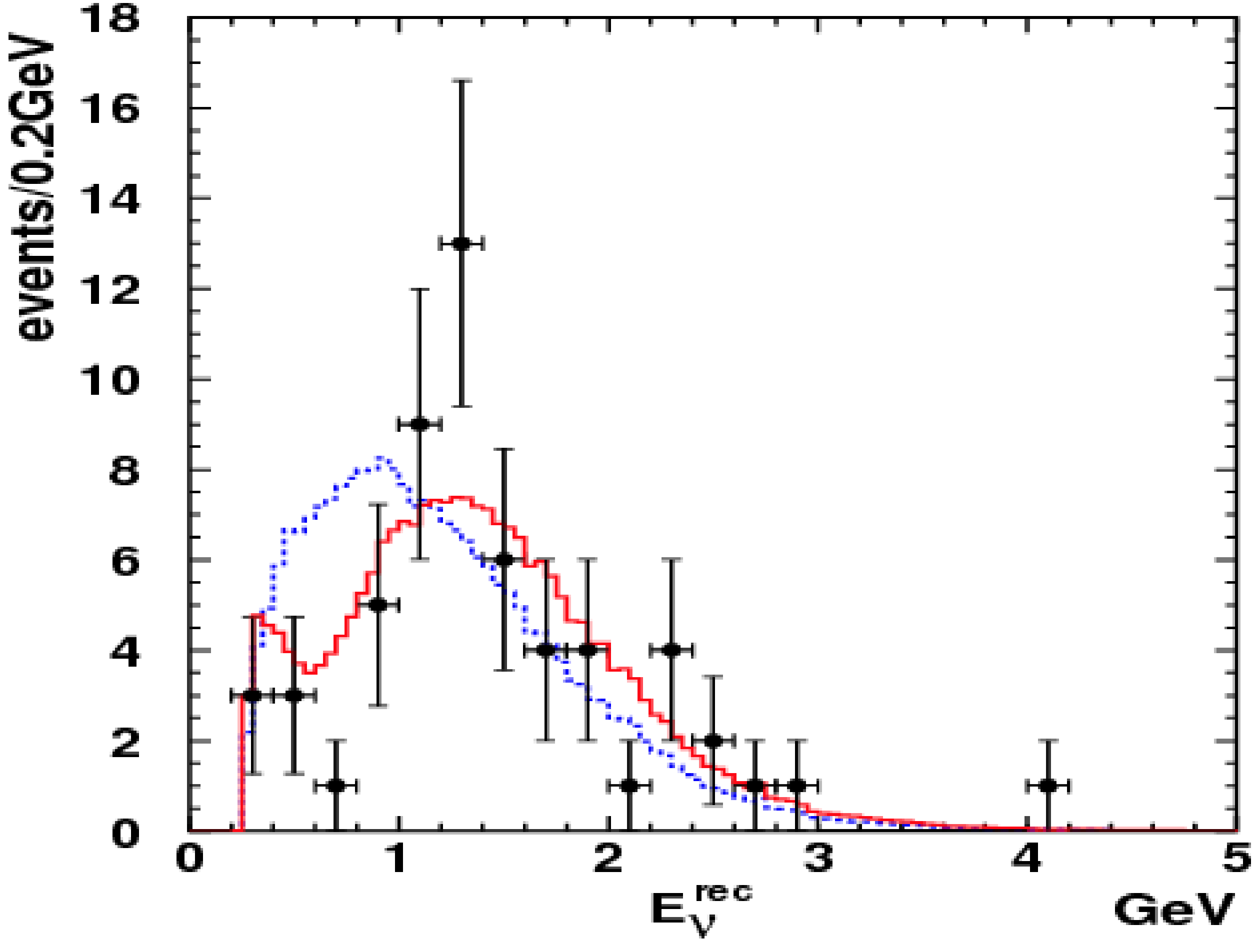
$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{\rm atm} \equiv \sin^2 2\theta_{23} = 1.0$;

$\Delta m^2_{31} = (1.9 - 2.9) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \geq 0.92$, 99% C.L.

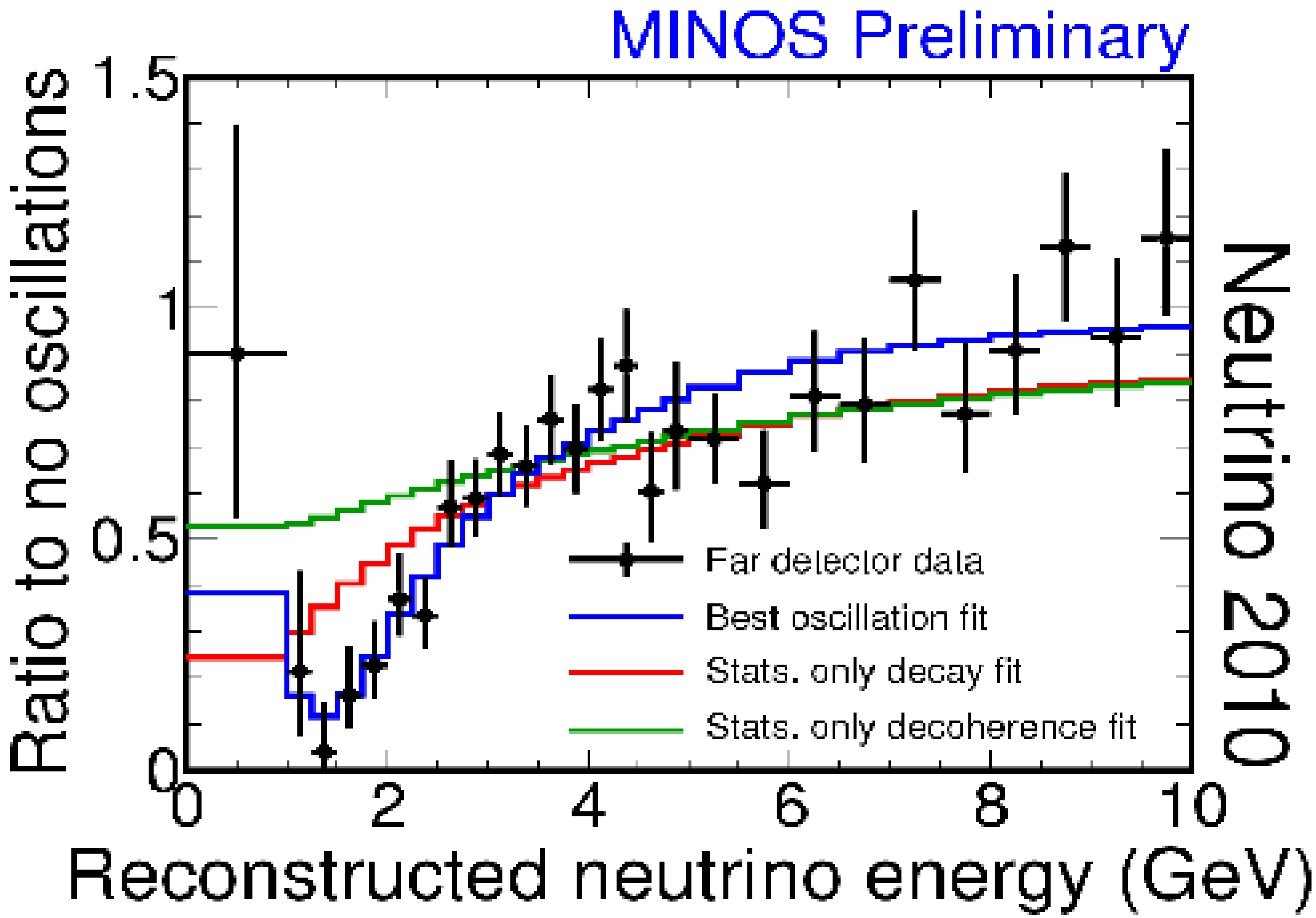
- sign of $\Delta m^2_{\rm atm}$ not determined. If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.

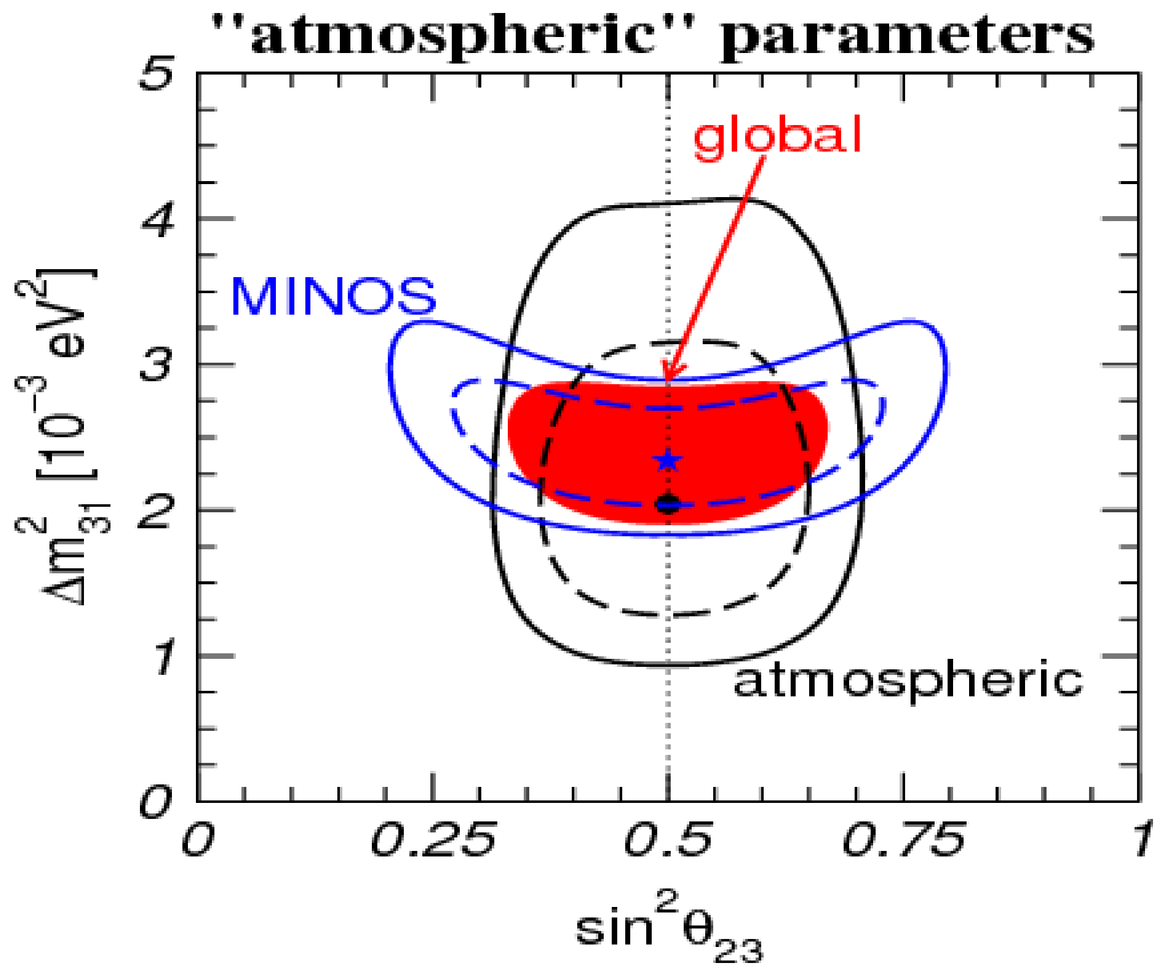
3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (NH); $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (IH).

K2K: ν_μ Spectrum (ν_μ “disappearance”)



MINOS: ν_μ Spectrum (ν_μ "disappearance")





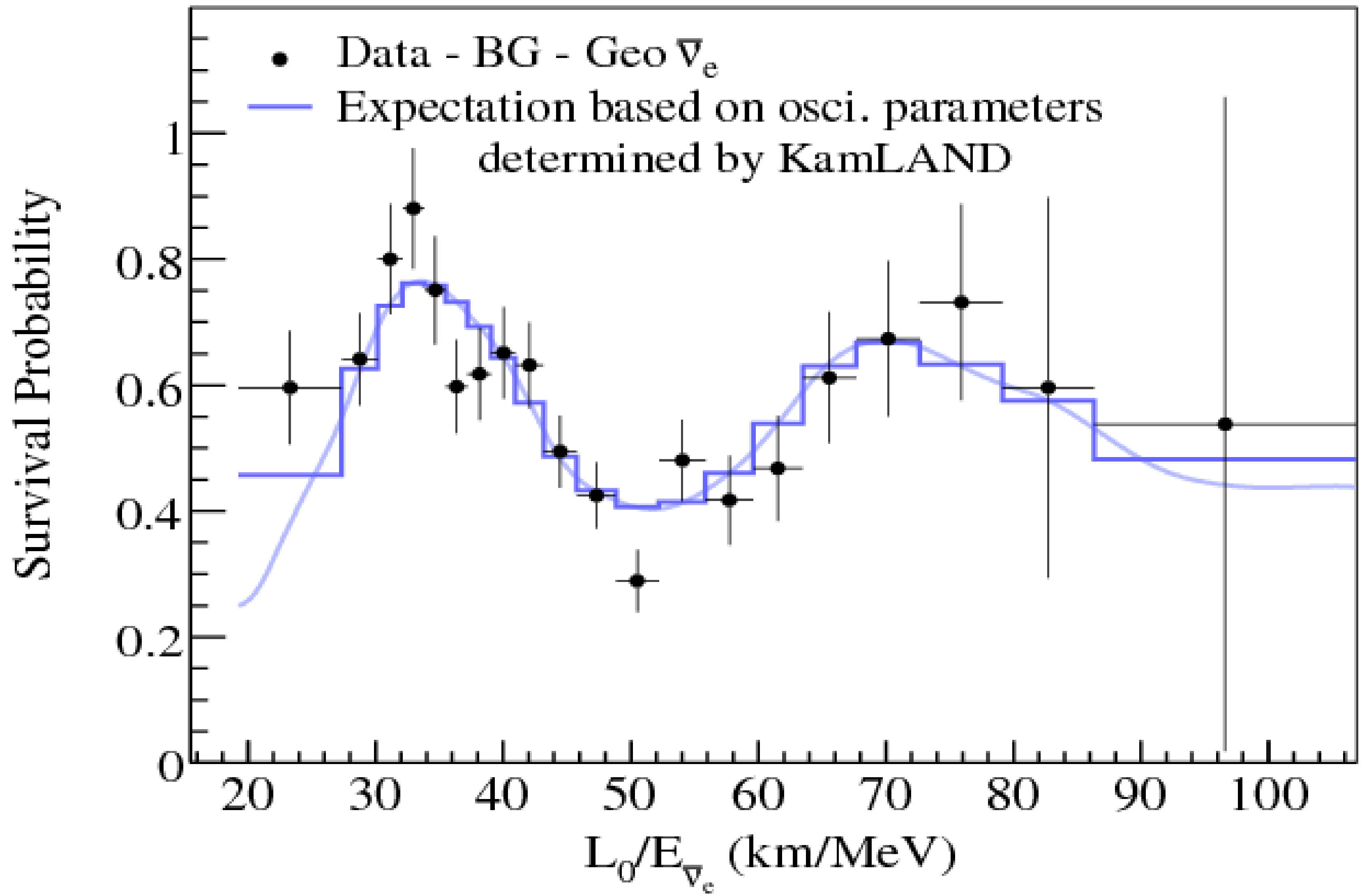
T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of Δm^2_{atm} not determined;

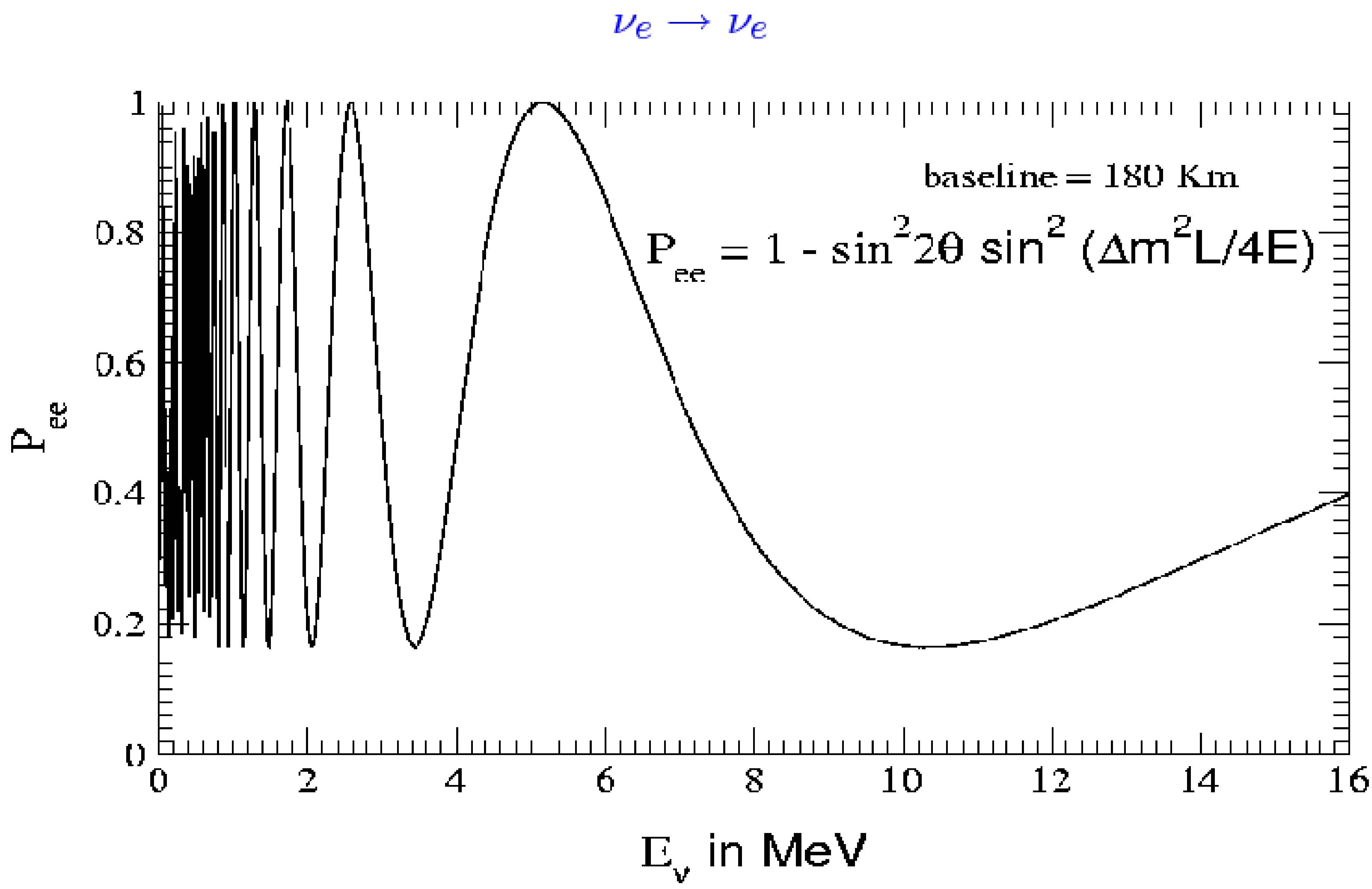
  3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

  $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



KamLAND: L/E−Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, E = (1.8 - 10) MeV)



$\nu_e \longrightarrow \nu_e$
baseline = 180 Km
$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L/4E)$
P_{ee}
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
E_ν in MeV

Solar Neutrinos: ν_e, $E \sim (0.26 - 14.4)$ MeV

Super-Kamiokande, $E \cong (5.0 - 14.4)$ MeV

$$R(SK) \propto \Phi^0_{\mathrm{E}}(\nu_e) \sum_{l=e,\mu,\tau} P(\nu_e \to \nu_l)\, \sigma(\nu_l e^- \to \nu_l e^-)$$
$$= \sigma(\nu_e e^- \to \nu_e e^-)\, [\Phi^0_{\mathrm{E}}(\nu_e) P(\nu_e \to \nu_e)$$
$$+ \Phi^0_{\mathrm{E}}(\nu_e)\, (1 - P(\nu_e \to \nu_e))\, \frac{\sigma(\nu_{\mu(\tau)} e^- \to \nu_{\mu(\tau)} e^-)}{\sigma(\nu_e e^- \to \nu_e e^-)}]$$
$$= \sigma(\nu_e e^- \to \nu_e e^-)\, [\Phi_{\mathrm{E}}(\nu_e) + 0.16(\Phi_{\mathrm{E}}(\nu_\mu) + \Phi_{\mathrm{E}}(\nu_\tau))]$$

$$P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau) = 1,$$
$$\sigma(\nu_\mu e^- \to \nu_\mu e^-) = \sigma(\nu_\tau e^- \to \nu_\tau e^-).$$

SNO, CC: $E \cong (5.0 - 14.4)$ MeV

$\nu_e + D \to e^- + p + p$

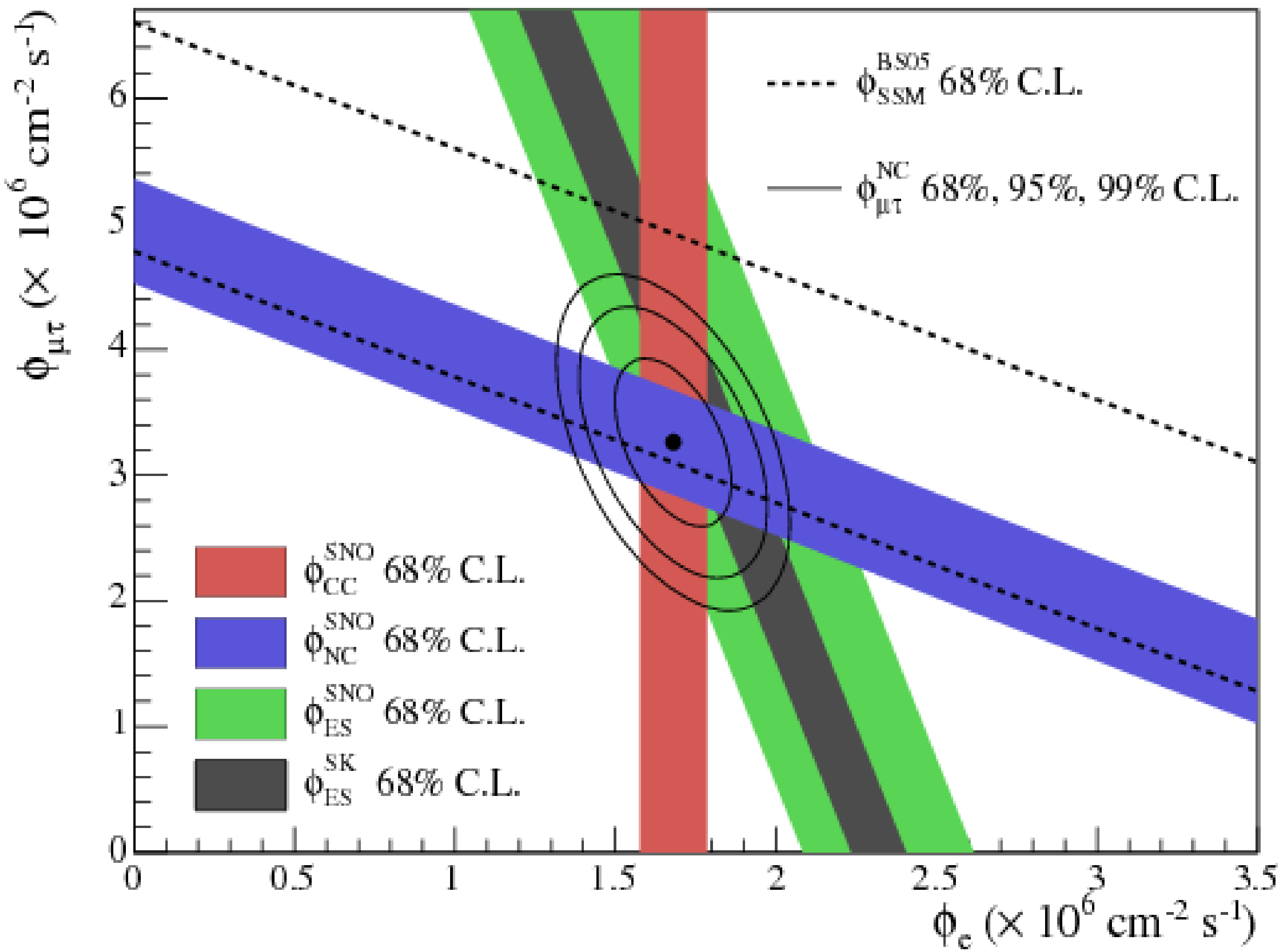
$R(SNO) \propto \sigma(\nu_e + D \to e^- + p + p)\, \Phi^0_{\mathrm{E}}(\nu_e) P(\nu_e \to \nu_e)$

$= \sigma(\nu_e + D \to e^- + p + p)\, \Phi_{\mathrm{E}}(\nu_e)$

SK: $\Phi^{\mathrm{SK}}(\nu_{\odot}) = \Phi_{\mathrm{E}}(\nu_e) + 0.16(\Phi_{\mathrm{E}}(\nu_\mu) + \Phi_{\mathrm{E}}(\nu_\tau))$

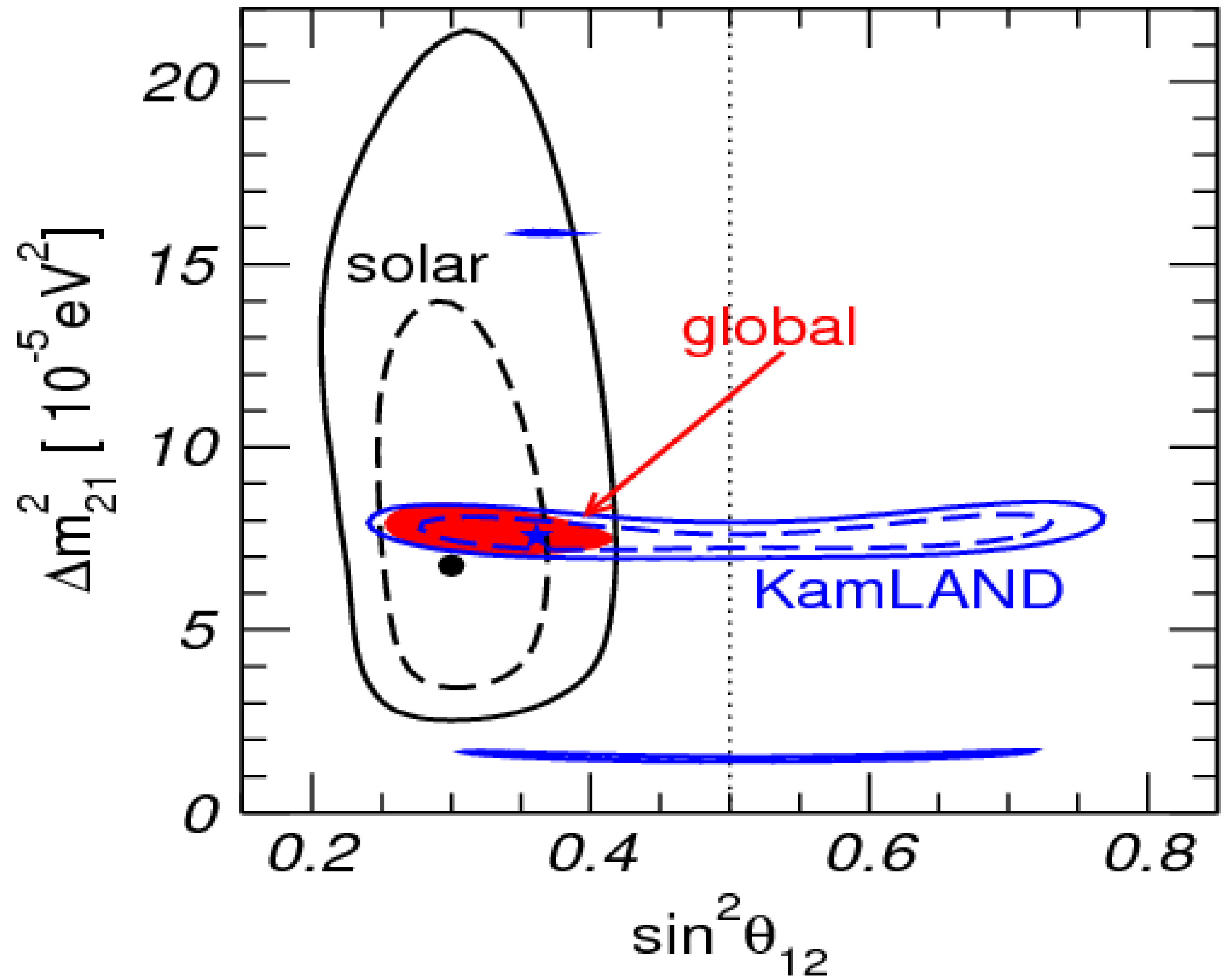
SNO CC: $\Phi^{\mathrm{SNO}}(\nu_{\odot}) = \Phi_{\mathrm{E}}(\nu_e)$

No oscillations: $\Phi_{\mathrm{E}}(\nu_\mu) + \Phi_{\mathrm{E}}(\nu_\tau) = 0$, $\Phi^{\mathrm{SK}}(\nu_{\odot}) = \Phi^{\mathrm{SNO}}(\nu_{\odot})$



$\phi_{\mu\tau}$ ($\times$ 10^6 cm^{-2} s^{-1})
ϕ_e ($\times$ 10^6 cm^{-2} s^{-1})
ϕ_{SSM}^{BS05} 68% C.L.
$\phi_{\mu\tau}^{NC}$ 68%, 95%, 99% C.L.
ϕ_{CC}^{SNO} 68% C.L.
ϕ_{NC}^{SNO} 68% C.L.
ϕ_{ES}^{SNO} 68% C.L.
ϕ_{ES}^{SK} 68% C.L.
0
0.5
1
1.5
2
2.5
3
3.5
0
1
2
3
4
5
6

"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the backgrpound of electrons, protons and neutrnos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

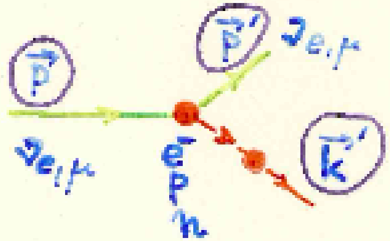
This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \to \nu_e) \cong \sin^2\theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

The presence of matter can change drastically the pattern of ν-oscillations

$$H_m = H_{vac} + H_{int}$$

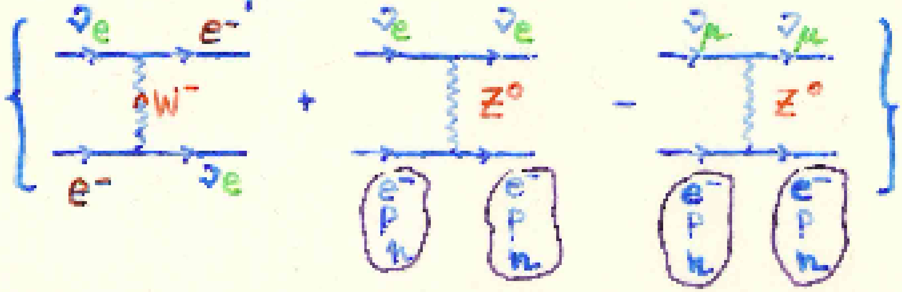


– negligible for the Sun

$$n(\nu_e) \neq 1, \quad n(\nu_\mu) \neq 1$$

$$n(\nu_e) - n(\nu_\mu) = \frac{2\pi}{p^2}\left[F^V_{\nu_e e^-}(0) - F^V_{\nu_\mu e^-}(0) \right] =$$

$$= + \frac{2\pi}{p^2} \{ \ldots \}$$



$$= -\frac{1}{p} \sqrt{2}\, G_F\, N_e$$

ν coherent scattering on e^-, p, n - effective potential (index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2} G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = - \sqrt{2} G_F N_e$$

$$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0 \text{ (leading order)}$$

L. Wolfenstein, 1978; V. Barger et al., 1980; P. Langacker *et al.*, 1983

$$V_{es} = V(\nu_e) - V(\nu_s) = \sqrt{2} G_F (N_e - \tfrac{1}{2} N_n)$$

$$\bar{V}_{es} = V(\bar{\nu}_e) - V(\bar{\nu}_s) = - \sqrt{2} G_F (N_e - \tfrac{1}{2} N_n) = - V_{es}$$

$$V_{\mu s} = V(\nu_\mu) - V(\nu_s) = \sqrt{2} G_F (-\tfrac{1}{2} N_n)$$

$$\bar{V}_{\mu s} = V(\bar{\nu}_\mu) - V(\bar{\nu}_s) = - \sqrt{2} G_F (-\tfrac{1}{2} N_n) = - V_{\mu s}$$

$V_{e\mu} \neq \bar{V}_{e\mu}$: CP, CPT violated

P. Langacker, S.T.P. *et al.*, 1987

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \qquad (1)$$

where $\alpha = \nu_e,\ \beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\ [\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

In matter, $H_m = H_0 + H_{int}$.

$H_0|\nu_{1,2}> = E_{1,2}|\nu_{1,2}>$, not eigenstates of H_m.

Consider first $N_e = const.$

$$H_m|\nu^m_{1,2}> = E^m_{1,2}|\nu^m_{1,2}>.$$

Then at $t = 0$ in matter

$$|\nu_e> \; = |\nu^m_1> \cos\theta_m + |\nu^m_2> \sin\theta_m,$$

$$|\nu_{\mu(\tau)}> \; = -|\nu^m_1> \sin\theta_m + |\nu^m_2> \cos\theta_m;$$

$$\sin 2\theta_m = \frac{\epsilon'}{\sqrt{\epsilon^2+\epsilon'^2}} = \frac{\tan 2\theta}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta}},$$

$$\cos 2\theta_m = \frac{1-N_e/N_e^{res}}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta}},$$

$$N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F} \cong 6.56\times 10^6 \; \frac{\Delta m^2[\text{eV}^2]}{E[\text{MeV}]} \cos 2\theta \, \text{cm}^{-3} \; \text{N}_\text{A} \, ,$$

$$E^m_2 - E^m_1 = \frac{\Delta m^2}{2E}\left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta\right)^{\frac{1}{2}}$$

$$P_m^{2\nu}(\nu_e \to \nu_\mu) = |A_\mu(t)|^2 = \frac{1}{2}\sin^2 2\theta_m \ [1 - \cos 2\pi \frac{L}{L_m}\ ],$$

$$L_m = \frac{E_2^m - E_1^m}{2\pi} = L^v \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{-\frac{1}{2}}.$$

The resonance condition: $N_e = N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$

At the resonance:

$\sin^2 2\theta_m = 1$, $\min(E_2^m - E_1^m)$, $L_m^{res} = L^v / \sin 2\theta$.

Limiting cases:

$N_e \ll N_e^{res}$: $\theta_m \cong \theta$, $E_{1,2}^m \cong E_{1,2}$, $L_m \cong L^v$.

$N_e \gg N_e^{res}$: $\theta_m \cong \frac{\pi}{2}$, $\nu_e \to \nu_\mu$ suppressed.

In this case: $|\nu_e\rangle \cong |\nu_2^m\rangle$, $|\nu_\mu\rangle = -|\nu_1^m\rangle$.

Antineutrinos: $N_e \to (-N_e)$

$\Delta m^2 \cos 2\theta > 0$: $\bar{\nu}_e \to \bar{\nu}_\mu$ suppressed by matter; $\nu_e \to \nu_\mu$ can be enhanced.

$\Delta m^2 \cos 2\theta < 0$: $\nu_e \to \nu_\mu$ suppressed by matter; $\bar{\nu}_e \to \bar{\nu}_\mu$ can be enhanced.

V. Barger et al., 1980; S.P. Mikheyev, A.Yu. Smirnov, 1985

Oscillations in matter (Earth, Sun) are neither CP- nor CPT- invariant.

P. Langacker, S.T.P., S. Toshev, G. Steigman, 1987

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

$$P^m(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta_m\ (1 - \cos 2\pi \frac{L}{L_{osc}^m}),\ \ L_{osc}^m \sim L_{osc}^{vac}$$

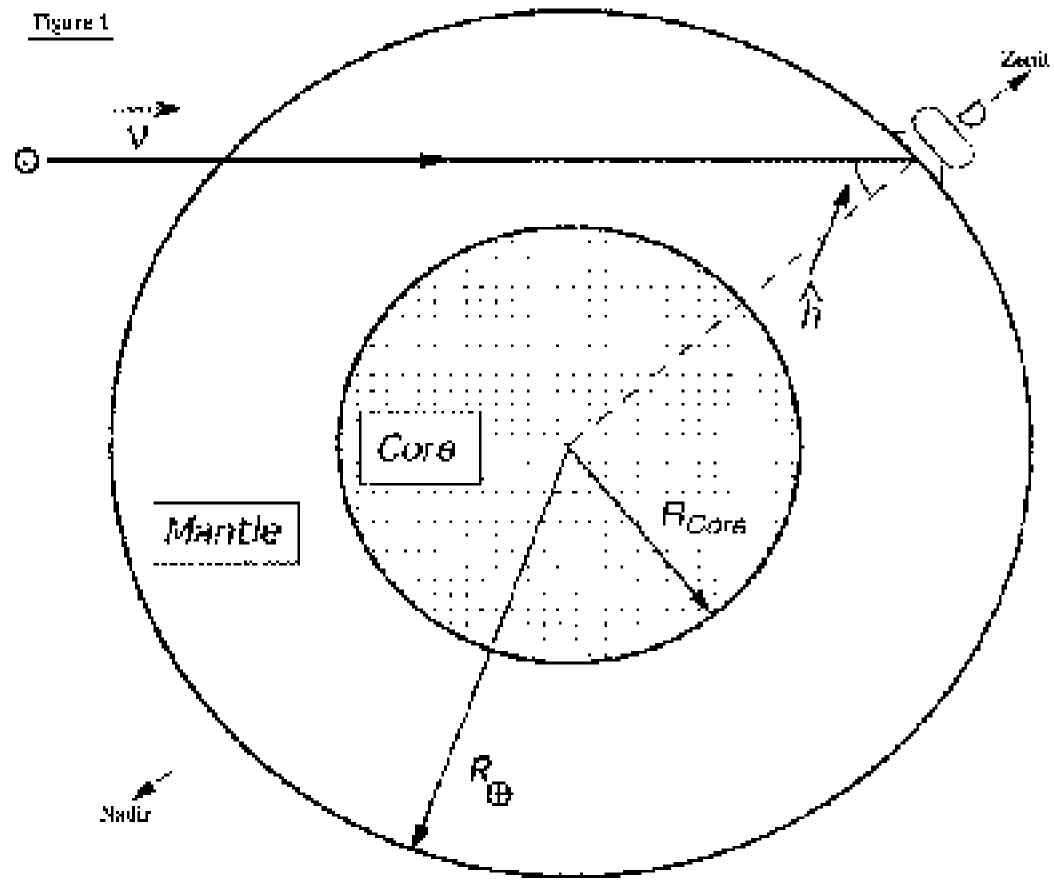
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(1-\frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta}\ \ , N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$N_e = N_e^{res}$: MSW (Mikheyev, Smirnov, Wolfenstein) resonance

$\Delta m^2 \cos 2\theta > 0$: $\nu_e \to \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$: $\bar{\nu}_e \to \bar{\nu}_\mu$

The Earth



Earth: $R_{core} = 3446$ km, $R_{mant} = 2885$ km

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

The Earth

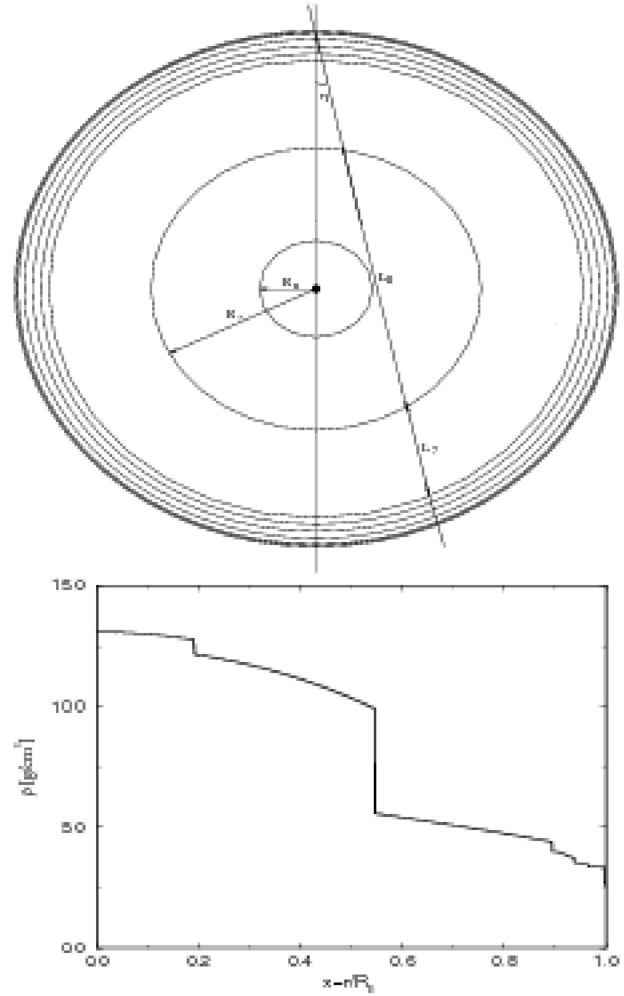
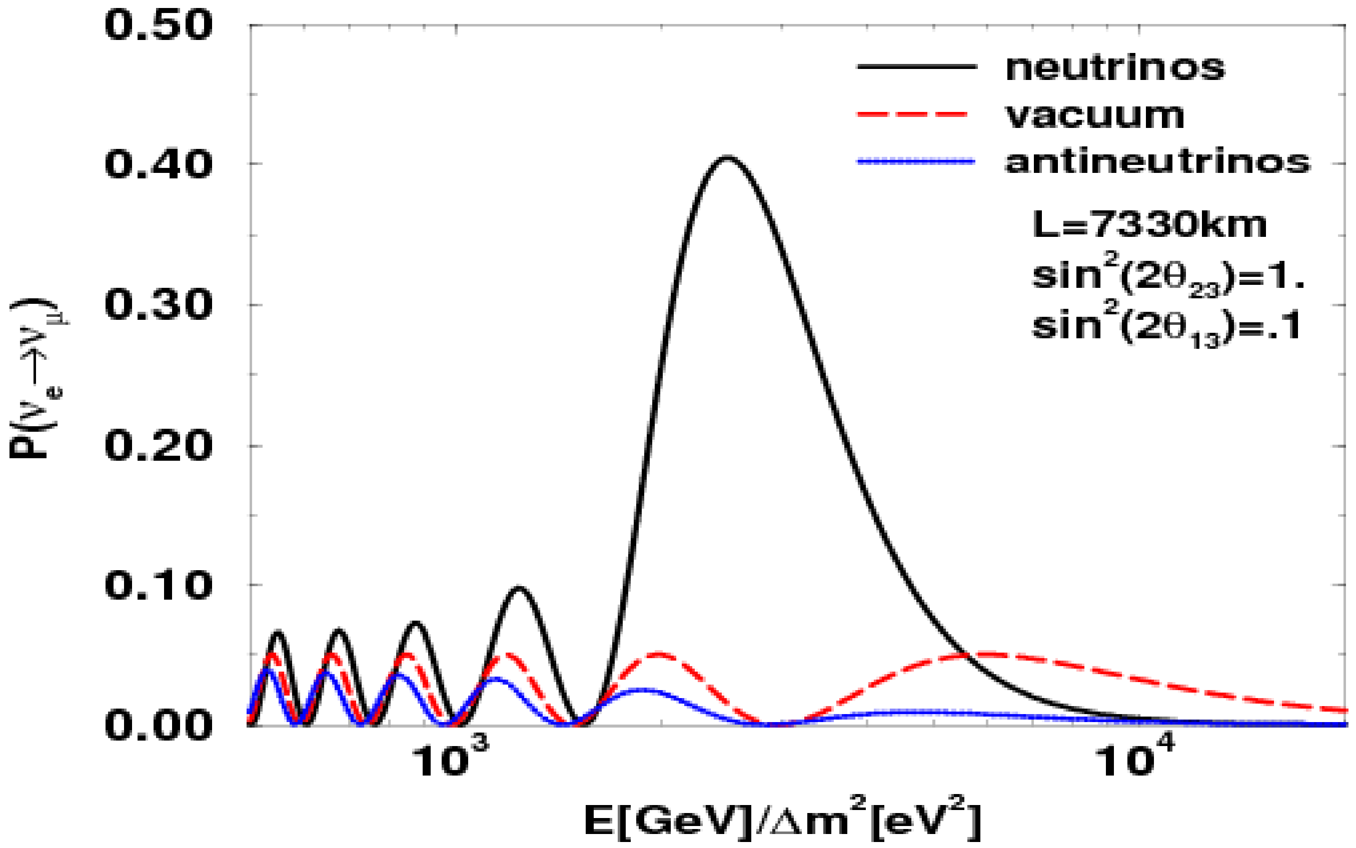


FIG. 1. Density profile of the Earth.

$R_c = 3446$ km, $R_m = 2885$ km; $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ GeV; $P^{3\nu} = \sin^2\theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;
$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^{v}/\sin 2\theta_{13} \cong 6250/0.32$ km; $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger, S. Pakvasa et al.,1980.

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ , \quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

3ν — mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ , \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

$$A_{\text{T(CP)}}^{(e,\mu)} = A_{\text{T(CP)}}^{(\mu,\tau)} = -A_{\text{T(CP)}}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa *et al.*, 1980

In vacuum: $A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin\left(\frac{\Delta m^2_{21}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E} L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$

CPT : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

In matter with constant density: $\mathrm{A}^{(e,\mu)}_{\mathrm{T}} = \mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} \mathrm{F}^{\mathrm{mat}}_{\mathrm{osc}}$

$$\mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} = \mathrm{J}^{\mathrm{vac}}_{\mathrm{CP}}\ \mathrm{R}_{\mathrm{CP}}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} \ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\left\{U_{e1}U_{e3}^{*}\right\}, \quad S_2 = \text{Im}\left\{U_{e2}U_{e3}^{*}\right\} \quad \text{(not unique);} \quad \text{or}$$

$$S_1' = \text{Im}\left\{U_{\tau 1}U_{\tau 2}^{*}\right\}, \quad S_2' = \text{Im}\left\{U_{\tau 2}U_{\tau 3}^{*}\right\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\left\{U_{e1}U_{e3}^{*}\right\} \neq 0$ and $\text{Re}\left\{U_{e1}U_{e3}^{*}\right\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P^{3\nu\ man}_m(\nu_e \to \nu_\mu) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

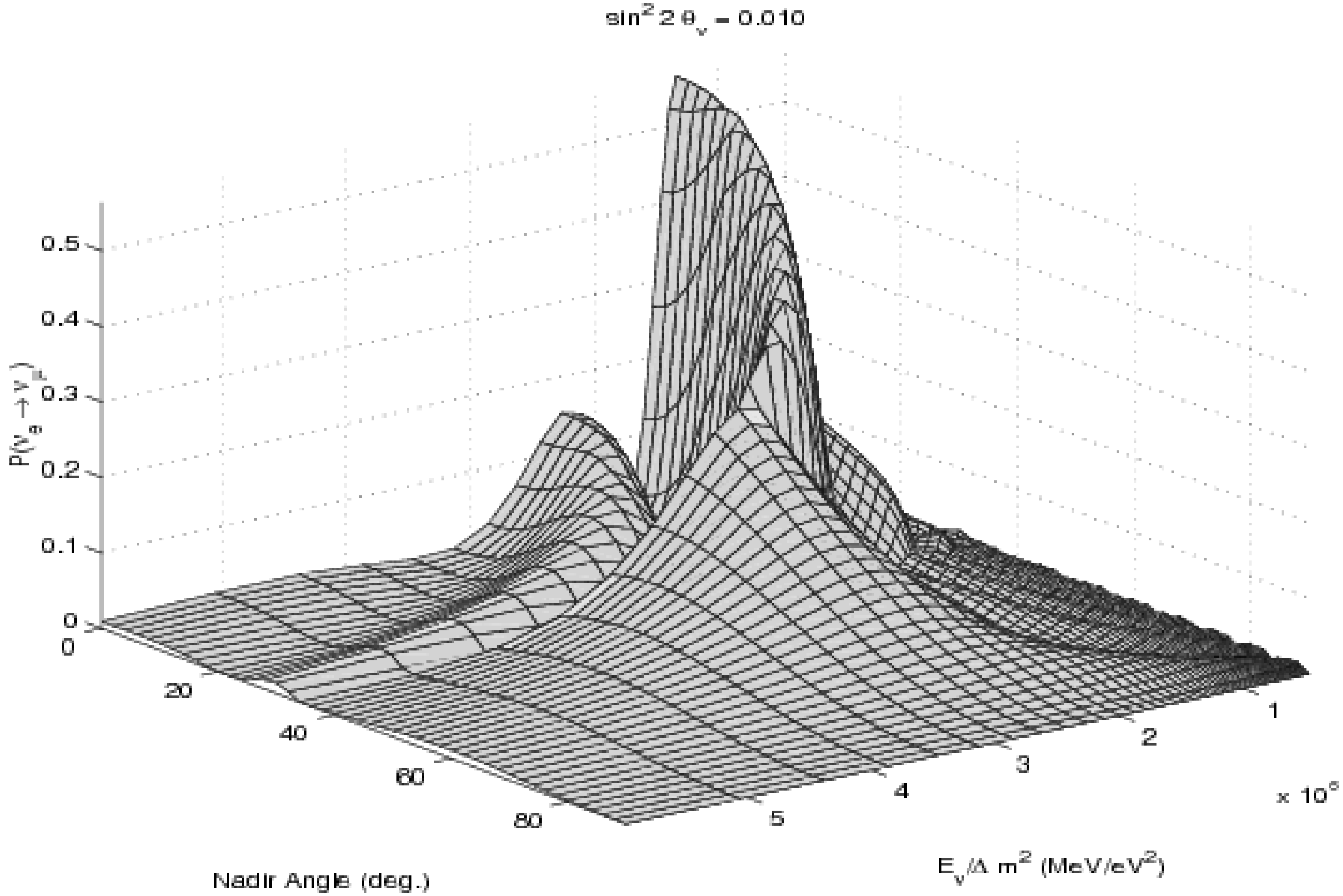
$$P_{\sin\delta} = \alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_F N^{man}_e\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_e \to \bar{\nu}_\mu$: $\delta,\;\; A \to (-\delta),\;\; (-A)$

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)

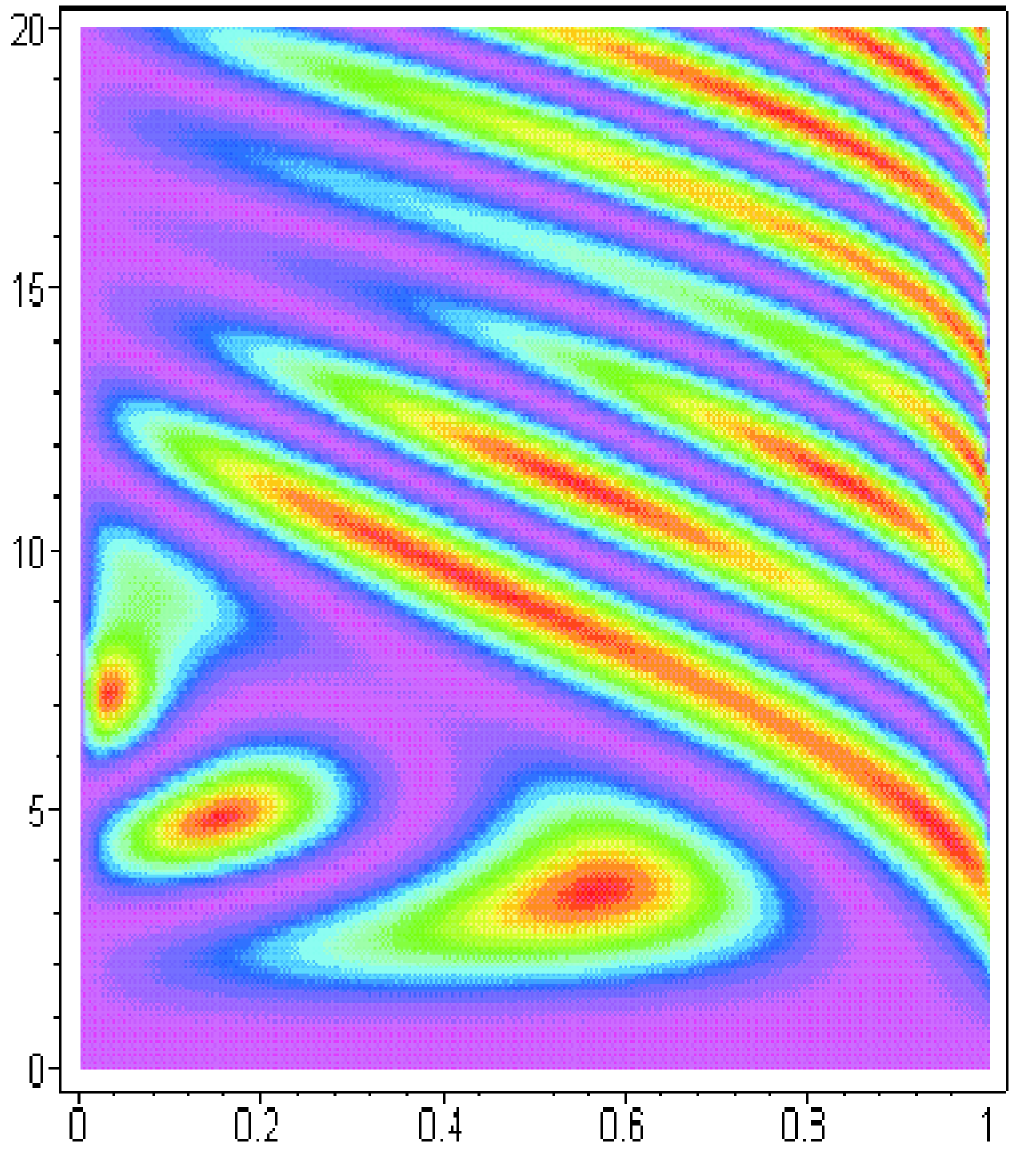


S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_v \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{\rm atm}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E$ $[10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

- **For Earth center crossing ν's ($\theta_n = 0$) and, e.g.** $\sin^2 2\theta_{13} = 0.01$, **NOLR occurs at** $E \cong 4$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

S.T.P., hep-ph/9805262

- **For the Earth core crossing ν's:** $P_{2\nu} = 1$ **due to NOLR when**

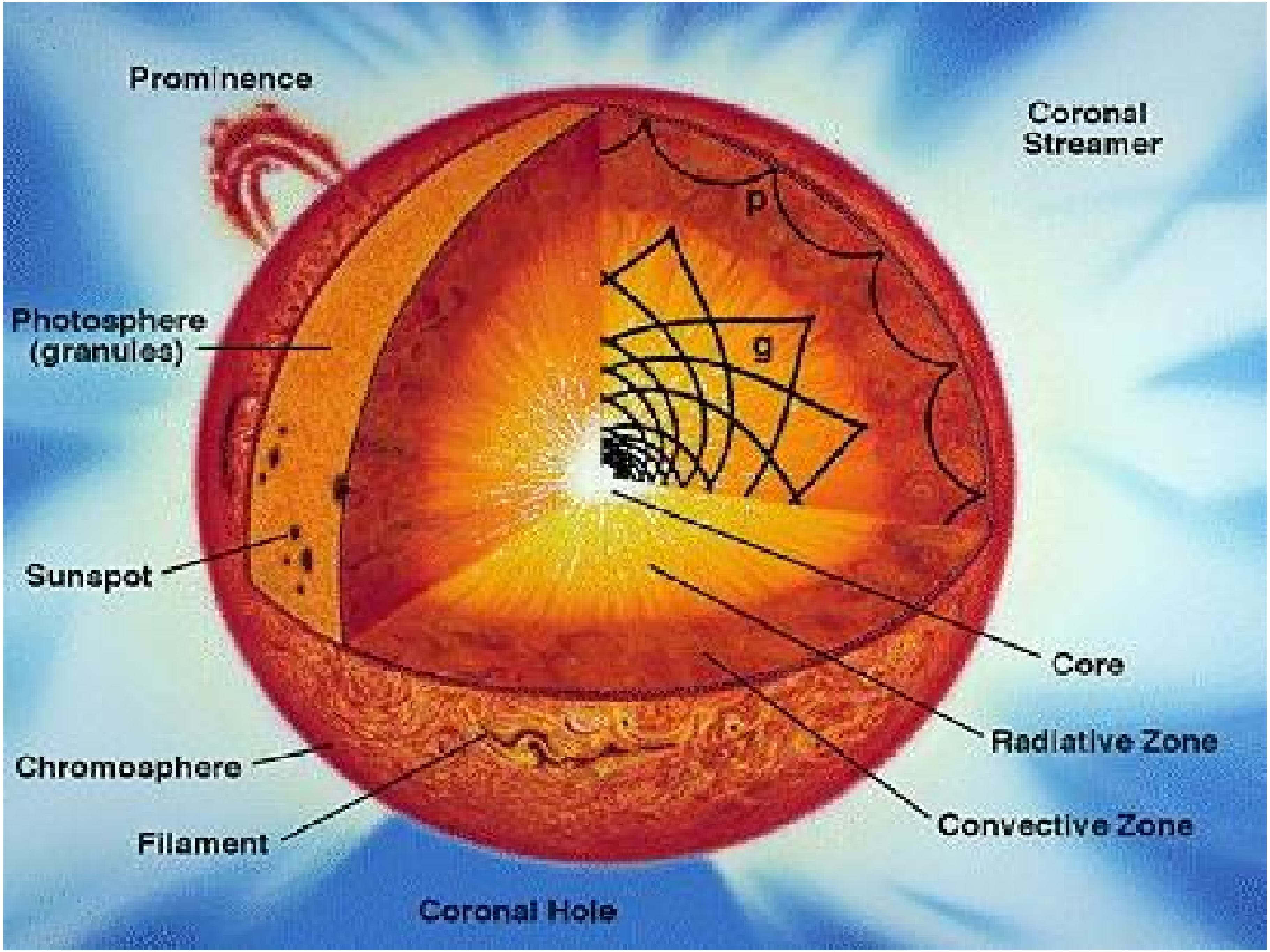
$$\tan \Phi^{\rm man}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\rm core}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta'_m}{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}$$

$\Phi^{\rm man}$ ($\Phi^{\rm core}$) **- phase accumulated in the Earth mantle (core),**
θ'_m (θ''_m) **- the mixing angle in the Earth mantle (core).**

$P_{2\nu} = 1$ **due to NOLR for** $\theta_n = 0$ **(Earth center crossing ν's) at,**
e.g. $\sin^2 2\theta_{13} = 0.034;\ 0.154$, $E \cong 3.5;\ 5.2$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).


Prominence
Coronal Streamer
p
g
Photosphere (granules)
Sunspot
Core
Radiative Zone
Chromosphere
Convective Zone
Filament
Coronal Hole

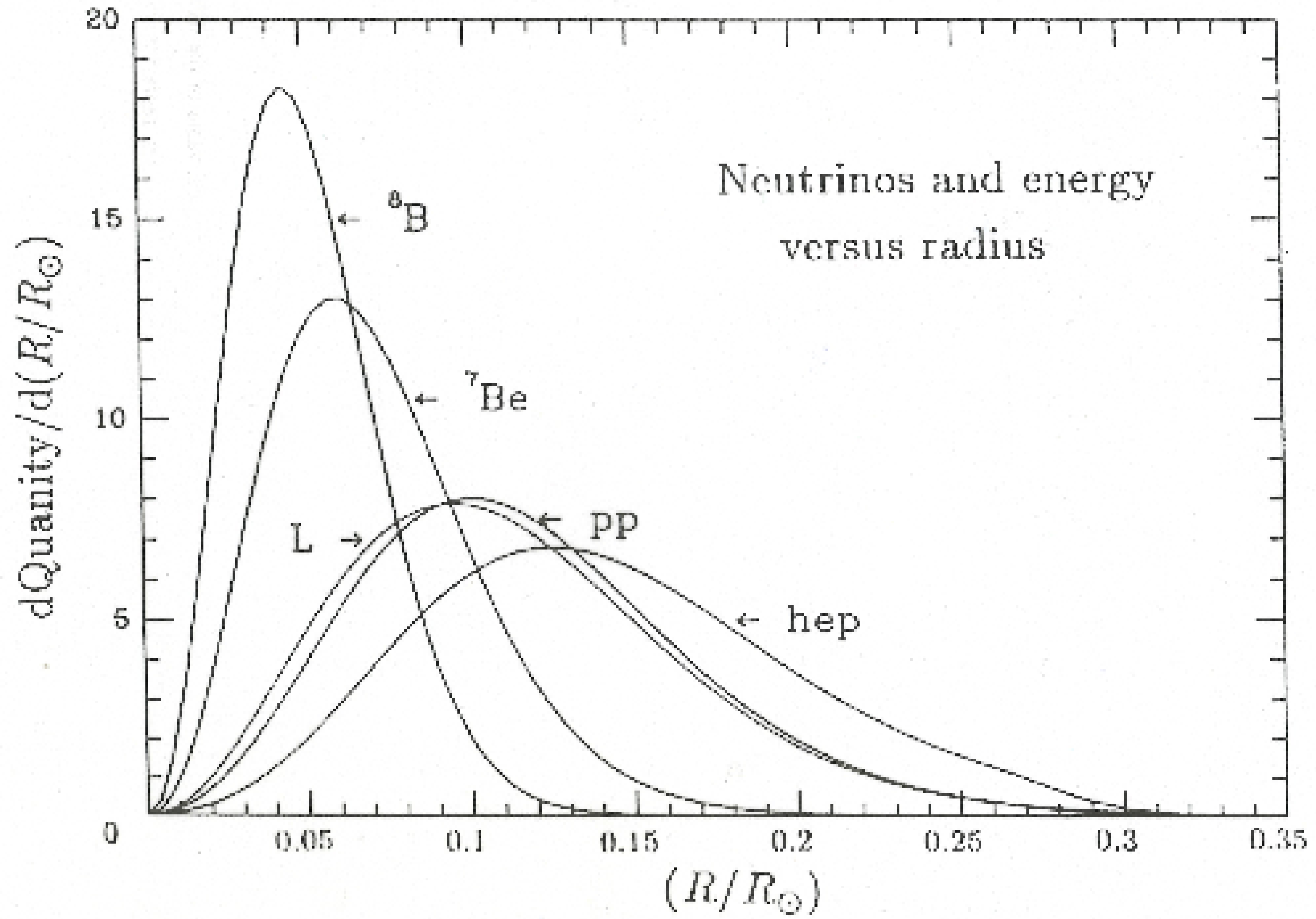
Solar Neutrino Production: pp Chain

REACTION	TERM. (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.420
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.442
${}^2H + p \rightarrow {}^3He + \gamma$	(100)	
${}^3He + {}^3He \rightarrow \alpha + 2\,p$	(85)	
or		
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	0.861 90% 0.383 10%
${}^7Li + p \rightarrow 2\,\alpha$		
or		
${}^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		< 15
${}^8Be^* \rightarrow 2\,\alpha$		
or		
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.000004)	≤ 18.8

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e.$$

- pp neutrinos, $E \leq 0.420$ MeV, $\bar{E} = 0.265$ MeV,
- ^7Be neutrinos, E=0.862 MeV (89.7% of the flux), 0.384 MeV (10.3%) ,
- ^8B neutrinos, $E \leq 14.40$ MeV, $\bar{E} = 6.71$ MeV,
- pep neutrinos, E=1.442 MeV,
- of ^{13}N, $E \leq 1.199$ MeV, $\bar{E} = 0.707$ MeV,
- of ^{15}O, $E \leq 1.732$ MeV, $\bar{E} = 0.997$ MeV.

J. N. Bahcall


Neutrinos and energy
versus radius
^{8}B
^{7}Be
L
pp
hep
dQuanity/d($R/R_\odot$)
($R/R_\odot$)
0
5
10
15
20
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Flux	BP'00	Cl–Ar	Ga–Ge
$\Phi_{pp} \times 10^{-10}$	$5.95(1\ ^{+0.01}_{-0.01})$	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	$1.40(1\ ^{+0.01}_{-0.01})$	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	$4.77(1\ ^{+0.09}_{-0.09})$	1.15	34.2
$\Phi_{B} \times 10^{-6}$	$5.93(1\ ^{+0.14}_{-0.15})$	6.76	14.2
$\Phi_{N} \times 10^{-8}$	$5.48(1\ ^{+0.19}_{-0.13})$	0.09	3.4
$\Phi_{O} \times 10^{-8}$	$4.80(1\ ^{+0.22}_{-0.15})$	0.33	5.5
Total		$8.55\ ^{+1.1}_{-1.2}$	$129.8\ ^{+9}_{-7}$

Solar Neutrinos ν_e, $E \sim$1 MeV: B. Pontecorvo 1946

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

R. Davis et al., 1967 - 1996: 615 t C_2Cl_4; 0.5 Ar atoms/day, exposure 60 days.

$$\nu + e^- \rightarrow \nu + e^-$$

Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);

$$\nu_e + D \rightarrow e^- + p + p \ , \qquad \text{SNO}$$

$$\nu_l + D \rightarrow \nu_l + n + p \, , \ l = e, \mu, \tau \qquad \text{SNO}$$

Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

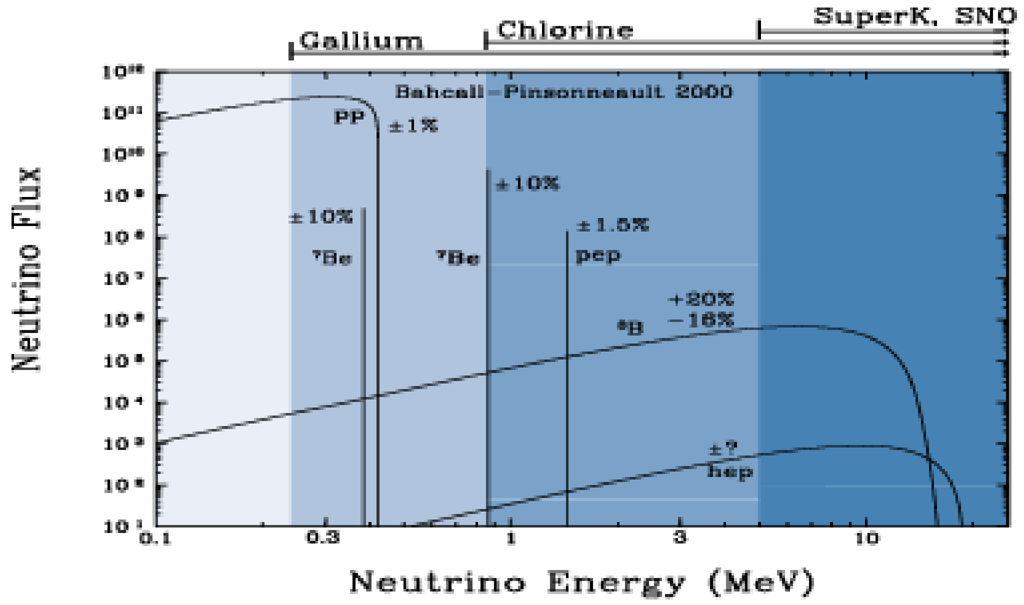


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

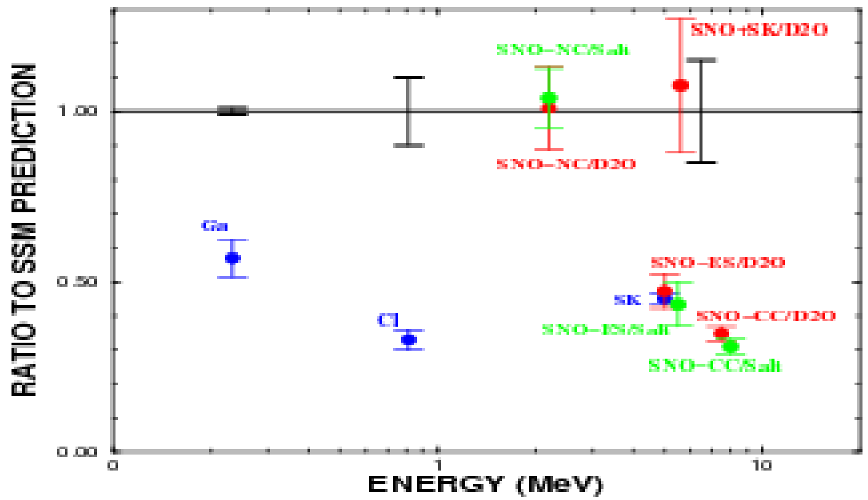
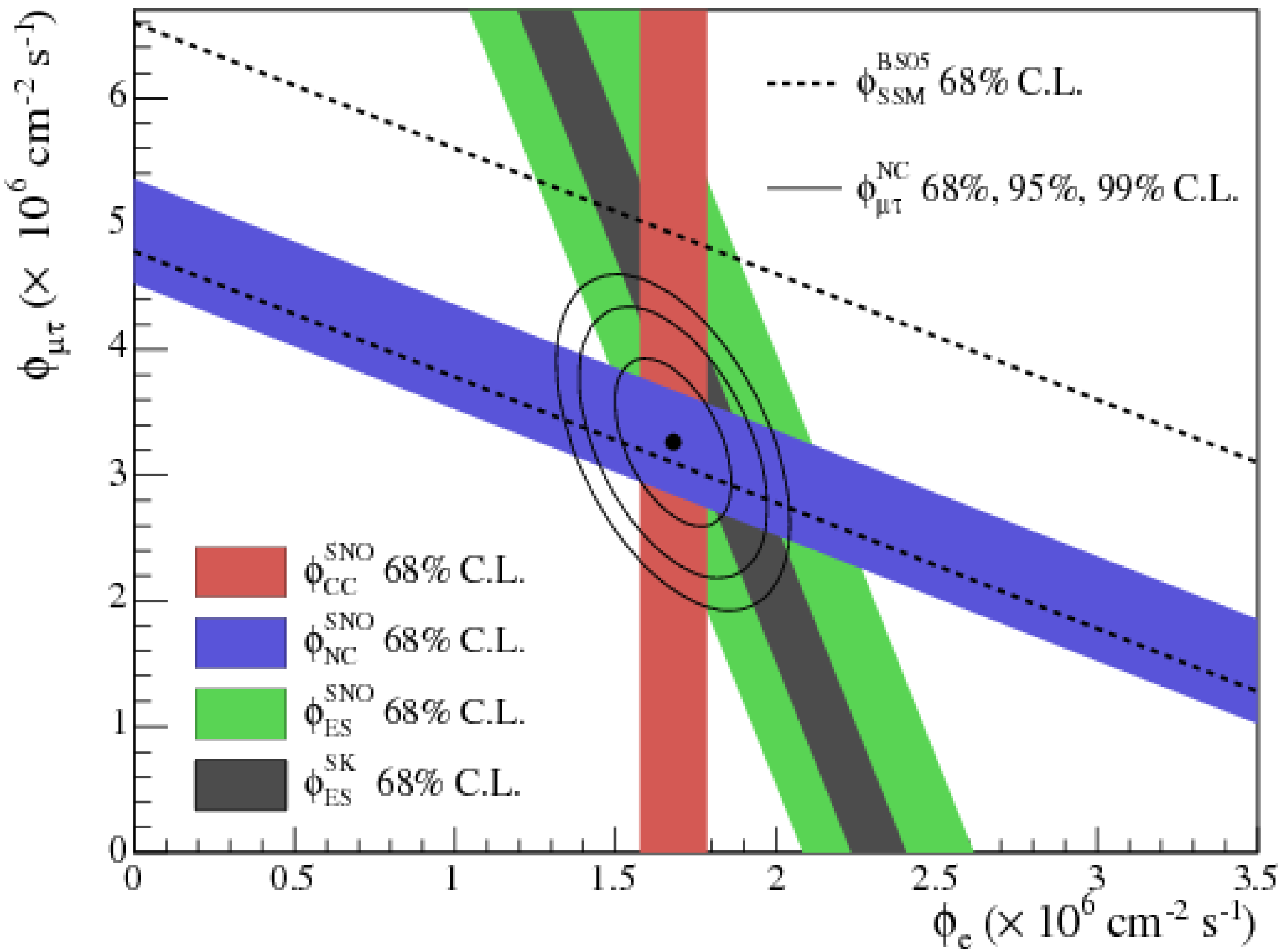


Figure 3: Comparison of measurements to Standard Solar Model predictions.

Flux	BP'00	Cl–Ar	Ga–Ge
$\Phi_{pp} \times 10^{-10}$	$5.95(1\ ^{+0.01}_{-0.01})$	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	$1.40(1\ ^{+0.01}_{-0.01})$	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	$4.77(1\ ^{+0.09}_{-0.09})$	1.15	34.2
$\Phi_{B} \times 10^{-6}$	$5.93(1\ ^{+0.14}_{-0.15})$	6.76	14.2
$\Phi_{N} \times 10^{-8}$	$5.48(1\ ^{+0.19}_{-0.13})$	0.09	3.4
$\Phi_{O} \times 10^{-8}$	$4.80(1\ ^{+0.22}_{-0.15})$	0.33	5.5
Total		$8.55\ ^{+1.1}_{-1.2}$	$129.8\ ^{+9}_{-7}$

Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted Rate at solar best-fit
Ga	0.52 ± 0.029	0.555	0.540
Cl	0.301 ± 0.027	0.356	0.345
SK(ES)	0.406 ± 0.014	0.394	0.395
SNO(CC)	0.274 ± 0.019	0.289	0.289
SNO(ES)	0.38 ± 0.052	0.386	0.386
SNO(NC)	0.895 ± 0.08	0.889	0.908

The observed rates w.r.t predictions from the latest Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of Δm^2_{21} and $\sin^2\theta_{12}$, obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.



φ_SSM^BS05 68% C.L.
φ_μτ^NC 68%, 95%, 99% C.L.
φ_CC^SNO 68% C.L.
φ_NC^SNO 68% C.L.
φ_ES^SNO 68% C.L.
φ_ES^SK 68% C.L.
φ_μτ (× 10^6 cm^-2 s^-1)
φ_e (× 10^6 cm^-2 s^-1)
0
1
2
3
4
5
6
0.5
1.5
2.5
3.5

MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \qquad (2)$$

where $\alpha = \nu_e,\ \beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\ [\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2} G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) = N_e(t_0)\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1 R_\odot$, $R_\odot = 6.96 \times 10^5$km

The region of $\nu_\odot$ production: $r \lesssim 0.2 R_\odot$

$20 \; N_A \; cm^{-3} \lesssim N_e(x_0) \lesssim 100 \; N_A \; cm^{-3}$

Suppose $N_e(x_0) \gg N_e^{res}$: $|\nu_e\rangle \;\cong\; |\nu_2^m\rangle$.

Possible evolution:

The system stays at this level; at the surface: $|\nu_2^m\rangle = |\nu_2\rangle$

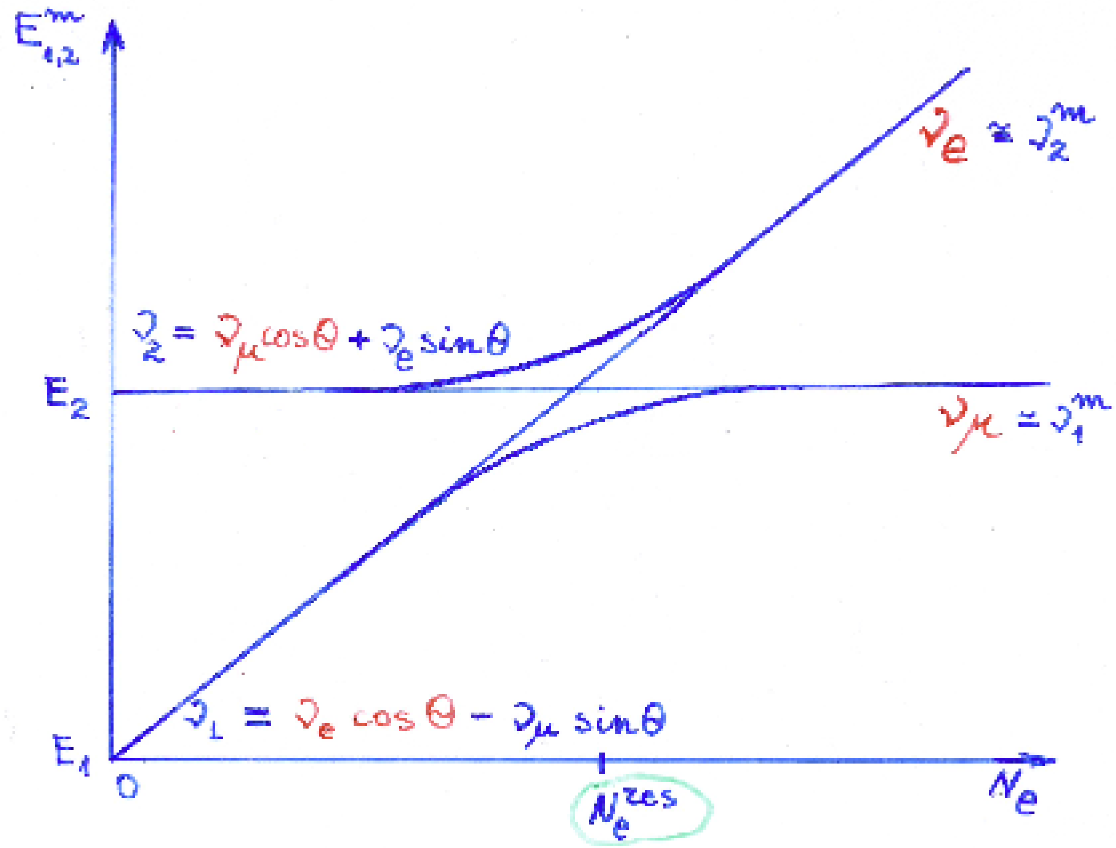
$P(\nu_e \to \nu_e) \cong |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta \, ,$ Adiabatic

At $N_e = N_e^{res}$, where $E_2^m - E_1^m$ is minimal, the system jumps to lower level $|\nu_1^m\rangle$; at the surface: $|\nu_1^m\rangle = |\nu_1\rangle$

$P(\nu_e \to \nu_e) \cong |\langle \nu_e | \nu_1 \rangle|^2 = \cos^2\theta \, ,$ Nonadiabatic

Type of transition: $P' \equiv P(\nu_2^m(t_0) \to \nu_1) \, ,$ jump probability

$\sin\theta \ll 1.$



1. $P(\nu_2^{\odot} \to \nu_1; t_\odot, t_0) \equiv P'$ - negligible : adiabatic transition
2. P' - nonnegligible : nonadiabatic

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a} \; e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'} \; A'_e(t, t_0),$$

$A'_e(t, t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{ Z\frac{d^2}{dZ^2} + (c - Z)\ \frac{d}{dZ} - a\right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0\ \frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\ \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$$\Psi(\vec{r}) = \frac{1}{r}\psi_{kl}(r)Y_{lm}(\theta', \phi'),$$

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2}\, e^{Z/2}\, \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2\,\frac{r}{a_0}\sqrt{-E_{kl}/E_I},\; a \equiv a_{kl} = l+1-\sqrt{-E_I/E_{kl}},\; c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\,\Phi(a-c+1, 2-c; Z)$; $\Phi(a',c';Z=0)=1$, $a',c' \neq 0,-1,-2,\ldots$.

$$A(\nu_e \to \nu_{\mu(\tau)}) = \frac{1}{2}\,\sin 2\theta\,\left\{\Phi(a-c, 2-c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}}\,\Phi(a-1, c; Z_0)\right\}.$$

Sun: $N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1R_\odot$, $R_\odot \cong 7\times 10^5$ km

The region of $\nu_\odot$ production:

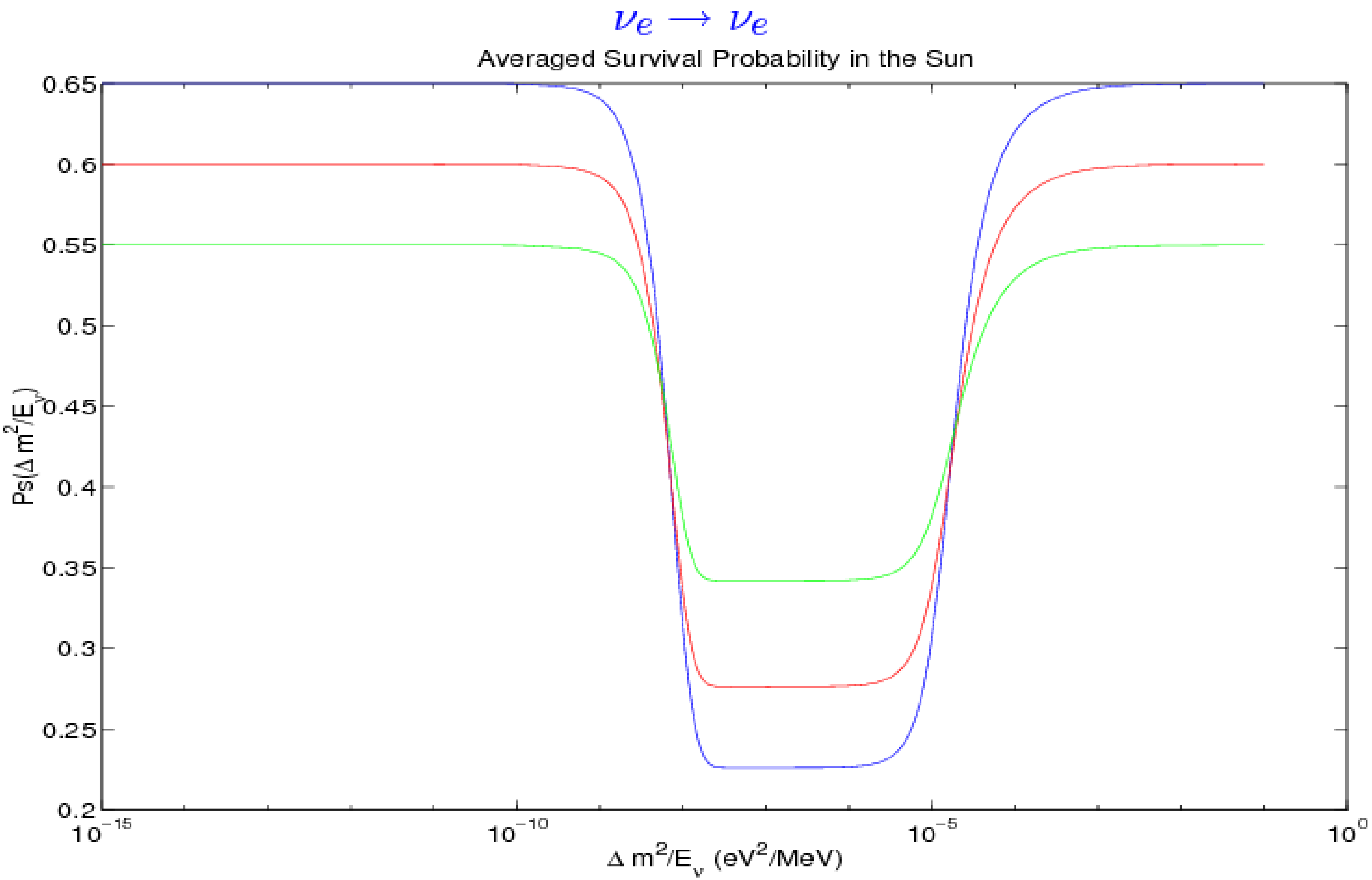
20 $N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$: $|Z_0| > 500$ (!)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \to \nu_e) = \tfrac{1}{2} + (\tfrac{1}{2} - P')\cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988


νe → νe
Averaged Survival Probability in the Sun
Ps(Δ m2/Eν)
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
10−15
10−10
10−5
100
Δ m2/Eν (eV2/MeV)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \to \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

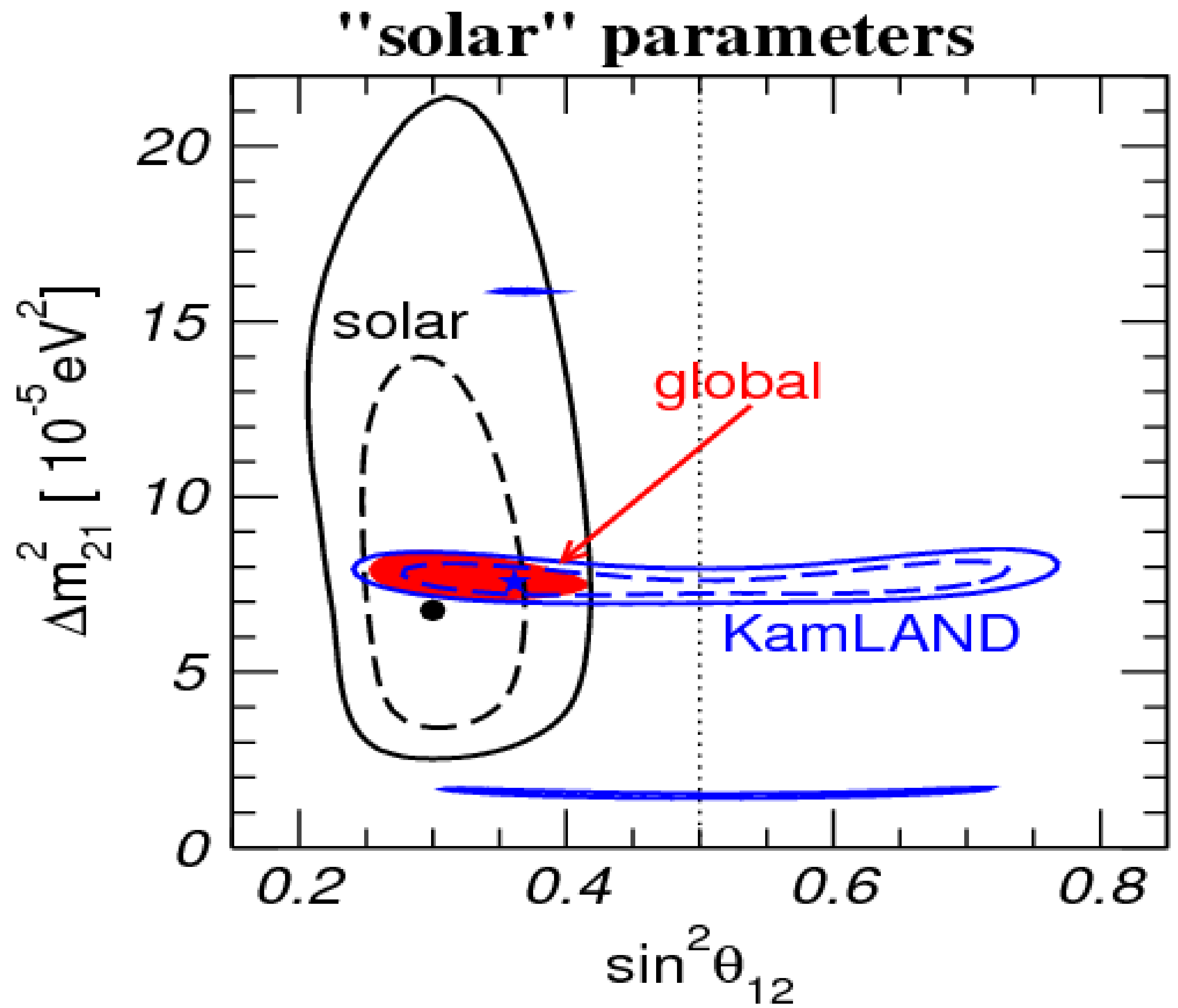
$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

Case 1: $\cos 2\theta_m^0 = -1$, $P' = 0$, $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$.

Case 2: $\theta_m^0 = \theta$, $P' = 0$, $\bar{P}(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$

Case 1: SNO, Super Kamiokande; $\bar{P} \cong 0.3$: $\cos 2\theta > 0$!

Case 2: pp neutrinos.



T. Schwetz, arXiv:0710.5027[hep-ph]

J.N. BAHCALL, M.C. GONZALEZ-GARCIA, C. PEÑA GARAY '01

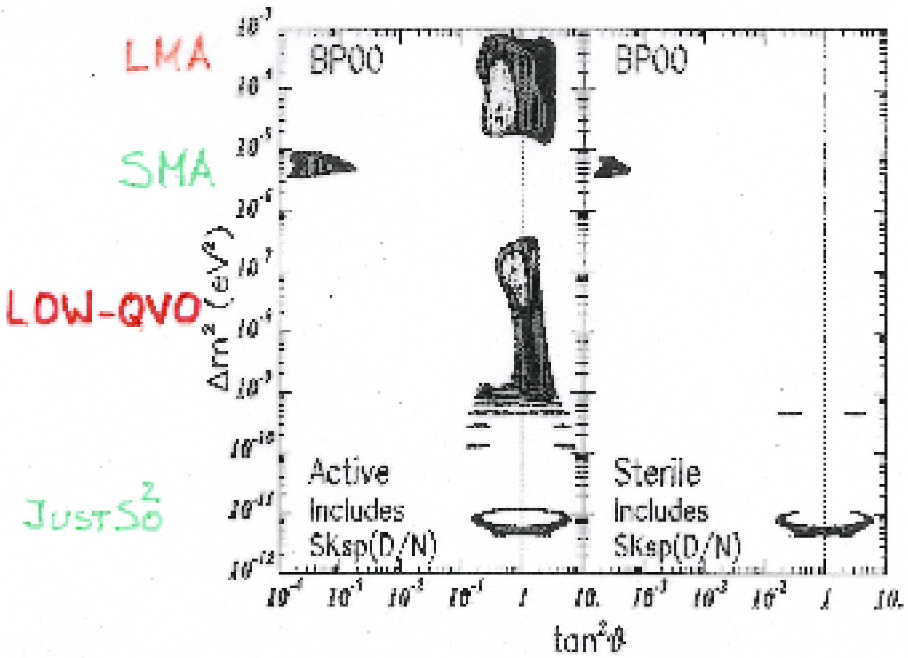


Figure 1: Global solutions including all available solar neutrino data. The input data include the total rates from the Chlorine [2], Gallium (averaged) [3, 5, 4], Super-Kamiokande [6], and SNO [1] experiments, as well as the recoil electron energy spectrum measured by Super-Kamiokande during the day and separately the energy spectrum measured at night. The C.L. contours shown in the figure are 90%, 95%, 99%, and 99.73% (3σ). The allowed regions are cutoff below 10^{-3} eV^2 by the Chooz reactor measurements [22]. The local best-fit points are marked by dark circles. The theoretical errors for the BP2000 neutrino fluxes are included in the analysis.

The reference scheme: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

3-flavour neutrino oscillation probabilities

$P(\nu_l \to \nu_{l'})$: $m_2^2 - m_1^2 \equiv \Delta m_{21}^2$, $m_3^2 - m_1^2 \equiv \Delta m_{31}^2$

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48$ $(2.44) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425$ (0.437), NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0239), NH (IH).

F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = -\Delta m^2_{32}(IO)$$

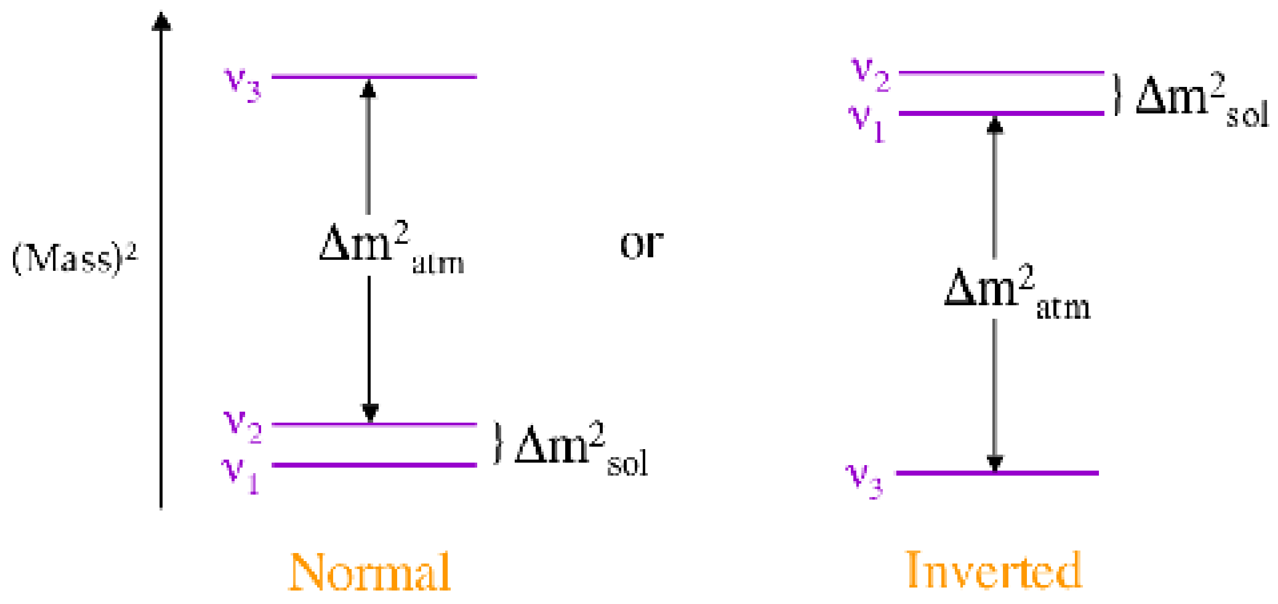
$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;

The (Mass)^2 Spectrum



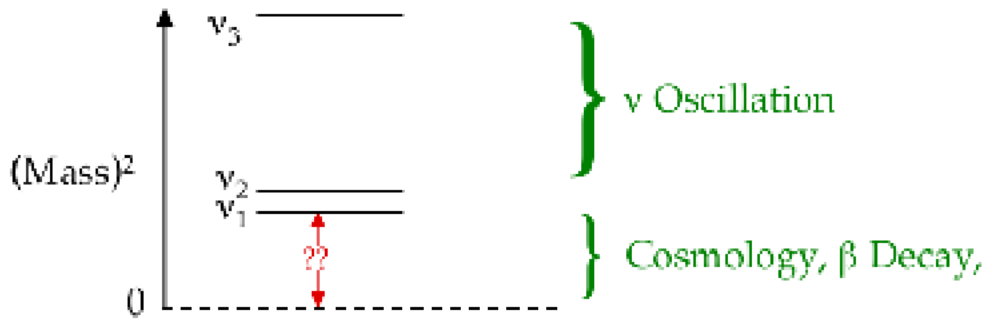
$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

Due to B. Kayser

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

4

Due to B. Kayser

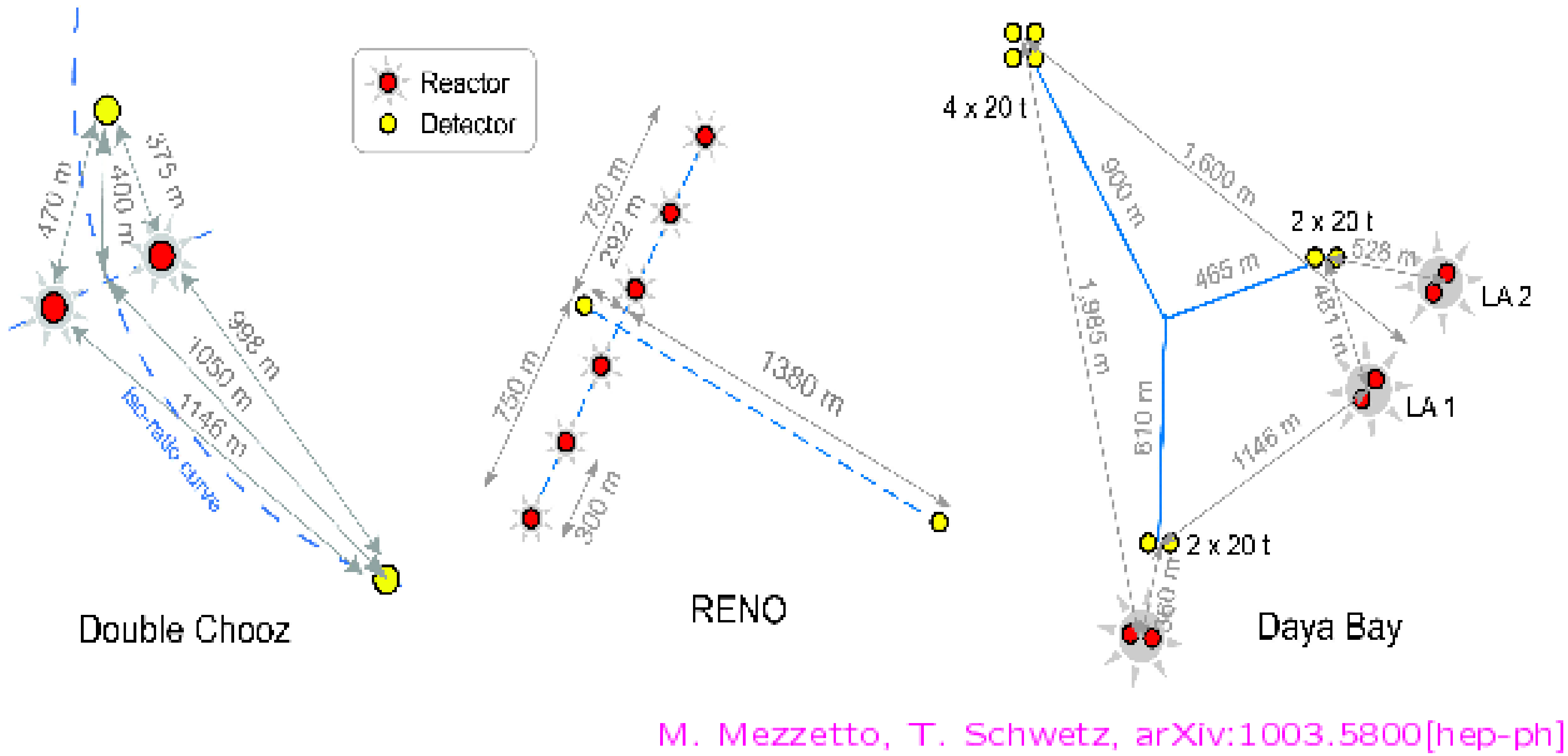
- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.

- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.

- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ and 2.9σ evidence for $\theta_{13} \neq 0$.

- Daya Bay, 23/08/2013: $\sin^2 2\theta_{13} = 0.090 \pm 0.009$.

- RENO, 12/09/2013 (TAUP 2013): $\sin^2 2\theta_{13} = 0.100 \pm 0.010\ (stat.) \pm 0.012$.


Reactor
Detector
470 m
400 m
375 m
998 m
1050 m
1146 m
iso-ratio curve
Double Chooz
750 m
292 m
750 m
300 m
1380 m
RENO
4 x 20 t
900 m
1,600 m
2 x 20 t
528 m
465 m
481 m
LA 2
LA 1
1,985 m
610 m
1146 m
2 x 20 t
360 m
Daya Bay
M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

T2K: Search for $\nu_\mu \to \nu_e$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ;
July, 2013 (28 events).

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.18$, best fit.

This value is by a factor of ~ 2 bigger than the value obtained in the Daya Bay and RENO experiments.


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

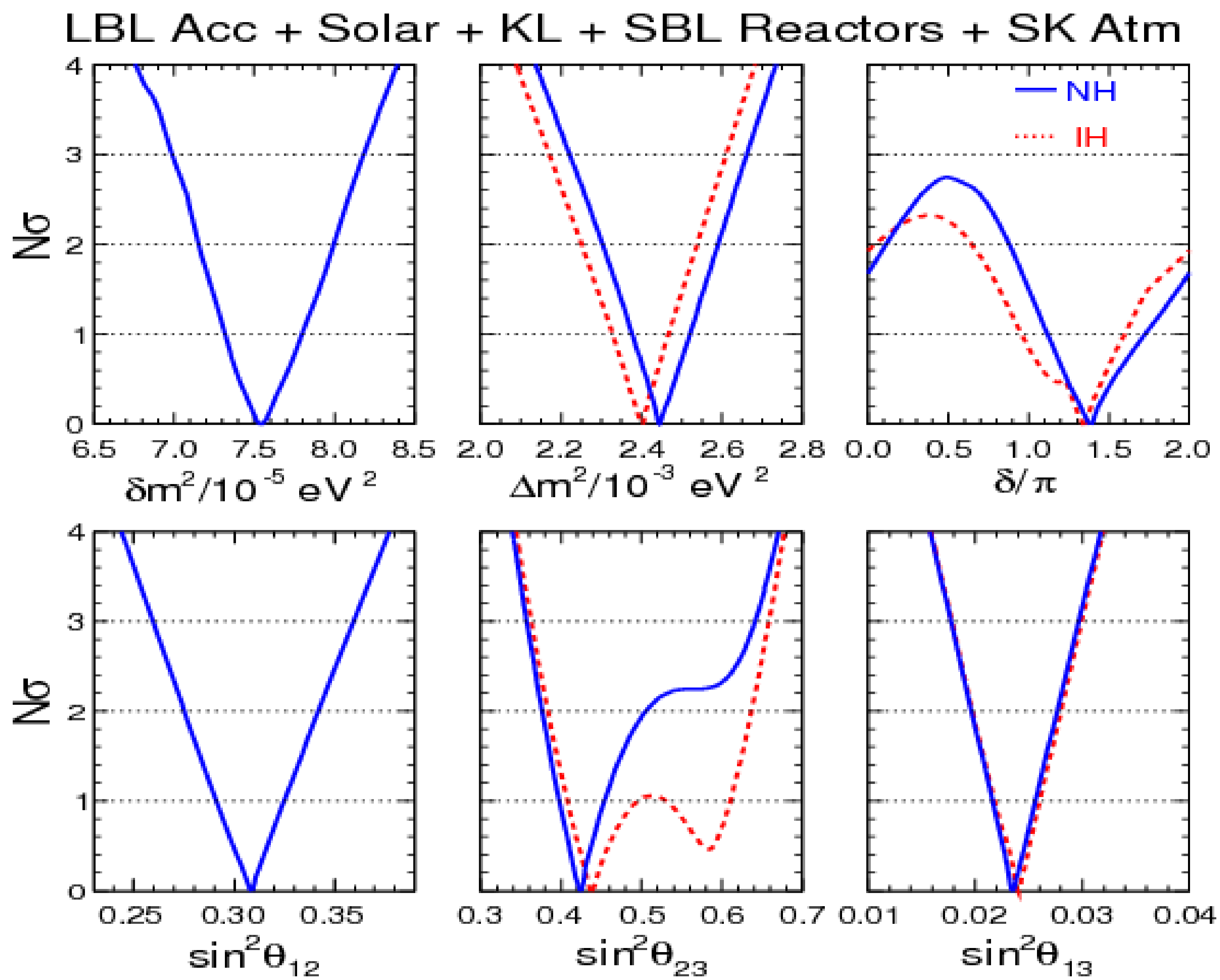
$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

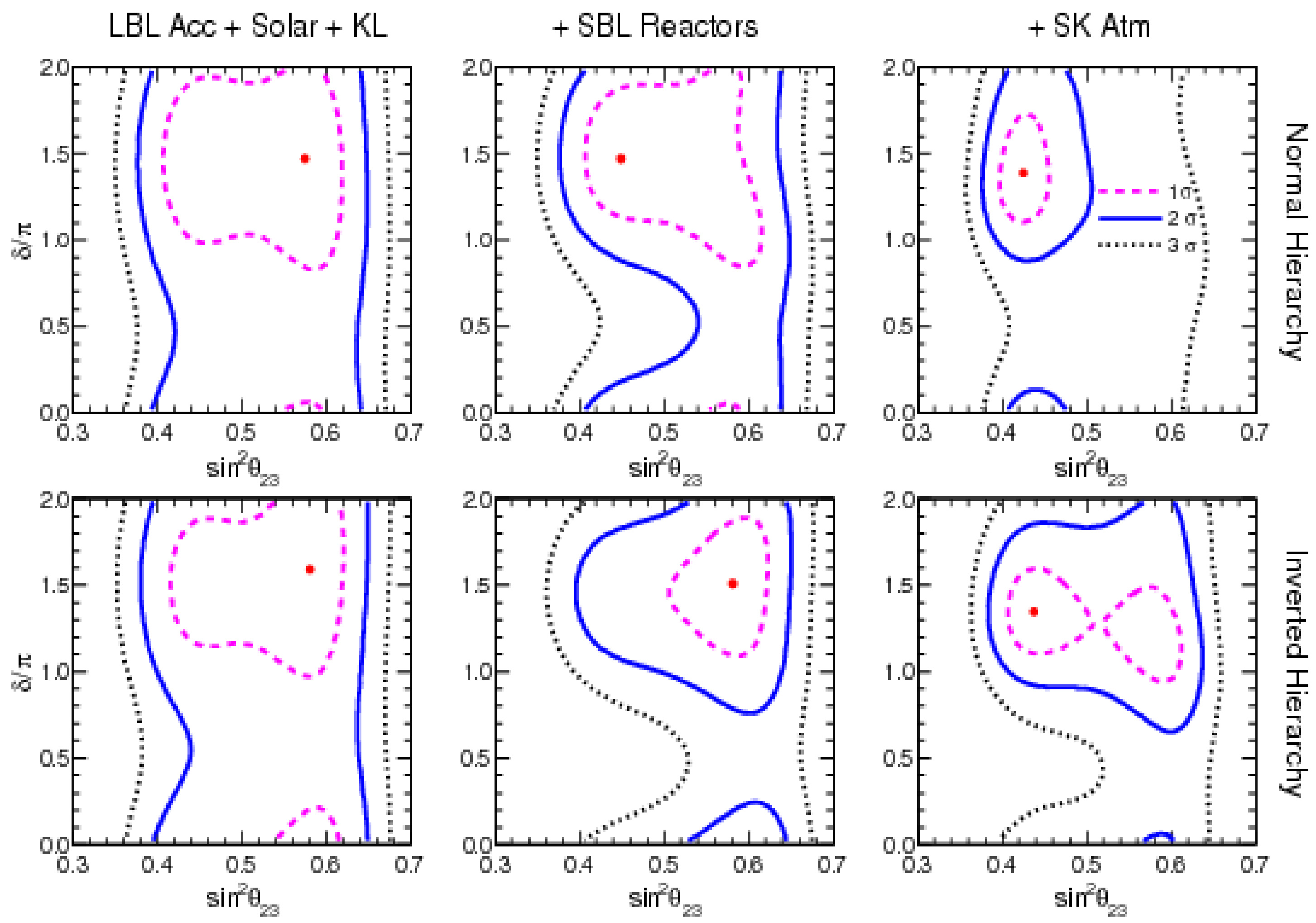
$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\ \ A = \sqrt{2}G_{\mathrm{F}}N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: δ, $A \to (-\delta)$, $(-A)$

$$
\begin{aligned}
J_{CP} &= \mathrm{Im}\left\{U_{e1}\,U_{\mu 2}\,U_{e2}^{*}\,U_{\mu 1}^{*}\right\} \\
&= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta
\end{aligned}
$$



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- **Dirac phase δ**: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- **Majorana phases α_{21}, α_{31}**:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|< m >|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, θ_{12}, $\Delta m^2_{\rm atm}$, θ_{23}, θ_{13}

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

  – the origin of the observed patterns of ν-mixing and ν-masses ;

  – the physical origin of CPV phases in U_{PMNS} ;

  – Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

  – Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

  – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

  – Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

- Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

  – Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) - far-reaching implications for the program of reserach in neutrino physics:

- For the determination of the type of $\nu-$ mass spectrum (or of $\text{sgn}(\Delta m^2_{\text{atm}})$) in neutrino oscillation experiments.

- For understanding the patern of the neutrino mixing and its origins (symmetry, etc.?).

- For the predictions for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).

Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;
- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13}\,\sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20, \quad \theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 - 0.11.$
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^{\dagger}(\theta_{ij}^{\ell}, \psi)\; Q(\phi, \varphi) U_{\text{tri,bim,LC}}\; P(\alpha_{21}, \alpha_{31})\,,$$

with

$$U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^{\dagger}(\theta_{ij}^{\ell}, \psi)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{bim,tri,LC}}\; P(\alpha_{21}, \alpha_{31})$ - from diagonalization of the ν mass matrix;
- $Q(\phi, \varphi)$, - from diagonalization of both the l^- and ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\text{tri(bim)}}$: Groups A_4, S_4, T', ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- U_{bim}: alternatively $U(1)$, $\quad L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

None of the symmetries leading to U_{tri}, U_{bim} or other approximate forms of U_{PMNS} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In the two cases of U_ν given by U_{tri}, or U_{bim}, the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\text{tri,bim}}$ and on the form of U_{lep}, one obtaines diifferent experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the obderved patterns of neutrino mixing and Δm^2_{ij}.

Predictions for δ

Assume:

- $U_{PMNS} = U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \psi)\ Q(\phi, \varphi) U_{\text{TBM,BM}}\ P(\alpha_{21}, \alpha_{31})$,
- $U^{\dagger}_{\text{lep}}$ - minimal, such that

i) $\sin\theta_{13} \cong 0.16$; BM: $\sin^2\theta_{12} \cong 0.31$;
ii) $\sin^2\theta_{23}$ can deviate significantly (by more than $\sin^2\theta_{13}$) from 0.5 (b.f.v. = 0.42-0.43) .

From i), ii) + $m_e << m_\mu << m_\tau$:

$$U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \psi) = R_{12}(\theta^{\ell}_{12})\, R_{23}(\theta^{\ell}_{23})\,,\quad Q(\phi, \varphi) = \text{diag}(1, e^{i\phi}, 1)$$

Leads to $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$ - new sum rules for δ!

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

For $U_{\rm TBM}$:

$$\cos\delta = \frac{\tan\theta_{23}}{3\sin 2\theta_{12}\sin\theta_{13}}\left[1 + \left(3\sin^2\theta_{12} - 2\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]$$

For $U_{\rm TBM}$ + b.f.v. of $\theta_{12},\theta_{23},\theta_{13}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \qquad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

T' model of lepton flavour: $U_{\rm TBM}$, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

For $U_{\rm BM}$:

$$\cos\delta = -\frac{1}{2\sin\theta_{13}}\cot 2\theta_{12}\tan\theta_{23}\left(1-\cot^2\theta_{23}\sin^2\theta_{13}\right).$$

For $U_{\rm BM}$ + b.f.v. of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.

- determination of the neutrino mass hierarchy;

- determination of the absolute neutrino mass scale (or min(m_j));

- determination of the status of the CP symmetry in the lepton sector.

The Absolute Neutrino Mass Scale and Mass Spectrum

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on $^3\mathrm{H} \rightarrow ^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
UNITED KINGDOM
IRELAND
Dublin
Belfast
Leeds
Manchester
Liverpool
Birmingham
Cardiff
London
Celtic Sea
Irish Sea
Sea
DENMARK
Copenhagen
Malmö
Bornholm
Kaliningrad
Vilnius
Minsk
Mahilyow
BELARUS
Hrodna
Homyel'
Brest
Kiev
Gdańsk
Hamburg
Bremen
Berlin
Poznań
Warsaw
POLAND
Łódź
Wrocław
Kraków
Amsterdam
Rotterdam
NETH.
Essen
Cologne
Leipzig
Brussels
Lille
BEL.
Bonn
GERMANY
Frankfurt am Main
Prague
CZECH REPUBLIC
Brno
SLOVAKIA
Bratislava
Guernsey (U.K.)
Jersey (U.K.)
Luxembourg
LUX.
Paris
Stuttgart
Munich
Strasbourg
Vienna
AUSTRIA
Budapest
HUNGARY
Rivne
L'viv
UKRAINE
Chernivtsi
Mykolayiv
Chişinău
Iaşi
MOLDOVA
Odesa
Cluj-Napoca
ROMANIA
Bucharest
Constanţa
Black Sea
Nantes
Bay of Biscay
FRANCE
MASSIF CENTRAL
Lyon
Bordeaux
Zurich
Bern
Geneva
SWITZ.
LIECH.
Ljubljana
SLOVENIA
Zagreb
Milan
Venice
Turin
Genoa
CROATIA
BOSNIA AND HERZEGOVINA
Sarajevo
Belgrade
Serbia
Varna
BULGARIA
Sofia
Skopje
Istanbul
Bursa
TURKEY
Bilbao
Toulouse
Andorra
Marseille
MONACO
Florence
SAN MARINO
Ligurian Sea
Porto
PORTUGAL
Madrid
Lisbon
SPAIN
Valencia
Barcelona
VATICAN CITY
Tirana
Thessaloniki
Naples
Balearic Sea
BALEARIC ISLANDS
Sardinia
Cagliari
Tyrrhenian Sea
GREECE
Aegean Sea
Athens
Ionian Sea
Sevilla
Gibraltar
Malaga
Mediterranean Sea
Palermo
Sicily
Rhodes
Crete
Strait of Gibraltar
Ceuta (SPAIN)
Alboran Sea
Melilla (SPAIN)
Oran
Algiers
Tunis
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + ΛCDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

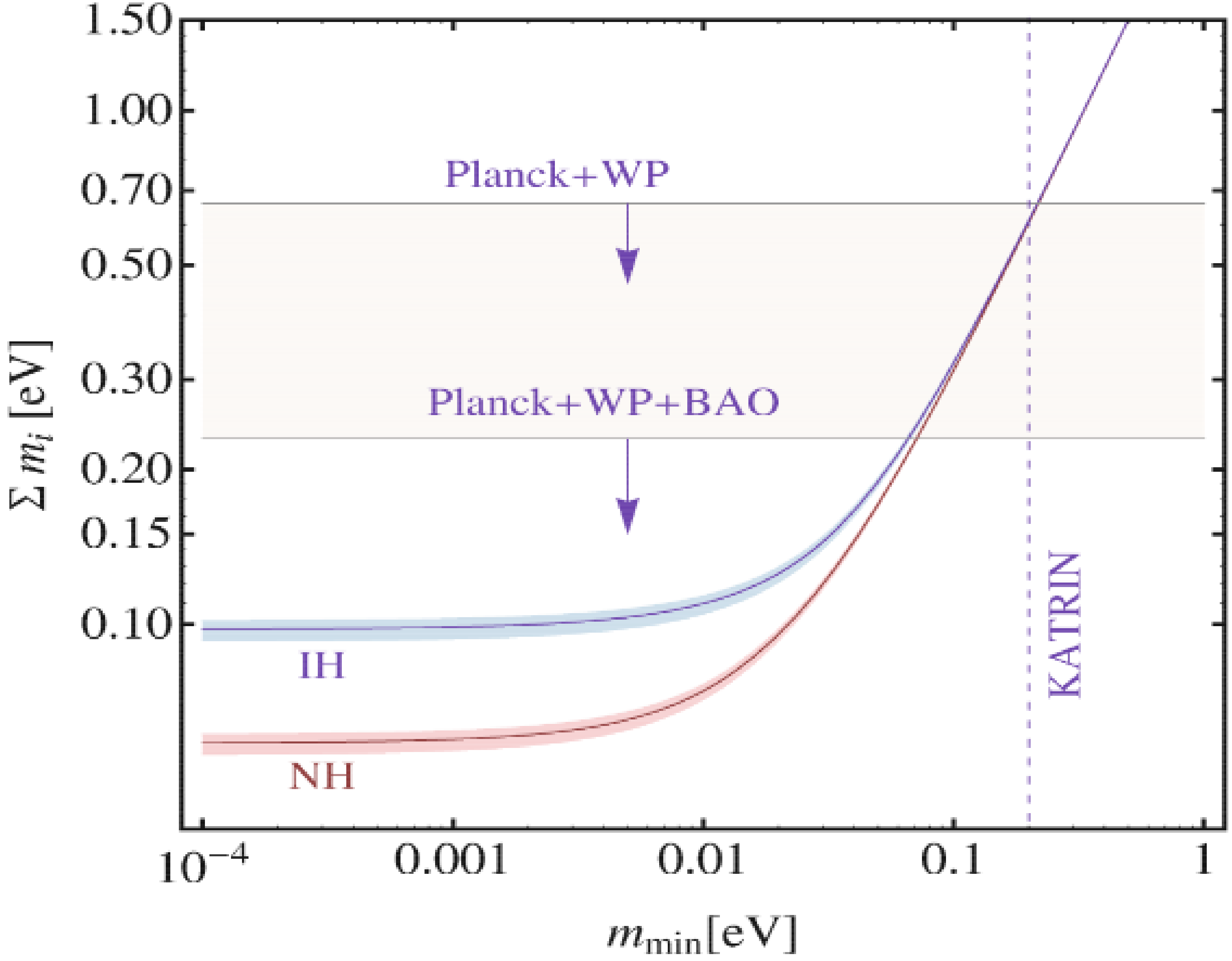
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04) \text{ eV}.$$

NH: $\sum_j m_j \leq 0.05$ eV (3σ);

IH: $\sum_j m_j \geq 0.10$ eV (3σ).

Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q, \; q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

The Nature of Massive Neutrinos I:

Majorana versus Dirac Massive Neutrinos

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ \left(\bar{\chi}_k(x)\right)^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\text{el}} = 0,\ L_l = 0,\ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

Special Properties of the Currents of $\chi(x)$–Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0)\,;$$

Has imortant implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0\,)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \; d_\chi = 0 \; (d_\Psi \neq 0,\; if\ CP\ is\ not\ conserved)$$

$\chi(x)$ cannot couple to a real photon (field) .

$\chi(x)$ couples to a virtual photon through an anapole moment :

$$(g_{\alpha\beta}\, q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5\; F_a(q^2)\,.$$

Properties of Currents Formed by $\chi_1(x)$, $\chi_2(x)$: $\chi_2 \to \chi_1 + \gamma$, $\chi_2 \to \chi_1\chi_1\chi_1$, etc.

$$\bar{\chi_1}(x)\gamma_\alpha(v - a\gamma_5)\chi_2(x) \quad (\bar{\chi_1}(x)\gamma^\alpha(1-\gamma_5)\chi_1(x)\,, ...) :$$

- CP is conserved: $v = 0$ $(a = 0)$ if $\eta_{1CP} = \eta_{2CP}$ $(\eta_{1CP} = -\eta_{2CP})$
- CP is not conserved: $v \neq 0$, $a \neq 0$

(Has imortant implications also, e.g. for SUSY neutralino phenomenology:
$e^+ + e^- \to \chi_1 + \chi_2$, $\chi_2 \to \chi_1 + l^+ + l^-$, etc.)

$$\bar{\chi_1}(x)\sigma_{\alpha\beta}(\mu_{12} - d_{12}\gamma_5)\chi_2(x) \quad (F^{\alpha\beta}(x)) :$$

- CP is conserved: $\mu_{12} = 0$ $(d_{12} = 0)$ if $\eta_{1CP} = \eta_{2CP}$ $(\eta_{1CP} = -\eta_{2CP})$
- CP is not conserved: $\mu_{12} \neq 0$, $d_{12} \neq 0$

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}^{\nu}_{M}(x) = -\frac{1}{2}\,\overline{\nu^{c}_{l'R}}(x)\; M_{l'l}\;\nu_{lL}(x) + h.c.\;,\;\; \nu^{c}_{l'R} \equiv C\;(\overline{\nu_{l'L}}(x))^{\mathsf{T}}$$

$\mathcal{L}^{\nu}_{M}(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- Dirac ν, Ψ, is equivalent to two Majorana ν's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,,\quad m_1 = m_2 = m_D > 0\,, \eta_{jCP} = i\rho_j\,, \rho_1 = -\rho_2\;\;(C\;(\overline{\chi_j})^{\mathsf{T}} = \rho_j \chi_j)$$

$$\text{Example ZKM } \nu:\;\; \nu_{eL}(x) = \Psi_L = \frac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}\,,\;\; \nu_{\mu L}(x) = \Psi^{C}_{L} = \frac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$$

- Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}^{\nu}_{M}(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to "leading order" $m_1 = m_2$, but due to "higher order" corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- Suppose: $m_1 = m_2$, $\rho_1 = -\rho_2$, but $\chi_{1,2}$ are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L}\cos\phi + \chi_{2L}\sin\phi = \Psi_L\cos\phi' + \Psi^{C}_{L}\sin\phi'$$

All Majorana effects are $\sim m_D \cos\phi' \sin\phi'$

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\tan\theta_{23} = M_{e\tau}/M_{e\mu}$,

$m_3 = 0$ - spectrum with IH, $m_1 = m_2$, $\chi_{1,2}$ - equivalent to one Dirac ν, Ψ .

Adding L'-breaking term, e.g. M_{ee}, $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, leads to $m_1 \neq m_2$ compatible with $\Delta m^2_\odot$.

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,, \quad m_1 \neq m_2 > 0\,, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - Majorana, maximal mixing.

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos\theta_C + \Psi_{2L} \sin\theta_C\,,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin\theta_C + \Psi_{2L} \cos\theta_C\,,$$

$\Psi_{1,2}$ - Dirac (composite), θ_C- the Cabbibo angle.

Determining the Nature of Massive Neutrinos

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ P_j e^{-i(E_j t - p_j x)} P^*_j = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j.

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

If ν_j- Majorana particles, U_{PMNS} contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (\text{A}, \text{Z}) \rightarrow \mu^+ + (\text{A}, \text{Z} - 2)$$

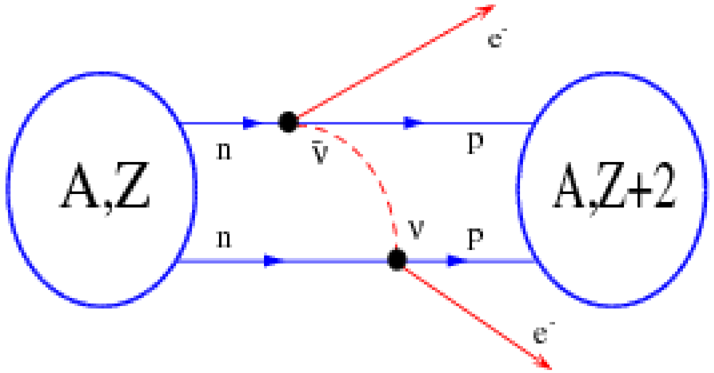
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\text{A,Z}) \rightarrow (\text{A}, \text{Z} + 2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

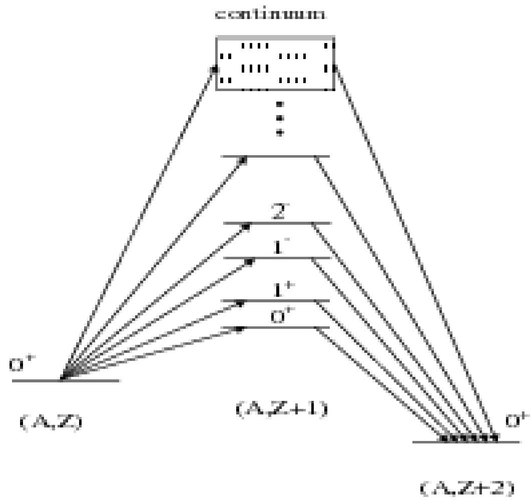
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay, cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|, \quad \theta_{12} \equiv \theta_\odot,\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$|<m>| \cong \left|\sqrt{\Delta m^2_{\odot}} \sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i\beta_M}\right|$, $m_1 \ll m_2 \ll m_3$ (NH),

$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}} \left|\cos^2\theta_{12} + e^{i\alpha} \sin^2\theta_{12}\right|$, $m_3 < (\ll) m_1 < m_2$ (IH),

$|<m>| \cong m \left|\cos^2\theta_{12} + e^{i\alpha} \sin^2\theta_{12}\right|$, $m_{1,2,3} \cong m \gtrsim 0.10$ eV (QD),

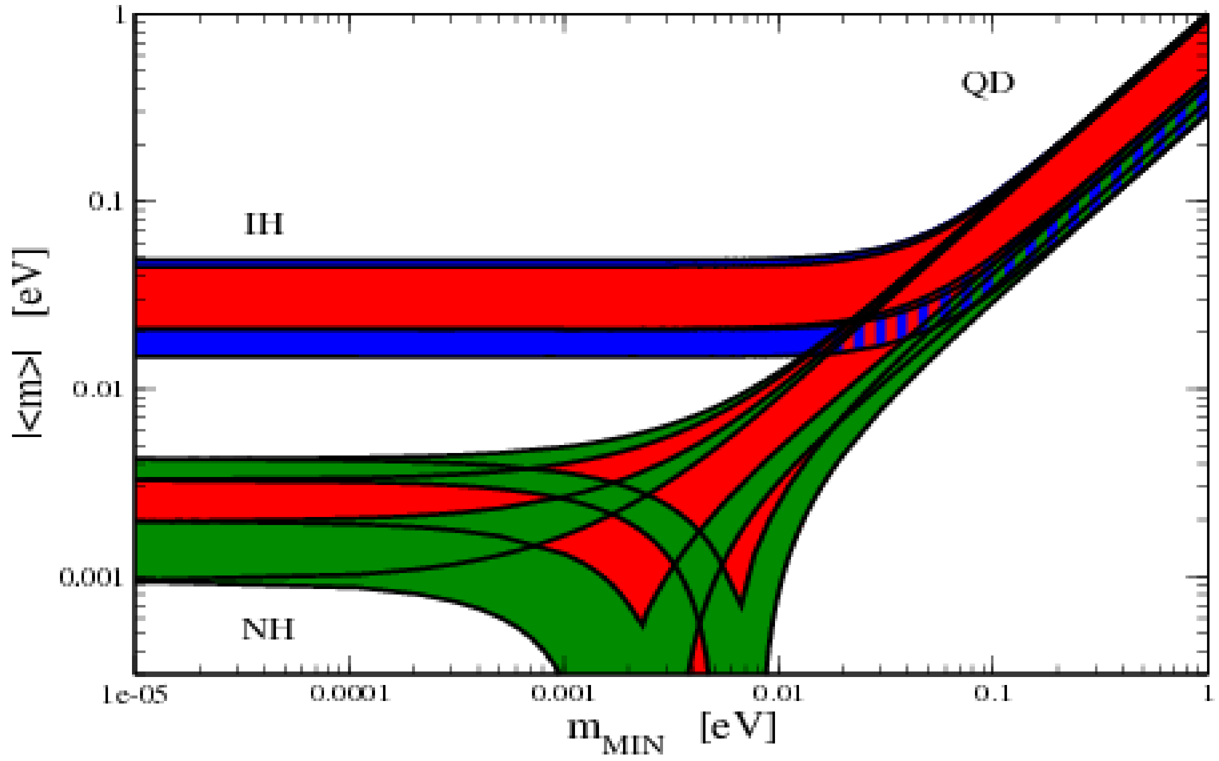
$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$|<m>| \lesssim 5 \times 10^{-3}$ eV, NH;

$\sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013$ eV $\lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055$ eV, IH;

$m \cos 2\theta_{12} \lesssim |<m>| \lesssim m$, $m \gtrsim 0.10$ eV, QD .



S. Pascoli, PDG, 2012

$\sin^2\theta_{13} = 0.0236 \pm 0.0042$; $\delta = 0$.

$1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$, $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$.

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|<m>|)$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|<m>| = (0.1 - 0.9)$ eV (99.73% C.L.); b.f.v.: $|<m>| = 0.33$ eV.

IGEX ^{76}Ge: $|<m>| < (0.33 - 1.35)$ eV (90% C.L.).

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|<m>| <$(0.45–0.96) eV, $|<m>| <$(0.18–0.64) eV (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$T(^{76}Ge) = 2.23^{+0.44}_{0.31} \times 10^{25} \text{ yr at } 90\% \text{ C.L.}$$

Results from 2012-2013:

$$T(^{136}Xe) > 1.6 \times 10^{25} \text{yr at } 90\% \text{ C.L., EXO}$$

$$T(^{136}Xe) > 1.9 \times 10^{25} \text{yr at } 90\% \text{ C.L., KamLAND} - \text{Zen}$$

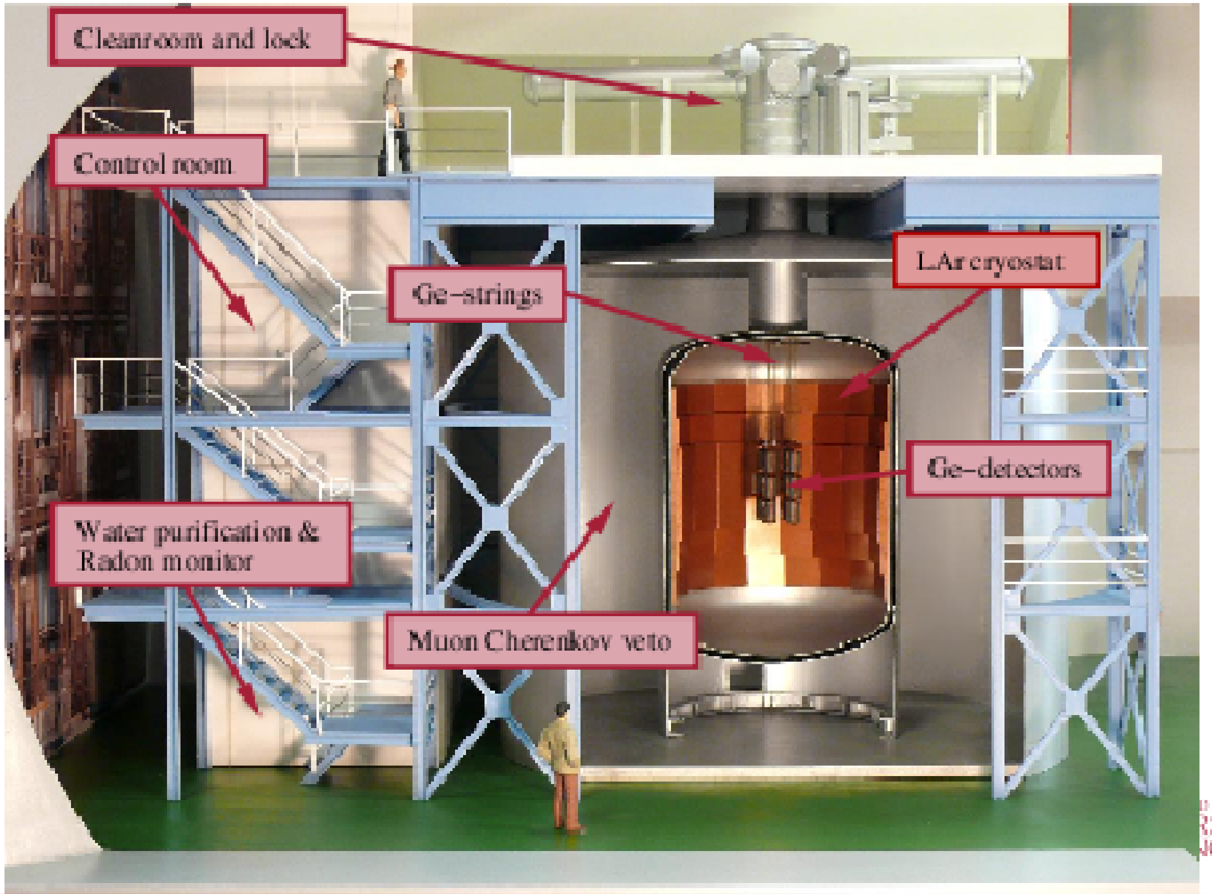
$$T(^{76}Ge) > 2.1 \times 10^{25} \text{yr at } 90\% \text{ C.L., GERDA.}$$

$$T(^{76}Ge) > 3.0 \times 10^{25} \text{yr at } 90\% \text{ C.L., GERDA} + \text{IGEX} + \text{HdM.}$$

Large number of experiments: $|<m>| \sim$ (0.01-0.05) eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
KamLAND-ZEN - ^{136}Xe;
EXO - ^{136}Xe;
SNO+ - ^{130}Te;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca;
SuperNEMO - ^{82}Se,...;
MAJORANA - ^{76}Ge;
COBRA - ^{116}Cd;
MOON - ^{100}Mo.

GERDA: Experimental Setup



Majorana CPV Phases and $|<m>|$

CPV can be established provided

- $|<m>|$ measured with $\Delta \lesssim 15\%$;
- $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

The Nature of Massive Neutrinos II:
Origins of Dirac and Majorana Massive Neutrinos

- Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L.

- Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^{\nu}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}_m^{\nu}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}^{\nu}_{m}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}^{\nu}_{m}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires $\nu_R(x)$ - $SU(2)_L$ singlet RH ν fields

$$\mathcal{L}^{\nu}_{D}(x) = -\,\overline{\nu_{l'R}}(x)\; M_{Dl'l}\; \nu_{lL}(x) + h.c.\;,\;\; M_D - complex$$

- $\mathcal{L}^{\nu}_{D}(x)$ conserves L: $L = const.$

$$M_D = V M_D^{diag} W^{\dagger}\,,\;\; V,U - \text{unitary}\;(bi - unitary\; transformation)\,,\;\; W \equiv U_{\text{PMNS}}$$

- ST + 3 $\nu_R(x)$ - RH ν fields: $n = 3$

$$\begin{aligned}\mathcal{L}_{Y}(x) &= Y^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^{T}(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,,\\ M_D &= \frac{v}{\sqrt{2}}Y^{\nu}\,,\;\; v = 246\ \text{GeV}\,.\end{aligned}$$

No explanation why $m(\nu_j) <<< m_l, m_q$.

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes $\mu^+ \to e^+ + \gamma$ decay, $\mu^- \to e^- + e^+ + e^-$ decay, $\tau^- \to e^- + \gamma$ decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \to e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U^*_{\mu j} \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$

$$M_W \cong 80 \text{ GeV, the } W^{\pm} - \text{mass}$$

S.T.P., 1976

"New Physics": $\nu_l \to \nu_{l'}$, $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$ oscillations.

- Majorana ν_j: Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$

$$\mathcal{L}^{\nu}_{M}(x) = \frac{1}{2}\, \nu^{\top}_{l'L}(x)\; C^{-1}\; M_{l'l}\; \nu_{lL}(x) + h.c.\;,\;\; C^{-1}\gamma_\alpha C = -\gamma^{\top}_\alpha$$

- If $M_{l'l} \neq 0$, $L_l \neq const.$, $L \neq const.$, $n = 3$
- $\nu_{lL}(x)-$fermions: $M = M^{\top}$, complex.

$$M^{diag} = U^{\top} M U\,,\;\; U - \text{unitary (congruent transformation)};\;\; U \equiv U_{\text{PMNS}}$$

$$\nu_j \equiv \chi_j(x) = U^{\dagger}_{jl}\nu_{lL}(x) + U^{*}_{jl}\nu^{c}_{lR} = C\;(\overline{\chi_j}(x))^{\top}\,,\quad m_j \neq 0\,,\;\; j = 1,2,3$$

CP-invariance: $M^* = M$, M - real, symmetric.

$M^{diag} = (m'_1, m'_2, m'_3)$: $m'_j = \rho_j m_j$, $m_j \geq 0$, $\rho_j = \pm 1$

χ_j: $m_j \geq 0$: $\eta_{CP}(\chi_j) = i\rho_j$

$\mathcal{L}^{\nu}_{M}(x)$ not possible in the ST: requires New Physics Beyond the ST

$(\beta\beta)_{0\nu}$-decay is allowed; typically also $BR(\mu \to e + \gamma)$, $BR(\mu \to 3e)$, $CR(\mu^- + \mathcal{N} \to e^- + \mathcal{N})$ can be "large", i.e., in the range of sensitivity of ongoing (MEG) and future planned experiments.

- Majorana ν_j: Dirac+Majorana Mass Term; requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$:

$$\mathcal{L}^{\nu}_{D+M}(x) = -\,\overline{\nu_{l'R}}(x)\, M_{Dl'l}\, \nu_{lL}(x) + \frac{1}{2}\,\nu^{\top}_{l'L}(x)\, C^{-1}\, M^{LL}_{l'l}\, \nu_{lL}(x) + \frac{1}{2}\,\nu^{\top}_{l'R}(x)\, C^{-1}\, (M^{RR})^{\dagger}_{l'l}\, \nu_{lR}(x) + h.c.\,,$$

$$M \;=\; \begin{pmatrix} M^{LL} & M_D \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad \left((M^{LL})^T = M^{LL},\; (M^{RR})^T = M^{RR}\right)$$

- If $M_{Dl'l} \neq 0$ and $M^{LL}_{l'l} \neq 0$ and/or $M^{RR}_{l'l} \neq 0$: $L_l \neq const.$, $L \neq const.$; $n = 6\ (> 3)$
- $M = M^{\top}$, complex.

$$M^{diag} = W^{\top} M W\,,\quad W - \text{unitary},\ 6 \times 6\,;\quad W^T \equiv (U^T \quad V^T)\,;\ U \equiv U_{\text{PMNS}}:\ 3 \times 6\,.$$

$$\nu_{lL}(x) = \sum_{j=1}^{6} U_{lj}\chi_j(x)\,,\quad \chi_j(x) - \text{Majorana}\,,\quad m_j \neq 0\,,\quad l = e, \mu, \tau\,;$$

$$\nu^{C}_{lL}(x) \equiv C\ (\overline{\nu_{lR}}(x))^{\top} = \sum_{j=1}^{6} V_{lj}\chi_j(x)\,,\quad \nu^{C}_{lL}(x):\ \text{sterile antineutrino}$$

$\mathcal{L}^{\nu}_{D+M}(x)$ possible in the ST + ν_{lR}: $M^{LL} = 0$

$(\beta\beta)_{0\nu}$-decay is allowed;
phenomenology depends on the relative magnitude of M_D and M^{RR}.

The Nature of Massive Neutrinos III:

The Seesaw Mechanisms of Neutrino Mass Generation

- Explain the smallness of $\nu-$masses.

- Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH νs' (heavy).

Type II seesaw mechanism: $\mathbf{H(x)}$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\mathbf{T(x)}$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$-decay, LFV processes, etc.) and New Physics at LHC.

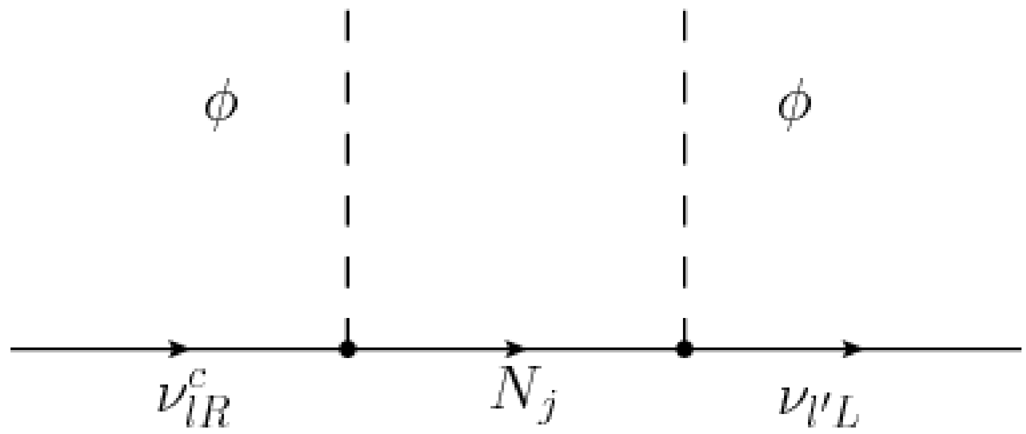
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^{\nu}/\sqrt{2}| << |M^{RR}|$.
- Diagonalising M^{RR}: N_j - heavy Majorana neutrinos, $M_j \sim$TeV; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large M_j, Majorana mass term for $\nu_{lL}(x)$:

$$M_{\nu} \cong v_u^2 \, (Y^{\nu})^T \, M_j^{-1} \, Y^{\nu} = U^*_{\text{PMNS}} \, m_{\nu}^{\text{diag}} \, U^{\dagger}_{\text{PMNS}} \, .$$

$v_u Y^{\nu} = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_{\nu} \sim 0.1$ eV.



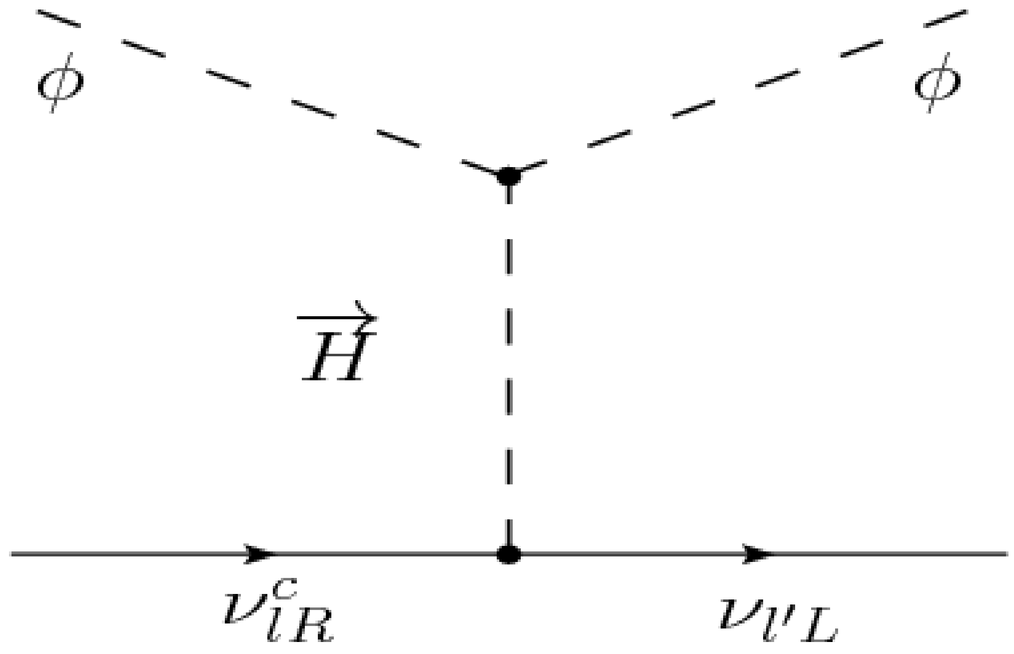
- $\nu_{l'R}(x)$: Majorana mass term at "high scale" ($\sim$TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)

$$\mathcal{L}_M^{\nu}(x) = +\,\frac{1}{2}\,\nu_{l'R}^{\top}(x)\;C^{-1}\;(M^{RR})_{l'l}^{\dagger}\;\nu_{lR}(x) + h.c. = -\frac{1}{2}\sum_j \bar{N}_j\,M_j\,N_j\ ,$$

- Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}}(x) &= \bar{Y}_{l'l}^{\nu}\,\overline{\nu_{l'R}}(x)\,\Phi^{T}(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,, \\
&= Y_{jl}^{\nu}\,\overline{N_{jR}}(x)\,\Phi^{T}(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,, \\
M_D &= \frac{v}{\sqrt{2}}\,Y^{\nu}\,, \quad v = 246\ \text{GeV}\,.
\end{aligned}$$

Type II Seesaw Mechanism

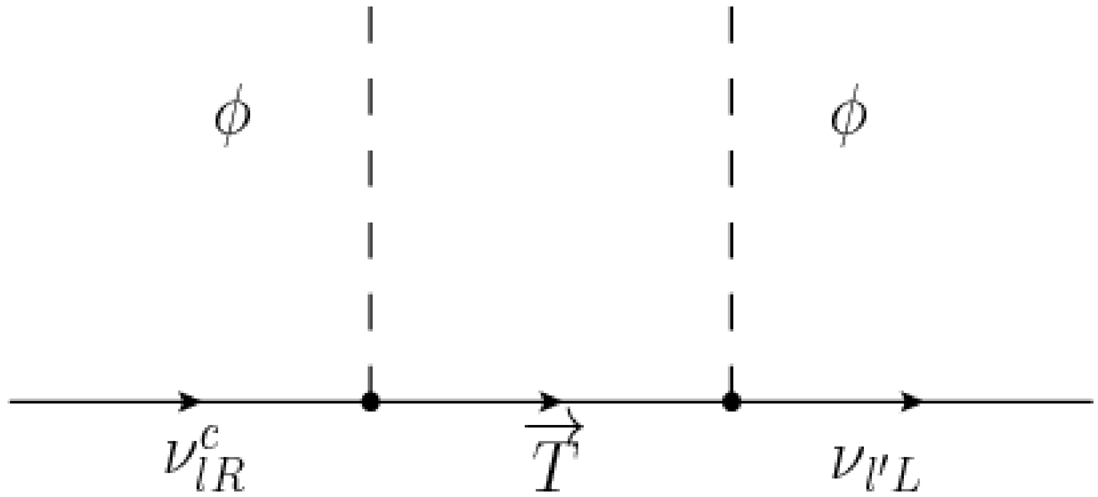


Due to I. Girardi

$$M_\nu \cong h\, v^2\, M_H^{-1} = U^*_{\text{PMNS}}\, m_\nu^{\text{diag}}\, U^\dagger_{\text{PMNS}}\ .$$

$h \sim 10^{-2}$, $v = 246$ GeV, $M_H \sim 10^{12}$ GeV: $M_\nu \sim 0.6$ eV.

Type III Seesaw Mechanism



$$M_\nu \cong v^2\ (Y_T)^T\, M_T^{-1}\, Y_T = U^*_{\text{PMNS}}\ m_\nu^{\text{diag}}\ U^\dagger_{\text{PMNS}}\ .$$

$v\,Y_T \sim 1$ GeV, $M_T \sim 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.

TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$m_\nu \simeq - M_D \hat{M}_N^{-1} M_D^T$, $\hat{M} = \text{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000)$ GeV.

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell}\, \gamma_\alpha\, (RV)_{\ell k} (1 - \gamma_5)\, N_k\, W^\alpha + \text{h.c.}\,, \quad (RV)_{\ell k} \equiv U_{\ell 3+k}\,,$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w} \overline{\nu_{\ell L}}\, \gamma_\alpha\, (RV)_{\ell k}\, N_{kL}\, Z^\alpha + \text{h.c.}$$

- **All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:** $M_2 = M_1(1 + z)$, $0 < z \ll 1$.

- **Only NH and IH ν mass spectra possible:** $\min(m_j) = 0$.

- Requirements: $|(RV)_{\ell k}|$ "sizable"

+ reproducing correctly the neutrino oscillation data:

$$\begin{aligned}
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2v^2}{M_1^2}\frac{m_3}{m_2+m_3}\left|U_{\ell 3}+i\sqrt{m_2/m_3}U_{\ell 2}\right|^2, \quad \text{NH},\\
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2v^2}{M_1^2}\frac{m_2}{m_1+m_2}\left|U_{\ell 2}+i\sqrt{m_1/m_2}U_{\ell 1}\right|^2 \cong \frac{1}{4}\frac{y^2v^2}{M_1^2}|U_{\ell 2}+iU_{\ell 1}|^2, \text{IH},\\
(RV)_{\ell 2} &= \pm i\,(RV)_{\ell 1}\sqrt{\frac{M_1}{M_2}}, \; \ell = e,\mu,\tau,
\end{aligned}$$

y- the maximum eigenvalue of Y^{ν}, $v_u \simeq 174$ GeV.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

4 parameters: M, z, y and a phase ω.

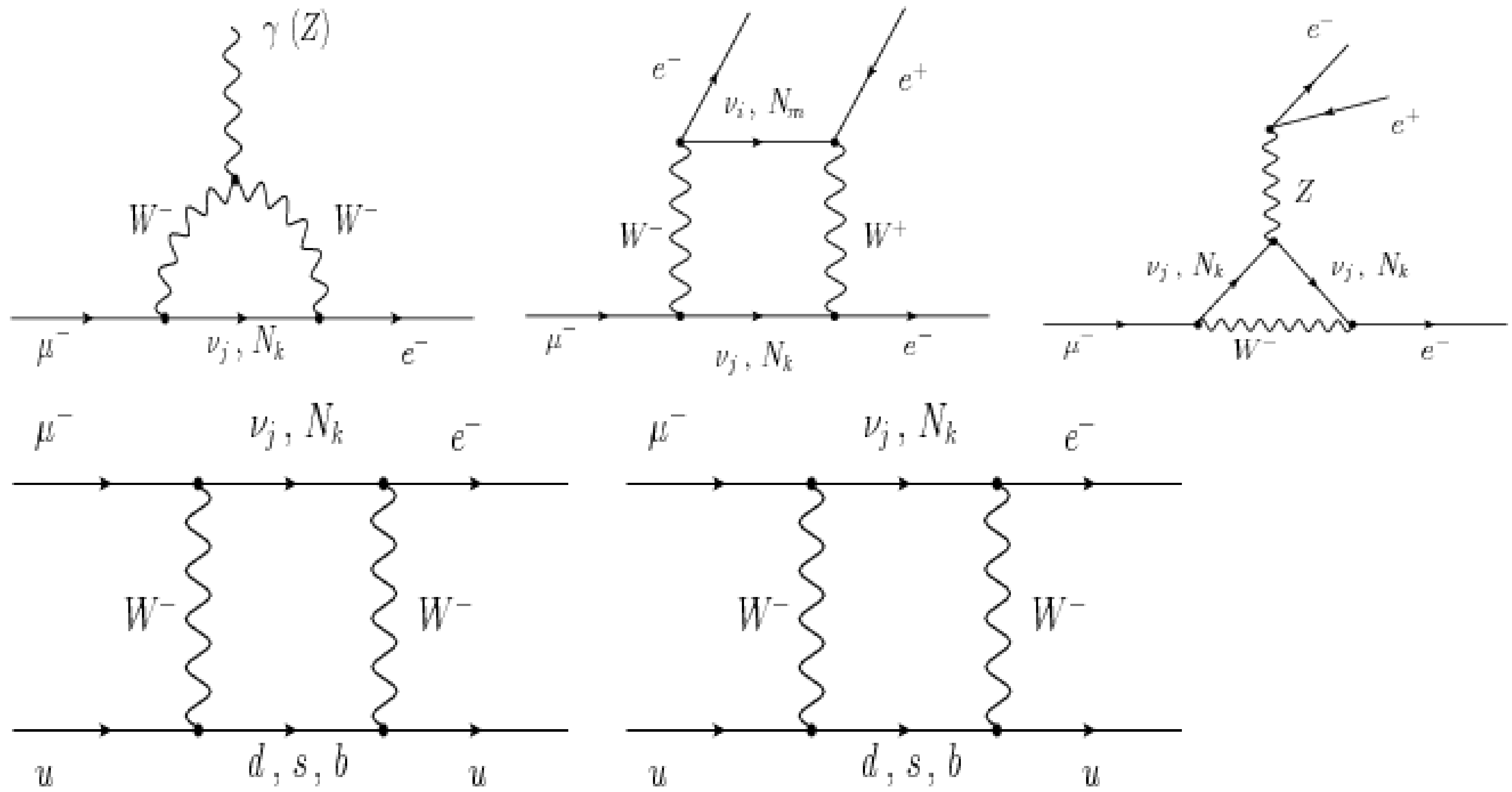
Low energy data:

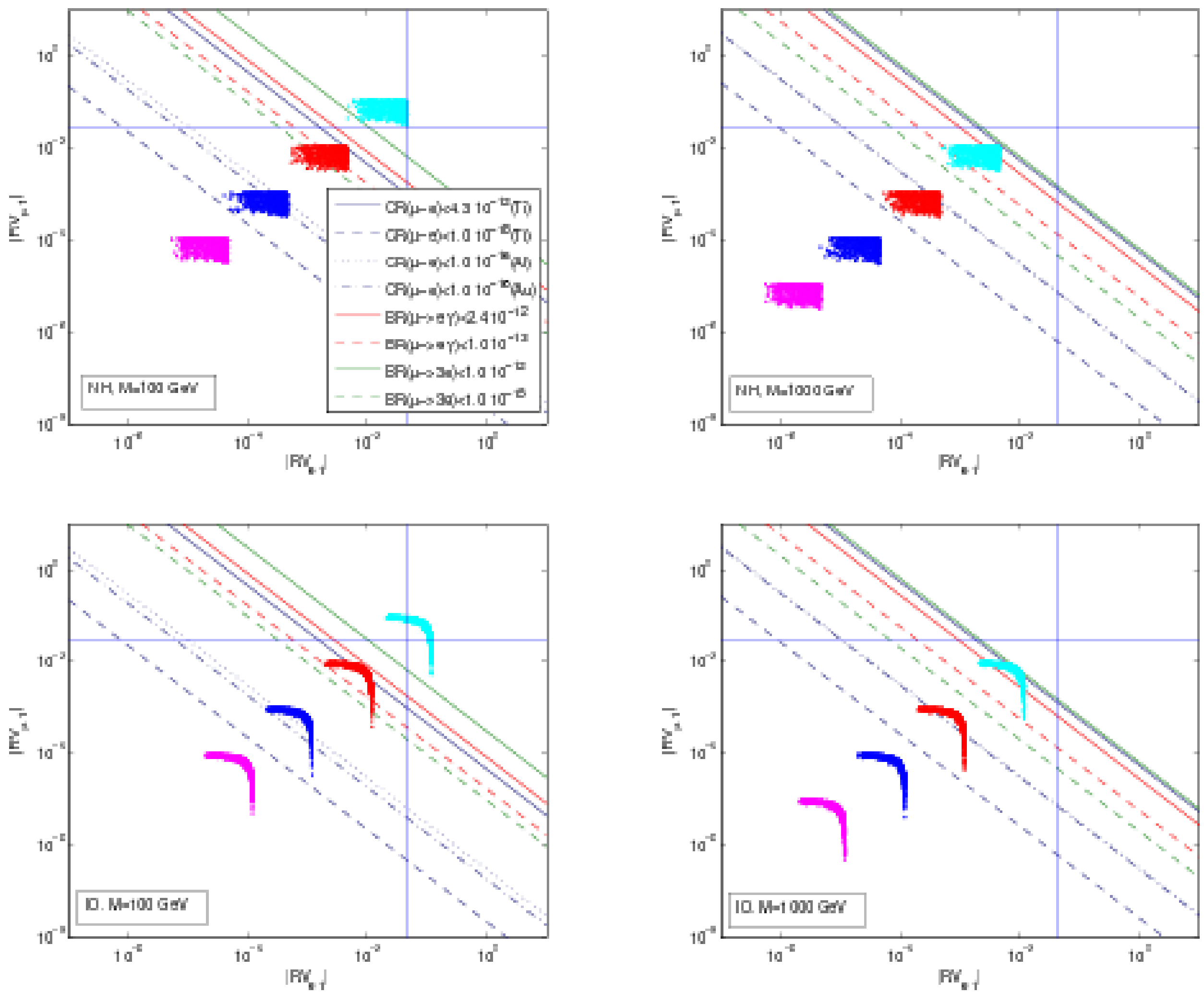
$$\begin{aligned}
|(RV)_{e1}|^2 &\lesssim 2\times 10^{-3},\\
|(RV)_{\mu 1}|^2 &\lesssim 0.8\times 10^{-3},\\
|(RV)_{\tau 1}|^2 &\lesssim 2.6\times 10^{-3}.
\end{aligned}$$

S. Antusch et al., 2008

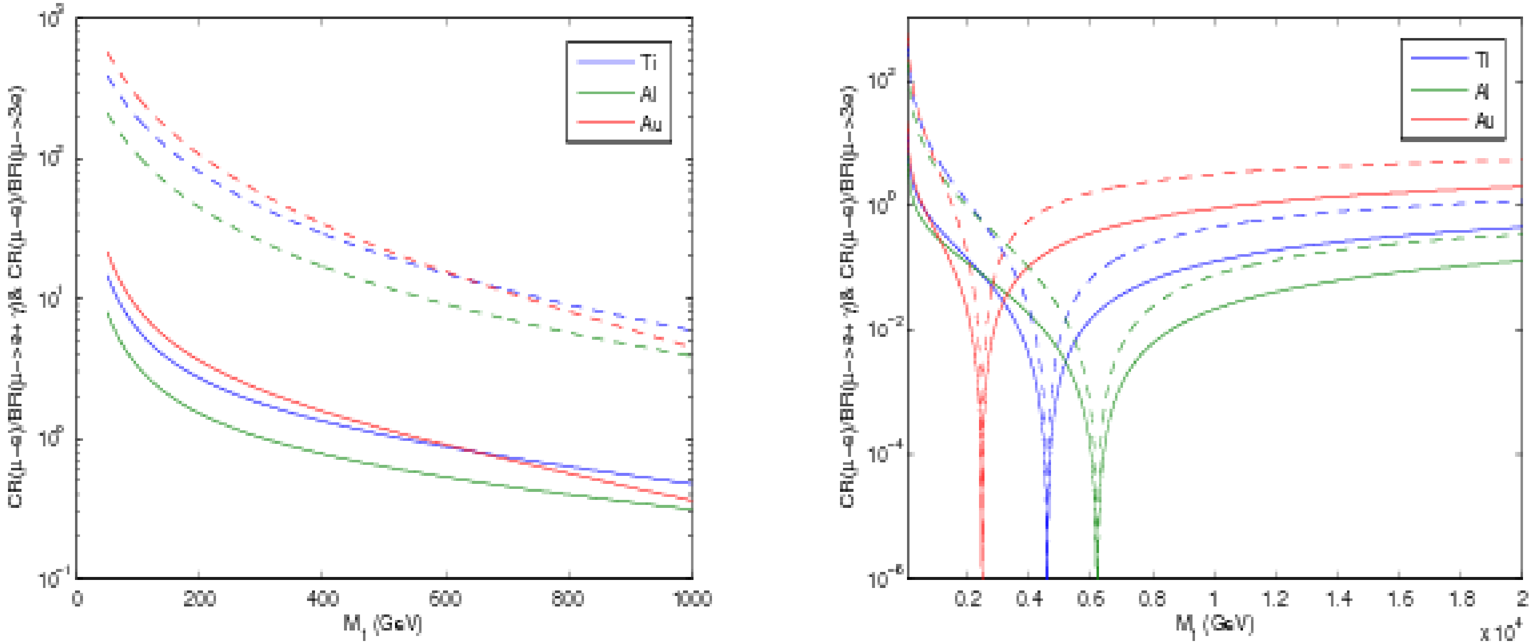
Observation of $N_{1,2}$ at LHC - problematic.

LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- + \mathcal{N} \to e^- + \mathcal{N}$: can proceed with exchange of virtual N_j:





Current limits and potential sensitivity to $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for $M_1 =$ 100 (1000) GeV and, $i)$ $y = 0.0001$ (magenta pts), $ii)$ $y = 0.001$ (blue pts), $iii)$ $y = 0.01$ (red pts) and $iv)$ $y = 0.1$ (cyan pts).



The ratio of the $\mu - e$ relative conversion rate and the branching ratio of the i) $\mu \to e\gamma$ decay (solid lines), ii) $\mu \to 3e$ decay (dashed lines), versus the type I see-saw mass scale M_1, for three different nuclei: ${}^{48}_{22}$Ti (blue lines), ${}^{27}_{13}$Al (green lines) and ${}^{197}_{79}$Au (red lines).

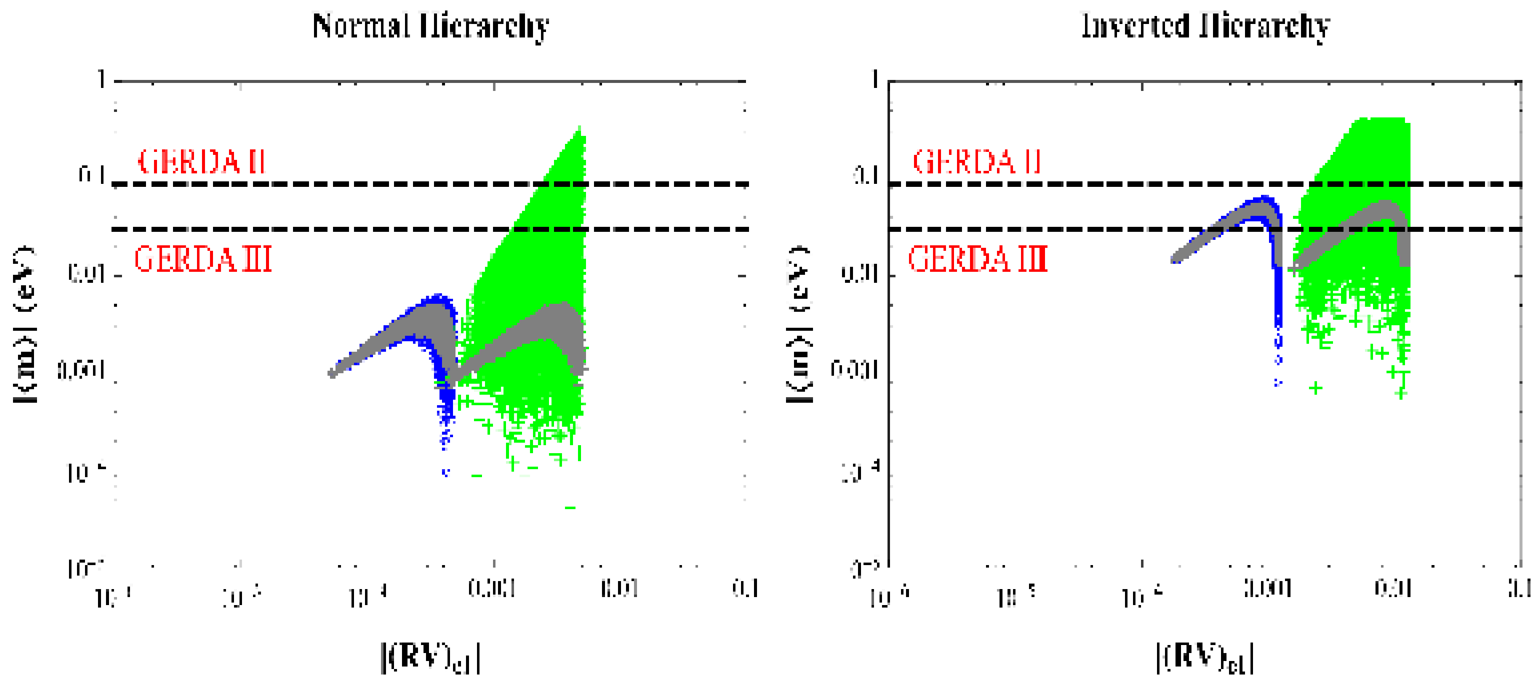
The exchange of virtual N_j gives a contribution to $|<m>|$:

$$|<m>| \cong \left| \sum_i (U_{PMNS})^2_{ei}\, m_i - \sum_k f(A, M_k)\,(RV)^2_{ek}\, \frac{(0.9\ \text{GeV})^2}{M_k} \right| ,$$

$$f(A, M_k) \cong f(A)\,.$$

For, e.g., ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te and ^{136}Xe, the function $f(A)$ takes the values $f(A) \cong 0.033,\ 0.079,\ 0.073,\ 0.085$ and 0.068, respectively.

- **The Predictions for $|<m>|$ can be modified considerably.**



$|<m>|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and $i)$ $y = 0.001$ (blue), $ii)$ $y = 0.01$ (green). The gray markers correspond to $|<m>^{\rm std}| = |\sum_i (U_{PMNS})^2_{ei} m_i|$.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- ν_j - Majorana particles.
- Diagonalisation of M_ν: $U_{\text{TBM}}\Phi$, $\Phi = \text{diag}(1,1,1(i))$
- U_{TBM} "corrected" by

$$U^{\dagger}_{\text{lep}} Q = R_{12}(\theta^{\ell}_{12})\, R_{23}(\theta^{\ell}_{23}) Q\,, \quad Q = \text{diag}(1, e^{i\phi}, 1)$$

T' model of lepton flavour: $U_{\rm TBM}$, $\delta \cong 3\pi/2$ or $\pi/2$.

- T': double covering of A_4 (tetrahedral symmetry group).

- T': $\mathbf{1}$, $1'$, $1''$; $\mathbf{2}$, $2'$, $2''$; $\mathbf{3}$.

- T' model: $\psi_{eL}(x)$,$\psi_{\mu L}(x)$,$\psi_{\tau L}(x)$ - triplet of T';
$e_R(x)$,$\mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T';
$\nu_{eR}(x)$,$\nu_{\mu R}(x)$,$\nu_{\tau R}(x)$ - a triplet of T';
the Higgs doublets $H_u(x)$, $H_d(x)$ - singlets of T'.

- The discrete symmetries of the model are $T' \times H_{CP} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2:

$$\text{NO spectrum A}: \quad (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3}\ \text{eV}$$

$$\text{NO spectrum B}: \quad (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3}\ \text{eV}$$

$$\text{IO spectrum}: \quad (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3}\ \text{eV}$$

$$\text{NO A}: \quad \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2}\ \text{eV}\,,$$

$$\text{NO B}: \quad \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2}\ \text{eV}\,,$$

$$\text{IO}: \quad \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2}\ \text{eV}\,,$$

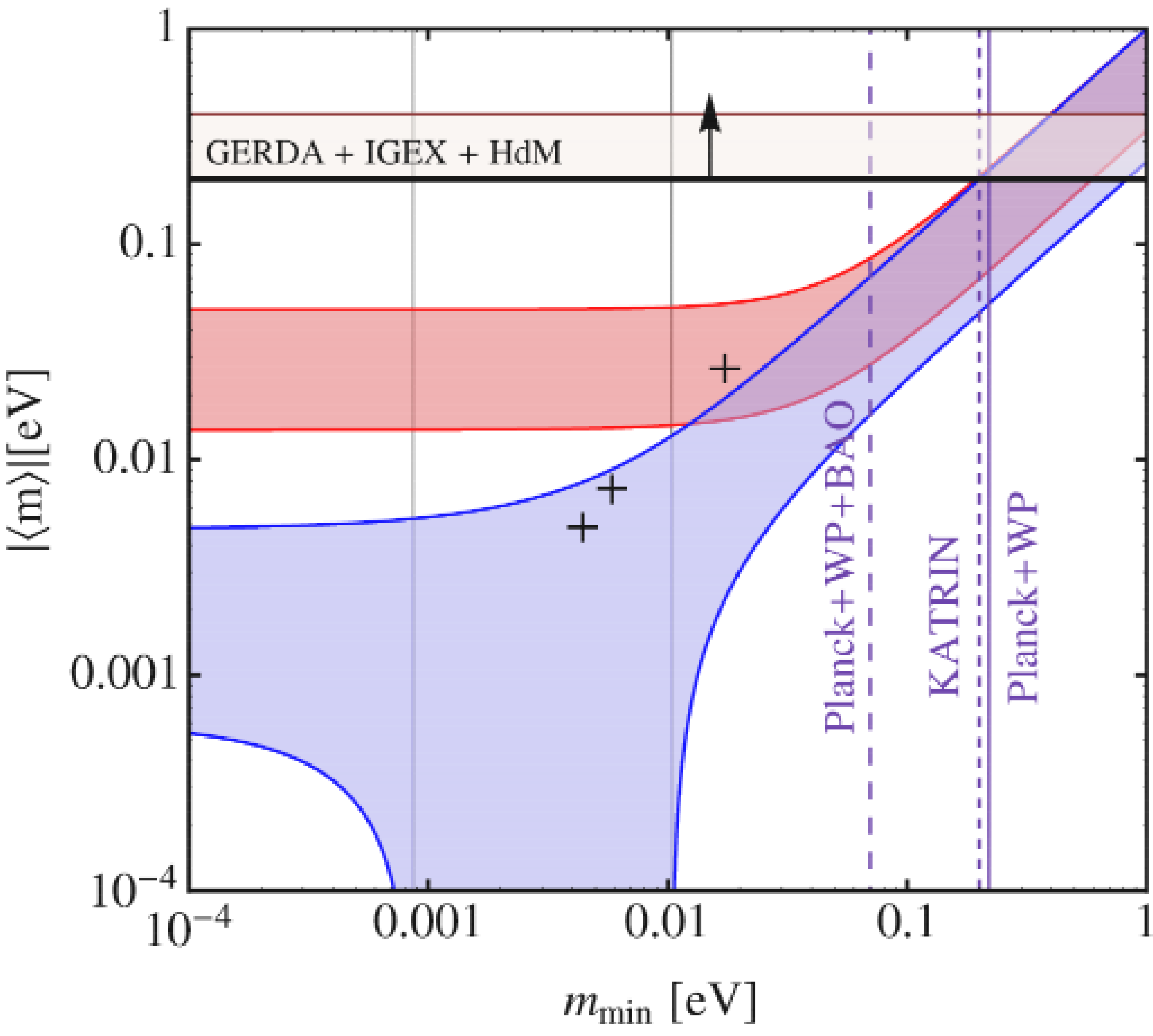
- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

Given the values of $\theta_{12}, \theta_{23}, \theta_{13}$, $\delta, \alpha_{21}, \alpha_{31}$ are predicted:

$\delta \cong 3\pi/2$ (266°) (or $\pi/2$ (94°));

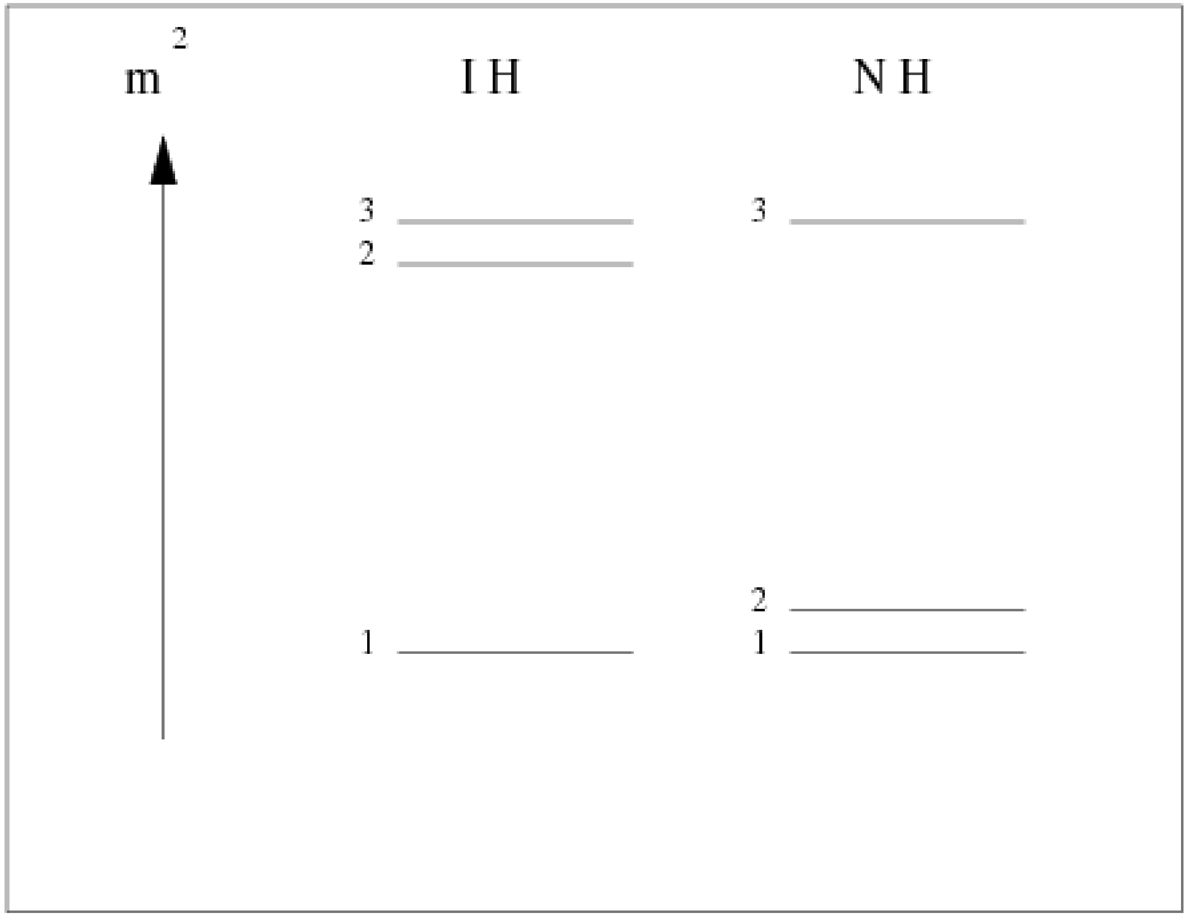
NO A: $\alpha_{21} \cong +47.0^\circ$ (or -47.0°) $(+2\pi)$,

$\alpha_{31} \cong -23.8^\circ$ (or $+23.8^\circ$) $(+2\pi)$.



GERDA + IGEX + HdM
Planck+WP+BAO
KATRIN
Planck+WP
|⟨m⟩|[eV]
1
0.1
0.01
0.001
10^-4
10^-4
0.001
0.01
0.1
1
m_min [eV]

Determining the ν−Mass Hierarchy (sgn(Δm^2_{atm}))



Our convention for IO: $m_3 < m_1 < m_2$.

Determining the $\nu-$Mass Hierarchy (sgn($\Delta m^2_{\rm atm}$))

- **Reactor $\bar{\nu}_e$ Oscillations in vacuum (Day Bay II (JUNO), RENO50).**

- **Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (HK, PINGU (IceCube), INO).**

- **LBL $\nu-$oscillation experiments (T2K, NOνA; LBNO, LBNE, $\nu-$factory); designed to search also for CP violation.**

- ^{3}H **β-decay Experiments (sensitivity to 5×10^{-2} eV) (NH vs IH).**

- $(\beta\beta)_{0\nu}$**−Decay Experiments; ν_j- Majorana particles (NH vs IH).**

- **Cosmology: $\sum_j m_j$ (NH vs IH).**

- **Atomic Physics Experiments: RENP.**

Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\rm NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\rm A} L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_\odot\left(1 - \cos\frac{\Delta m^2_\odot L}{2E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\sin^2\theta_\odot \sin\frac{\Delta m^2_\odot L}{4E_\nu}\sin\left(\frac{\Delta m^2_{\rm A} L}{2E_\nu} - \frac{\Delta m^2_\odot L}{4E_\nu}\right),$$

$$P_{\rm IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\rm A} L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_\odot\left(1 - \cos\frac{\Delta m^2_\odot L}{2E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\cos^2\theta_\odot \sin\frac{\Delta m^2_\odot L}{4E_\nu}\sin\left(\frac{\Delta m^2_{\rm A} L}{2E_\nu} - \frac{\Delta m^2_\odot L}{4E_\nu}\right),$$

$\theta_\odot = \theta_{12}, \Delta m^2_\odot = \Delta m^2_{21} > 0$; $\sin^2\theta_{12} \leq 0.36$ at 3σ;
$\Delta m^2_{\rm A} = \Delta m^2_{31} > 0$, NH spectrum,
$\Delta m^2_{\rm A} = \Delta m^2_{23} > 0$, IH spectrum

M. Piai, S.T.P., hep-ph/0112074;

The reactor $\bar{\nu}_e$ detected via

$$\bar{\nu}_e + p \to e^+ + n\,.$$

The visible energy of the detected e^+:

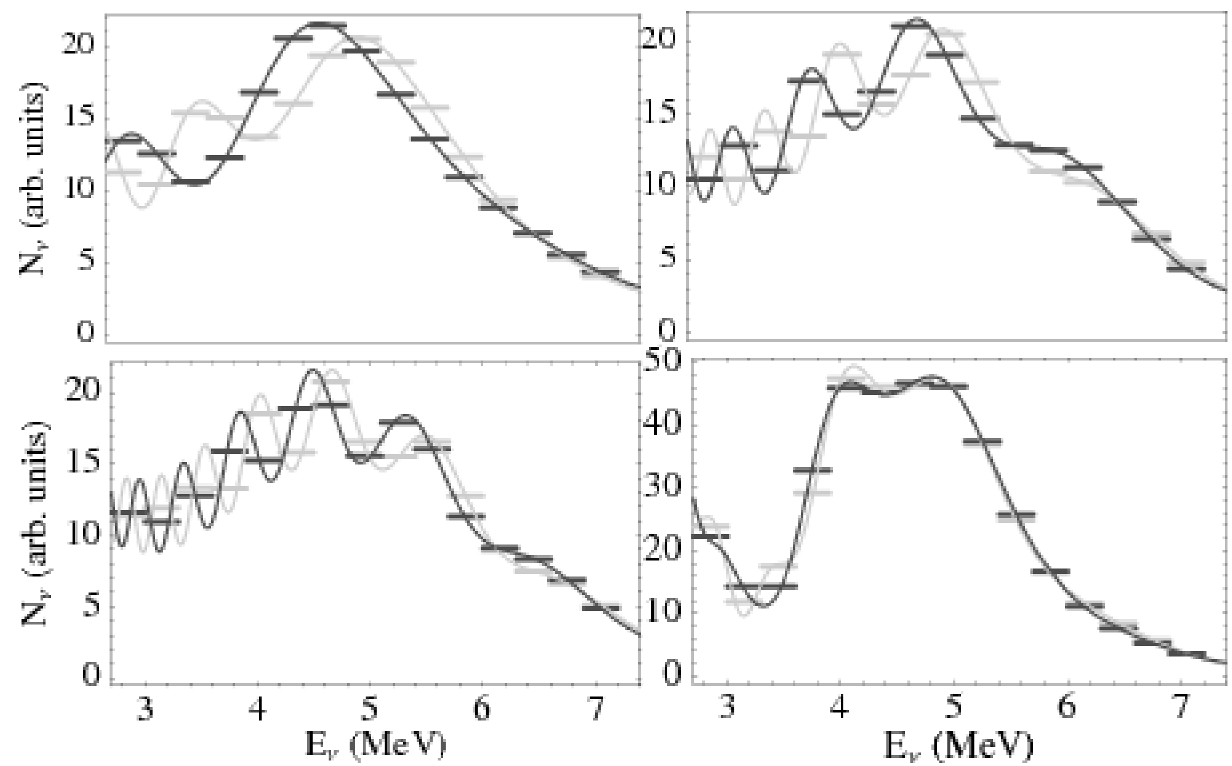
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8\ \text{MeV}\,.$$

The measured event rate spectrum vs. L/E_m:

$$N(L/E_m) = \int R(E, E_m)\Phi(E)\sigma(\bar{\nu}_e p \to e^+ n; E) P_{\bar{e}\bar{e}}^{NO(IO)} dE\,.$$

$$|P_{\text{NO}}(\bar{\nu}_e \to \bar{\nu}_e) - P_{\text{IO}}(\bar{\nu}_e \to \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$\cos 2\theta_{12} \cong 0.38$; 3σ: $\cos 2\theta_{12} \geq 0.28$; $\sin^2 2\theta_{13} \cong 0.09$.



M. Piai, S.T.P., 2001

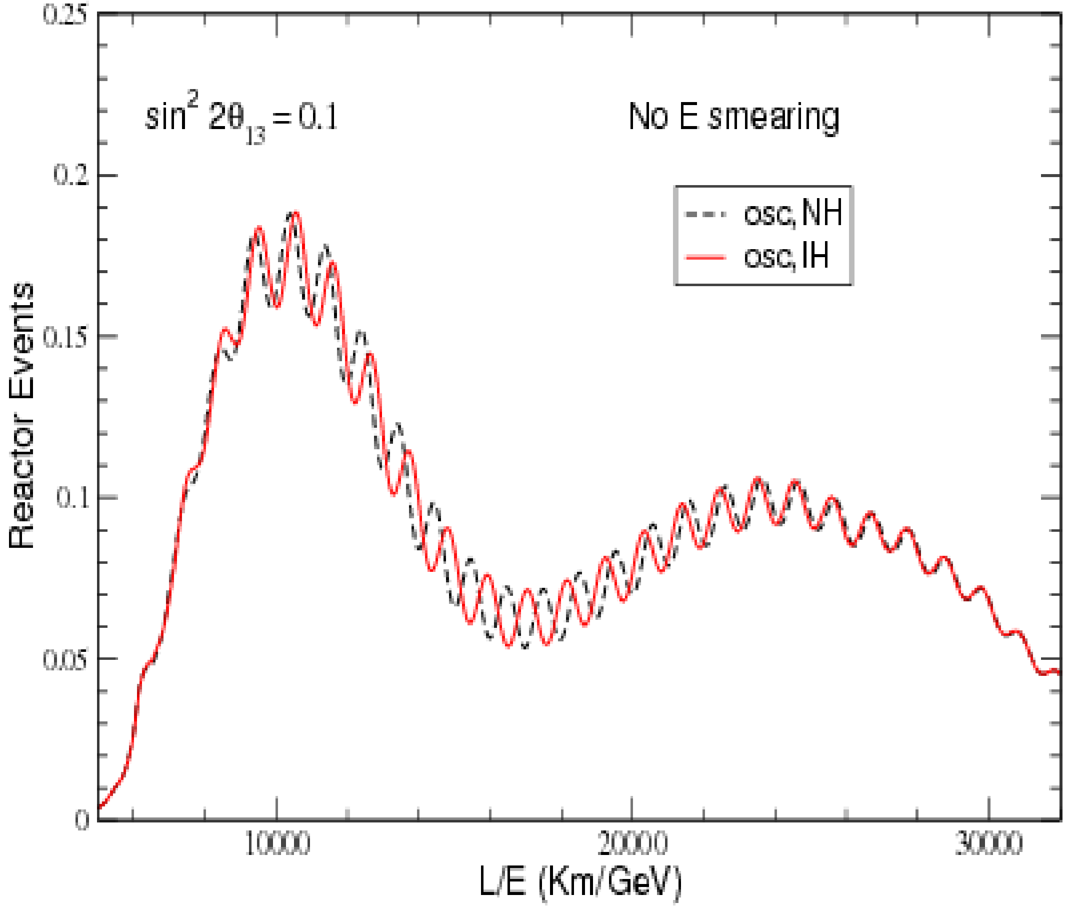
$\sin^2\theta_{13} = 0.05$, $\Delta m^2_{21} = 2\times 10^{-4}$ eV^2; $\Delta m^2_{\rm A} = 1.3;\ 2.5;\ 3.5\times 10^{-3}$ eV^2

$L = 20$ km, $\Delta E_\nu = 0.3$ MeV.

$\Delta m^2_{21} = 2\times 10^{-4}$ eV^2: $L = 20$ km;

$\Delta m^2_{21} = 7.6\times 10^{-5}$ eV^2: $L \cong 53$ km.

NH – light grey; IH – dark grey

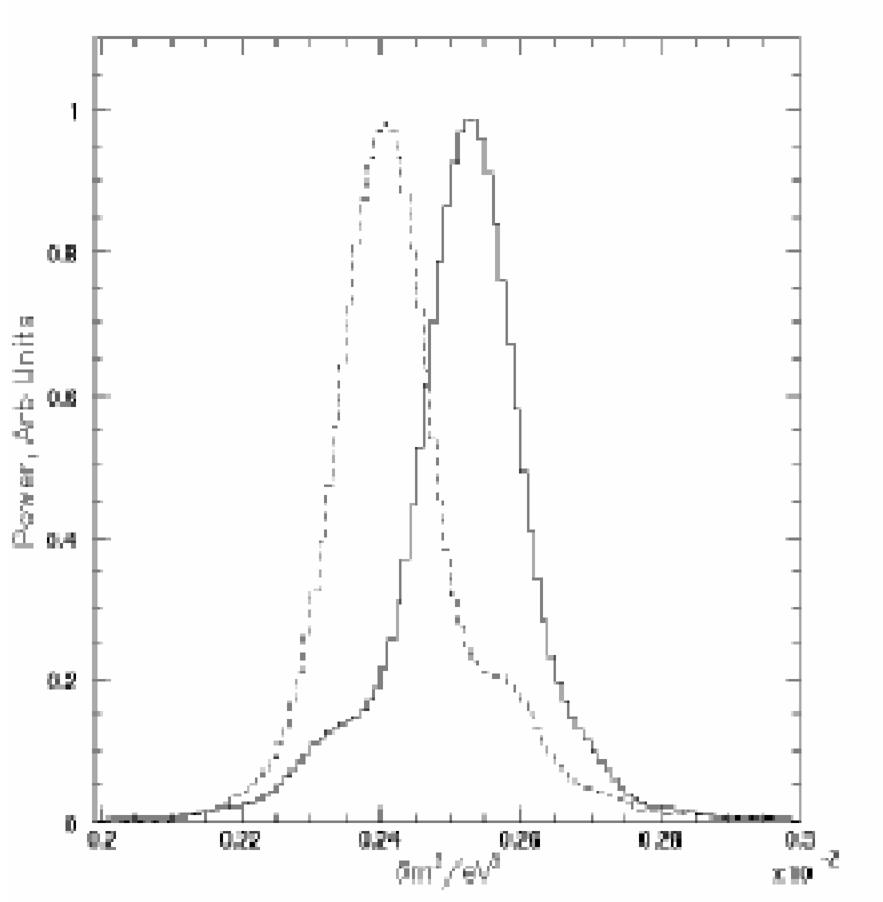


P. Ghoshal, S.T.P., arXiv:1011.1646

Fourier Analysis:

$$\text{NO}: \quad \cos^2\theta_{12}\sin^2\Delta + \sin^2\theta_{12}\sin^2(\Delta - \Delta_{21}),$$
$$\text{IO}: \quad \sin^2\theta_{12}\sin^2\Delta + \cos^2\theta_{12}\sin^2(\Delta - \Delta_{21}),$$

$$\Delta \equiv \Delta_{31}(NH) = |\Delta_{32}(IH)|;$$
$$\sin^2\theta_{12} \cong 0.31, \quad \cos^2\theta_{12} \cong 0.69.$$



J.Learned et al., 2007

Very challenging; requires:

- energy resolution $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}}$;
- relatively small energy scale uncertainty;
- relatively large statistics ($\sim$ (300 − 1000) kT GW yr);
- relatively small systematic errors;
- subtle optimisations (distance, number of bins, effects of "interfering distant" reactors).

Two experiments planned with $L \cong 50$ km: Daya Bay II (20 kT), RENO50 (18 kT). Can measure also $\sin^2\theta_{12}$, Δm^2_{21} and $|\Delta m^2_{31}|$ with remarkably high precision. Can be used for detection of SN neutrinos as well.

Atmospheric Neutrino Experiments on $\mathrm{sgn}(\Delta m^2_{31})$

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2\, P_{2\nu}\,, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2\, P_{2\nu}\,,$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4\; P_{2\nu} - 2c_{23}^2 s_{23}^2\; \left[1 - Re\; (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\right]\;,$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; N_e)$: 2-$\nu$ $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,
$\nu'_\tau = s_{23}\, \nu_\mu + c_{23}\, \nu_\tau$; $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$, $E_\nu \gtrsim 2$ GeV;

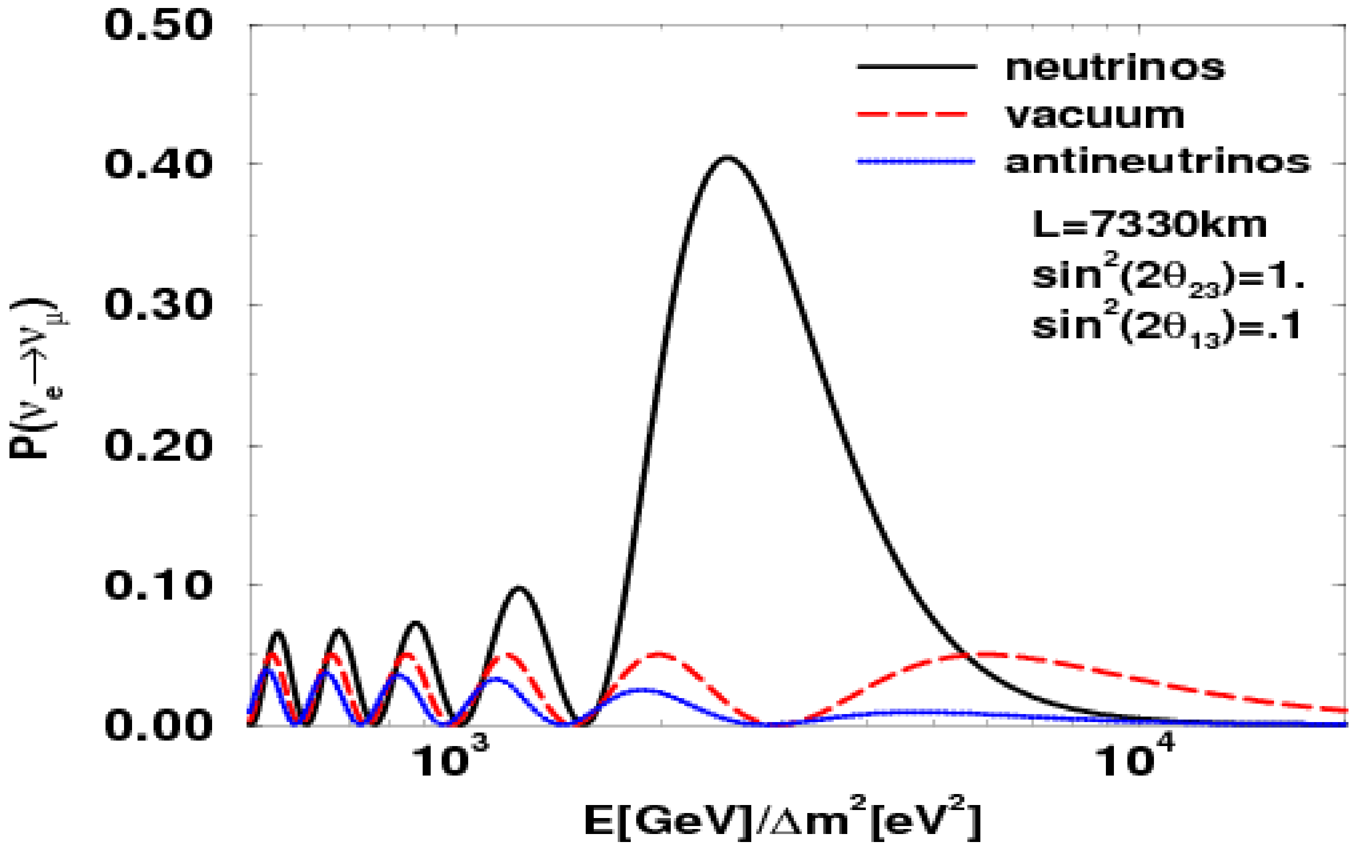
κ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2-ν amplitude.

NO: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced, $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

IO: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced, $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$−suppressed

No charge identification (SK, HK, IceCube-PINGU); event rate (DIS regime):
$[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ GeV; $P^{3\nu} = \sin^2\theta_{23} P^{2\nu}_m = 0.5 P^{2\nu}_m$;
$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^\nu/\sin 2\theta_{13} \cong 6250/0.32$ km; $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

Hyper Kamiokande (10SK), INO, IceCube-PINGU;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU (?)

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-, no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

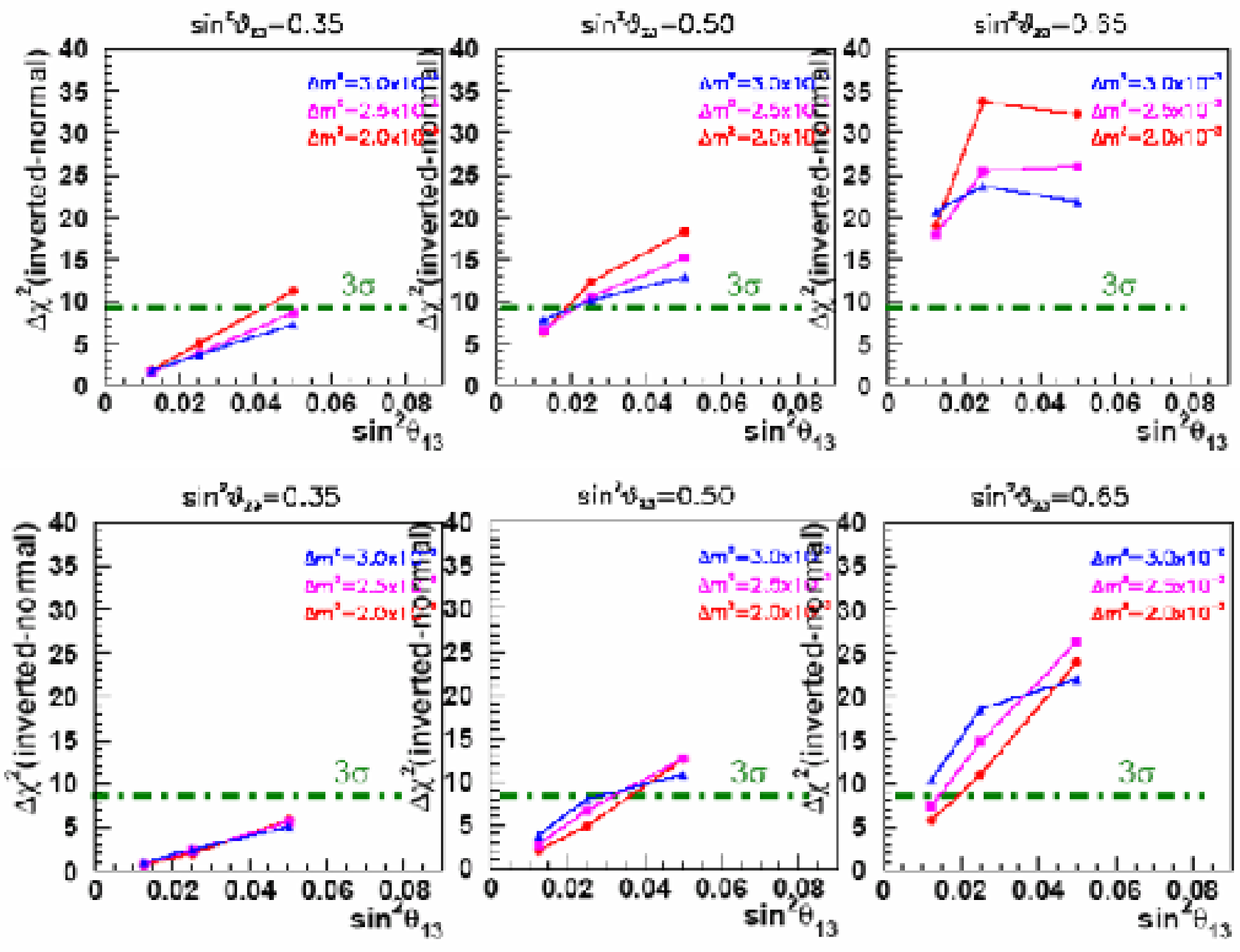
Water-Cerenkov detector: Hyper Kamiokande (10SK)

Sensitivity depends critically on θ_{23}, the “true” hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

$$P(\nu_\mu \to \nu_e) \cong \sin^2\theta_{23}\, \sin^2 2\theta_{13}^m\, \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

No charge identification (SK, HK, IceCube-PINGU); event rate (DIS regime):
$[2\sigma(\nu_l + N \to l^- + X) + \sigma(\bar{\nu}_l + N \to l^+ + X)]/3$

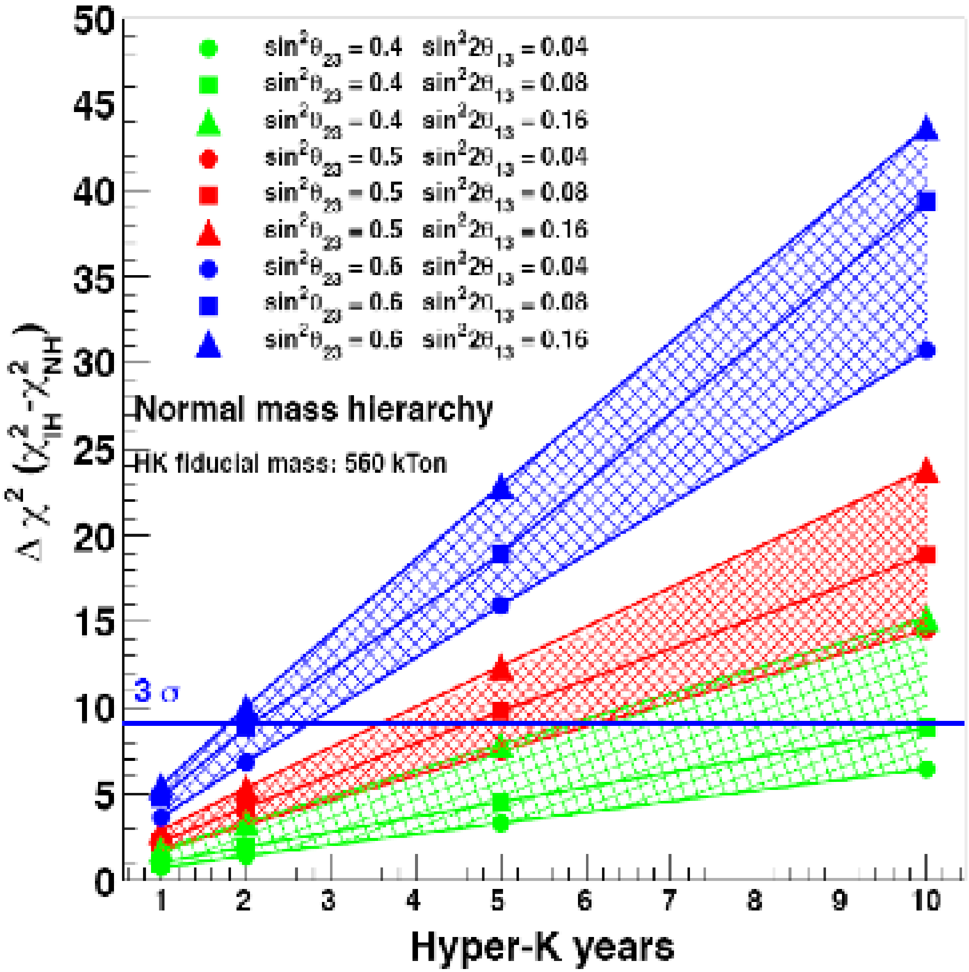


Water-Cerenkov detector, 1.8 MTy (HK = 10SK)

Critical dependence on θ_{23}, "true hierarchy".

T. Kajita et al., 2004

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

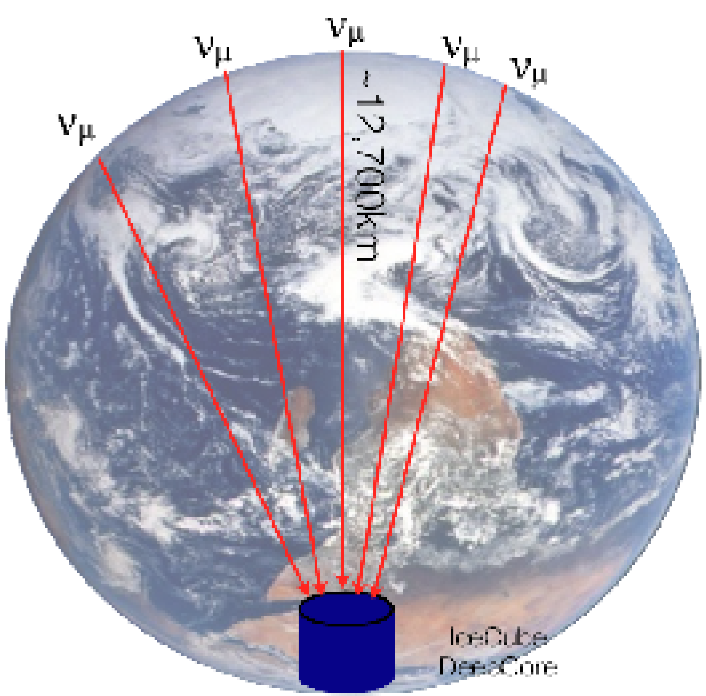


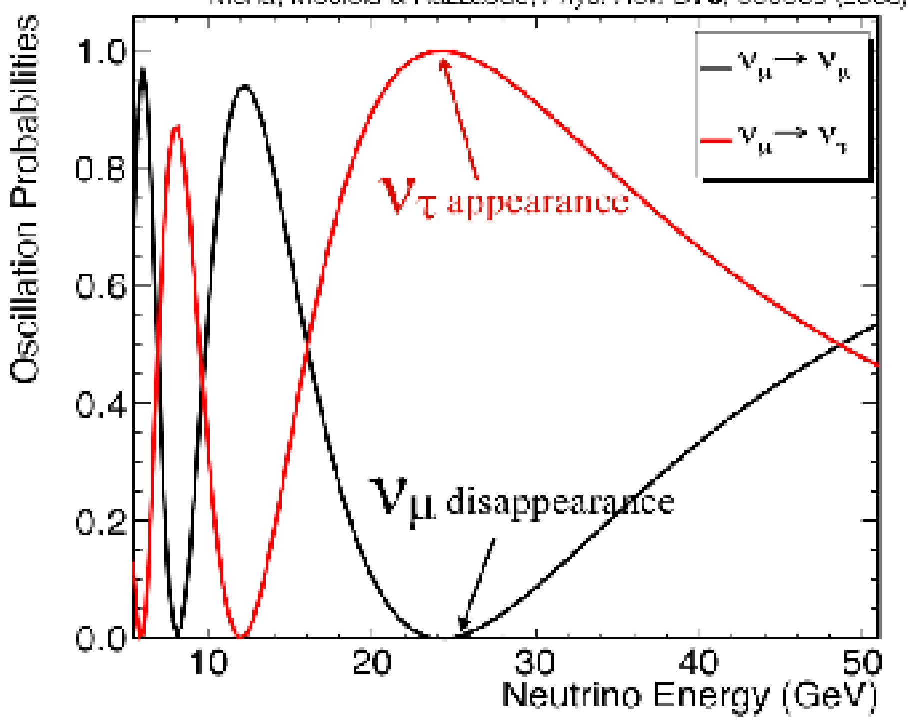
Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data. θ_{23} and θ_{13} are assumed to be known as indicated in the figure.

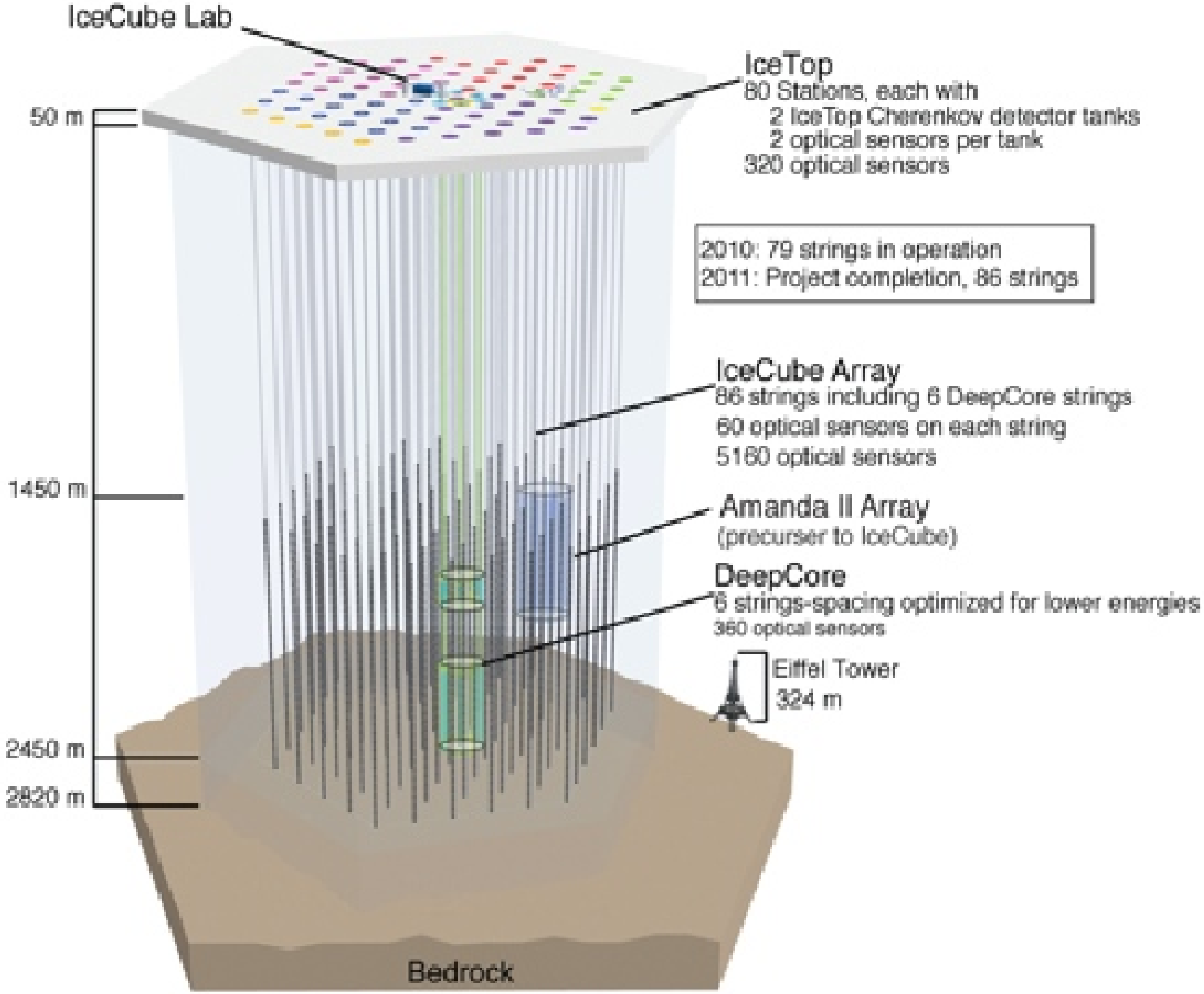
K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

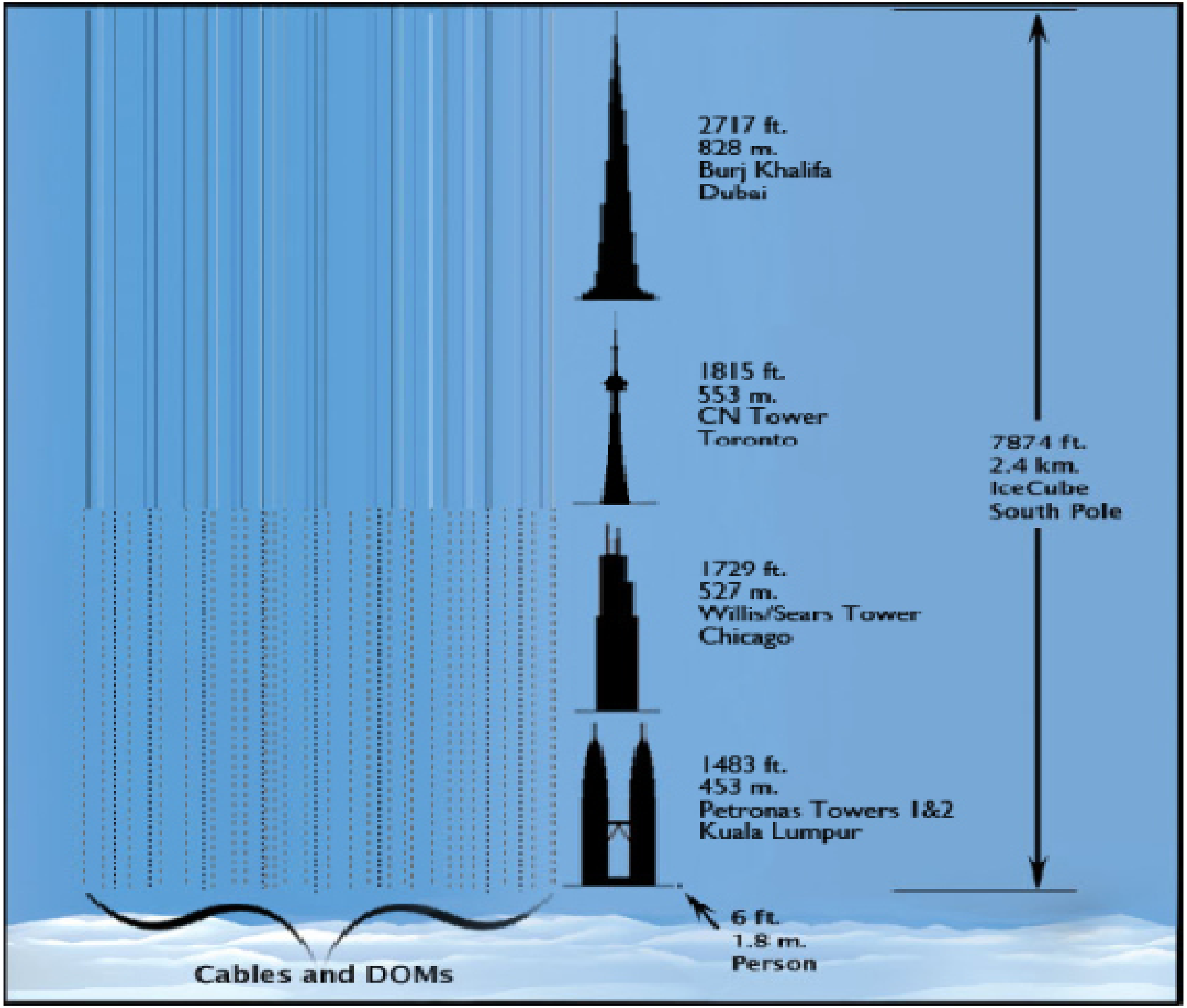
Neutrino Oscillation Source

- Northern Hemisphere ν_μ oscillating over one earth radii produces ν_μ (ν_τ) oscillation minimum(maximum) at ~25 GeV
  - Covers all possible terrestrial baselines
  - "Beam" is free and never turns off





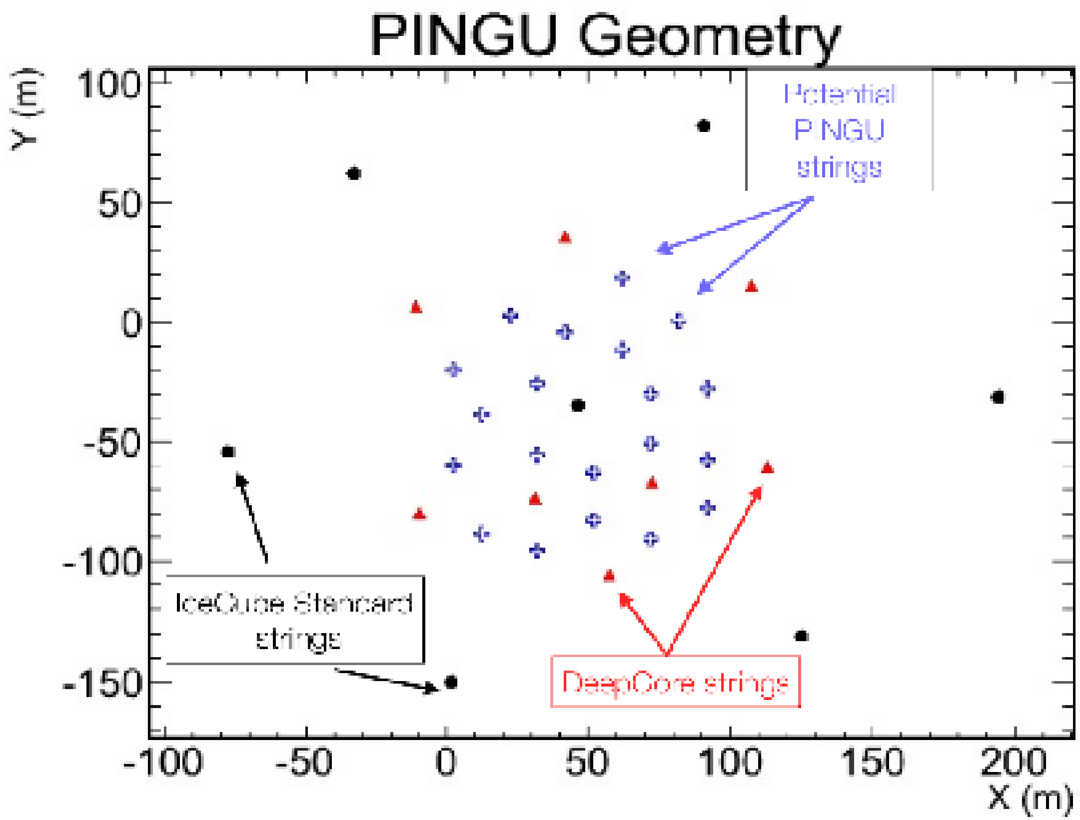

IceCube Lab
IceTop
80 Stations, each with
2 IceTop Cherenkov detector tanks
2 optical sensors per tank
320 optical sensors
2010: 79 strings in operation
2011: Project completion, 86 strings
IceCube Array
86 strings including 6 DeepCore strings
60 optical sensors on each string
5160 optical sensors
Amanda II Array
(precursor to IceCube)
DeepCore
6 strings-spacing optimized for lower energies
360 optical sensors
Eiffel Tower
324 m
50 m
1450 m
2450 m
2820 m
Bedrock



2717 ft.
828 m.
Burj Khalifa
Dubai
1815 ft.
553 m.
CN Tower
Toronto
1729 ft.
527 m.
Willis/Sears Tower
Chicago
1483 ft.
453 m.
Petronas Towers 1&2
Kuala Lumpur
7874 ft.
2.4 km.
IceCube
South Pole
6 ft.
1.8 m.
Person
Cables and DOMs

PINGU: Possible Geometry

- ~20 strings within DeepCore volume w/ short string-string spacing
  - IC-IC: 125m
  - DC-DC: ~80m
  - PINGU-PINGU: <= 26m
- Shorter DOM-DOM spacing
  - IC-IC: 17m
  - DC-DC: 7m
  - PINGU-PINGU: <= 5m
- R & D for future water/ice cerenkov



Koskinen, BNL, 2012

Future LBL Neutrino Oscillation Experiments on sgn(Δm^2_{31}) (the Hierarchy) and CP Violation

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the backgrpound of electrons, protons and neutrnos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \to \nu_e) \cong \sin^2\theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$\sin^2 2\theta_{13}^m$, ΔM_{31}^2 depend on the matter potential
$V_{eff} = \sqrt{2}\, G_F\, N_e$,

For antineutrinos V_{eff} has the opposite sign:
$V_{eff} = -\sqrt{2}\, G_F, N_e$.

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
$\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
$\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ —suppressed

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_{\mathrm{F}}N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: $\delta,\;\; A \to (-\delta),\;\; (-A)$

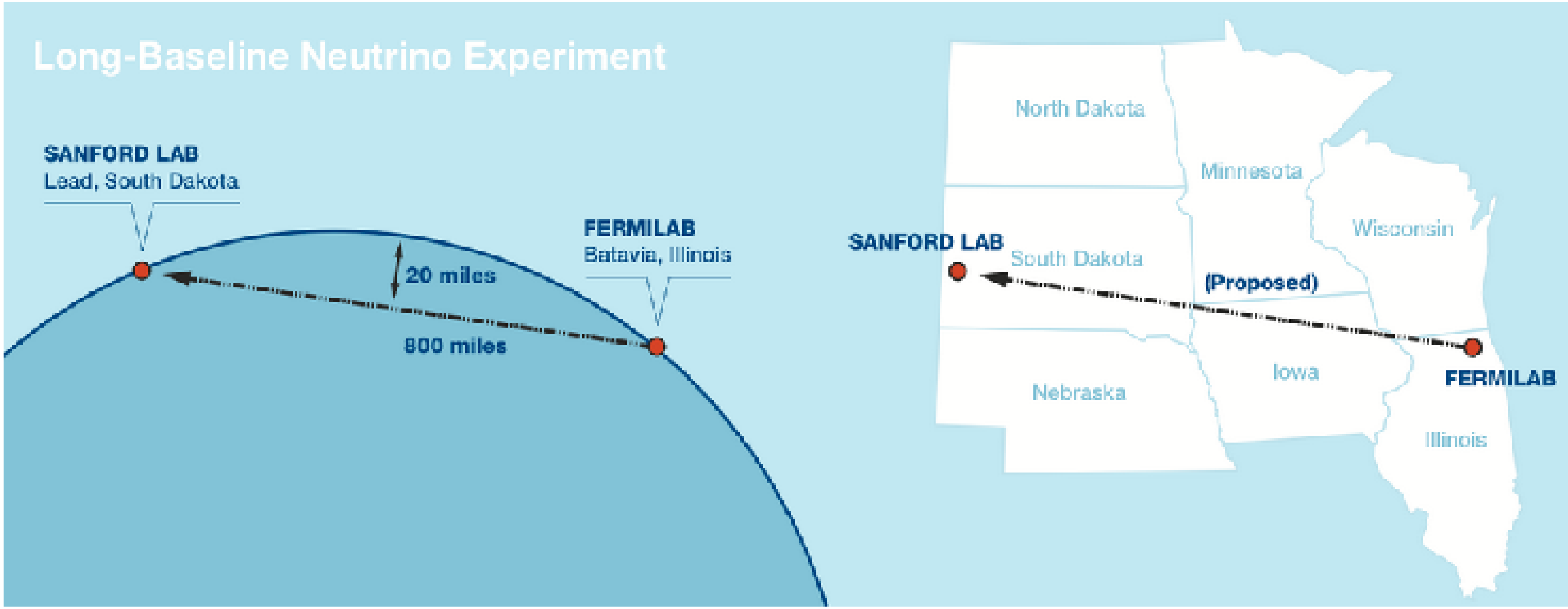
LBL Oscillation Experiments NOνA, LBNE, LBNO

NOνA: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2013.

LBNE: Fermilab-DUSEL, $L = 1290$ km, 700 KW wide band ν beam (first and second osc. maxima at $E = 2.4$ GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with 15% to 30% PMT covarage, or multiple 17 kt fiducial volume LAr detecors; plans to run 5 years with ν_μ and 5 years with $\bar{\nu}_\mu$; 2023 (?)

LAGUNA-LBNO: CERN-Pyhasalmi, $L = 2290$ km, wide band ν_μ 1.6 MW super beam (first and second osc. maxima at $E \cong 4$ GeV and 1.5 GeV); 440 kt Water Cherenkov, or 100 kt LAr, or 50 kt liquid scintillator detector; 2023 (?)


Ontario
NOvA
Soudan
Minnesota
Wisconsin
735 kilometers
Iowa
Fermilab
Illinois


Long-Baseline Neutrino Experiment
SANFORD LAB
Lead, South Dakota
20 miles
800 miles
FERMILAB
Batavia, Illinois
North Dakota
Minnesota
Wisconsin
SANFORD LAB
South Dakota
(Proposed)
Iowa
FERMILAB
Nebraska
Illinois

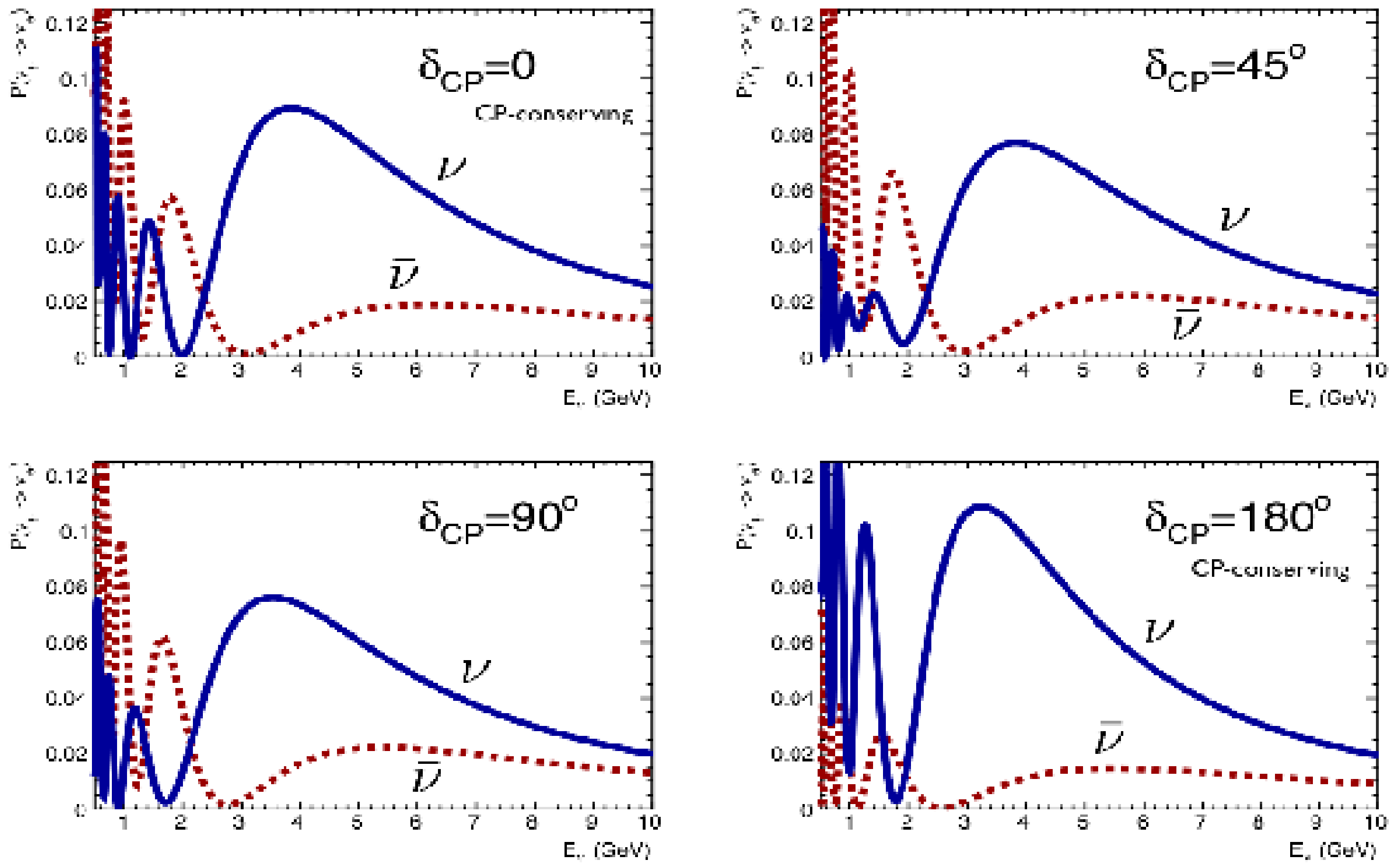
2100 km from RAL, 1500 km from DESY, and 1160 km from Protvino.

CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★Normal mass hierarchy

L=2300 km

$\sin^2(2\theta_{13}) = 0.09$

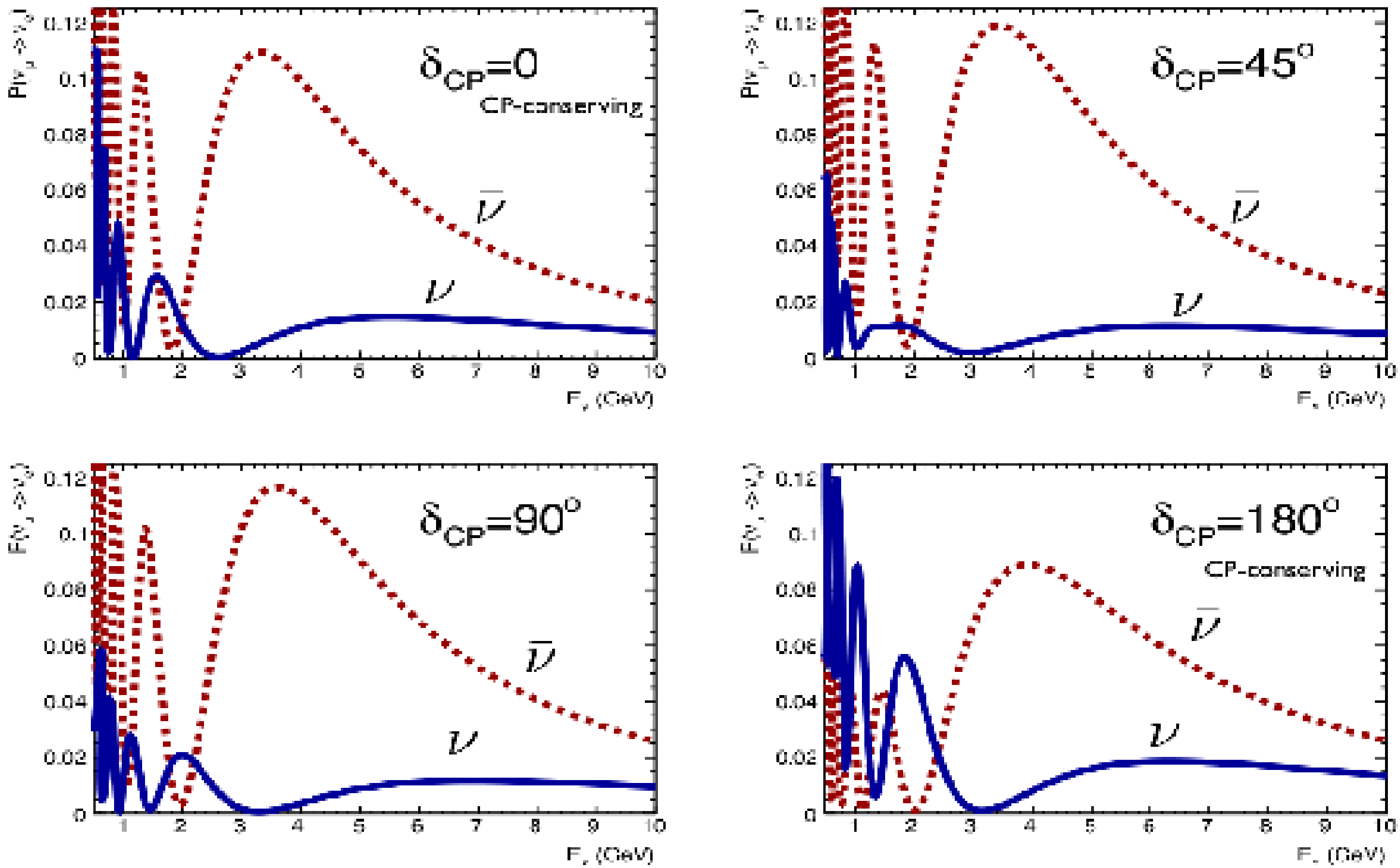


CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★Inverted mass hierarchy

L=2300 km

$$\sin^2(2\theta_{13}) = 0.09$$



Instead of Conclusions

The future of neutrino physics is bright.