

Coupling Moduli to Inflation

Astroparticle workshop
Warsaw February 2007

Outline

Moduli

Mutated chaotic inflation

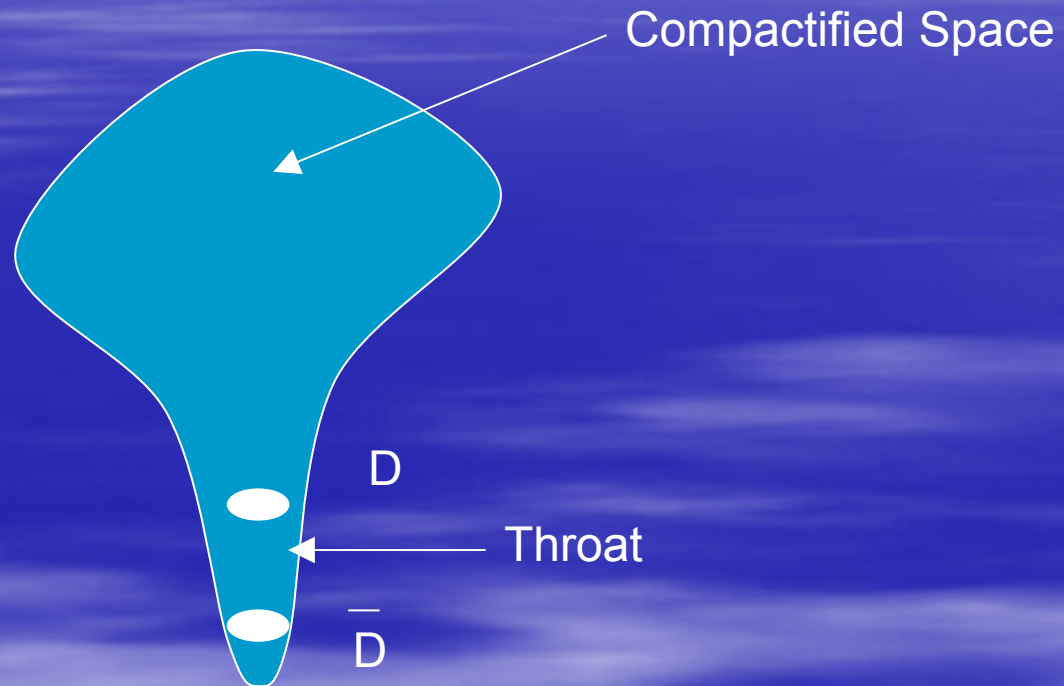
F-term hybrid inflation

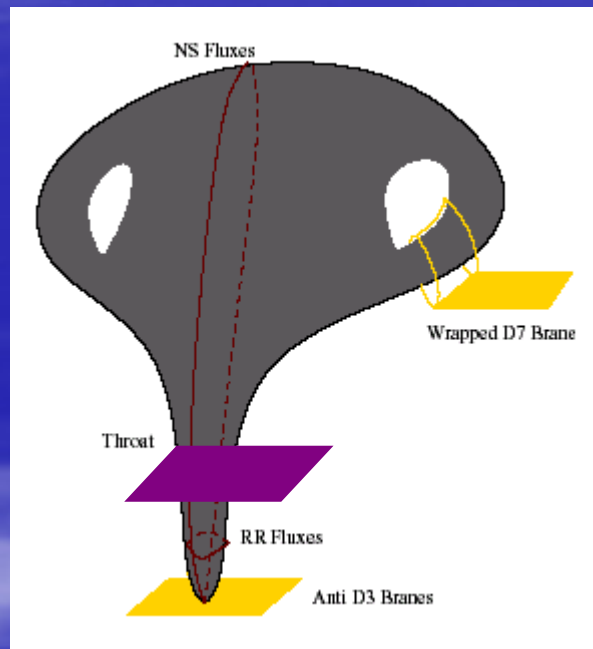
D-term hybrid inflation

Moduli Fields

- **Moduli fields** parameterise **flat directions** of the scalar potential in supersymmetric models
- Flat directions ubiquitous in susy gauge theories (even the MSSM)
- Flat directions can have a direct geometric origin, e.g. **shape moduli** (Kahler or complex structure moduli)
- The **dilaton** is a moduli as string perturbation theory does not fix its value.
- Moduli can have a dynamical origin, e.g. **brane distances** of BPS configurations (no force).

Compactification





The KKLT Scenario

- In type IIB string theory **compactified** on a Calabi-Yau manifold (10d to 4d), there are 3 types of moduli: the Kahler moduli, the complex structure moduli and the dilaton.
- Introducing **fluxes**, the complex structure moduli and the dilaton are stabilised.

$$\int F_3 = 4\pi^2\alpha' M, \quad \int H_3 = -4\pi^2\alpha' N$$

- This is due to the superpotential:

$$W = \int \Omega \wedge G_3, \quad G_3 = F_3 - \tau H_3, \quad \tau = C_0 + ie^{-\phi}$$

- This leaves the **Kahler moduli** as flat directions.
- In general, only one Kahler moduli **T**: the size of the compact manifold: How to stabilise it??

No scale?

- The **Kahler potential** of the T modulus is of the no-scale type.

$$K = -3 \ln(T + \bar{T})$$

- Once the dilaton and complex structure moduli have been stabilised, the resulting superpotential is constant

$$W = W_0$$

- Non-perturbative effects appear on D7 branes wrapped around 4-cycles of the CY compactification:

$$W = W_0 + Ae^{-aT}$$

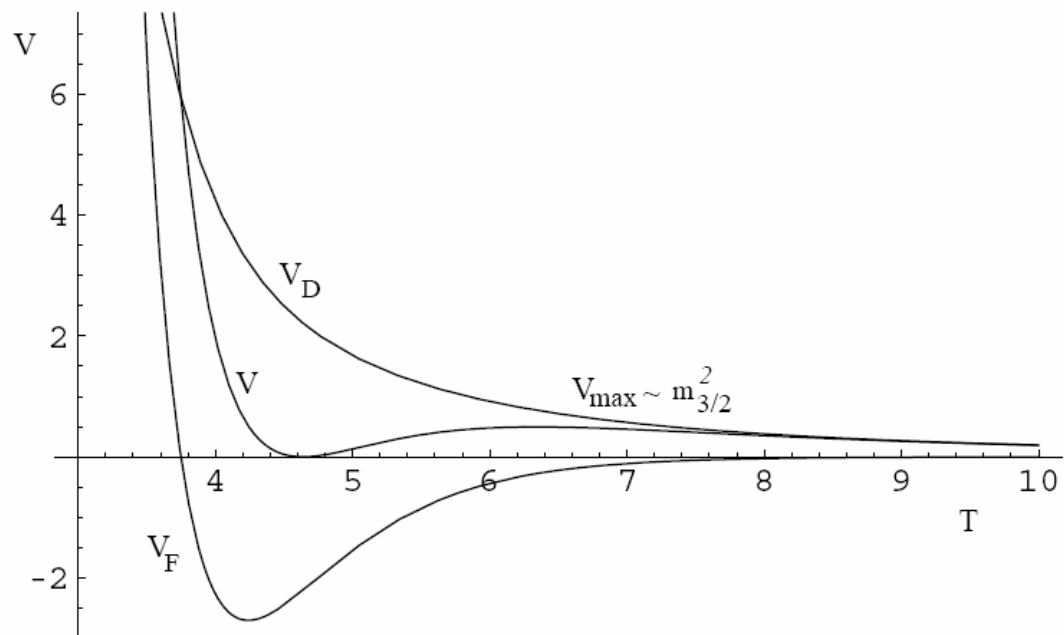
- The non-perturbative superpotential is nothing but an SQCD effect with a gauge function:

$$\frac{1}{g^2} = \Re T$$

- The size moduli T picks up a scalar potential with a negative minimum.
- The minimum describes an anti de Sitter space.

Uplifting

- Of course, we do not live in AdS space!!.
- Need to “**uplift**” the potential to get a vanishing (nearly) cosmological constant.
- Originally, “uplifting” by adding an **anti-brane**. Badly non-supersymmetric (mutated chaotic and F-term inflation).
- Lifting by **F-terms**, coupling to metastable susy breaking sectors.
- Lifting by **D-terms**: tricky due to gauge invariance and anomalies... the route we will follow for D-term inflation.



Inflation and Supergravity

- Inflation requires **flat potentials** $\epsilon = m_p^2 \left(\frac{V'}{V}\right)^2 \leq 1$

- Almost scale invariant spectrum $n_s - 1 = -6\epsilon + 2\eta$

requiring $\eta = m_p^2 \frac{V''}{V} \ll 1$

- In global susy, easy to obtain **chaotic inflation** $K = \phi\bar{\phi}, W = \frac{1}{2}m\phi^2 \rightarrow V = m^2|\phi|^2$

- In supergravity the potential depends on $G = K + \ln |W|^2$

K Kahler potential, **W** superpotential

$$V = e^G (G_i G^i - 3) + V_D$$

easy to obtain **Hybrid inflation**, tricky **Chaotic inflation**

KKLT Stabilisation and Inflation

- In string theory \longrightarrow **Moduli** (volume of the compactifying manifold)

- Stabilisation $W = W_0 + Ae^{-\beta T}$

- Inflation \longrightarrow motion of D3 branes $K = -3 \ln(T + \bar{T} - \phi\bar{\phi})$

- **Brane-antibrane** potential lifts the potential

$$V = \frac{1}{3\Delta} \left| \frac{\partial W}{\partial T} \right|^2 \left(1 + \frac{1}{3\Delta} \phi\bar{\phi} \right) - \frac{1}{\Delta^2} \left(W \frac{\partial \bar{W}}{\partial \bar{T}} + \bar{W} \frac{\partial W}{\partial T} \right) + \frac{E}{\Delta^2}$$

- Around the stabilised modulus

$$V = V_{inf} + 2H_{inf}^2 |\Phi|^2$$

mass of the inflaton of the Hubble scale

large value of $\eta = O(1)$

needs fine-tuning (KKLMMT scenario)

The flatness problem

- In supergravity models of inflation, the scalar potential

$$\exp(\kappa K(\phi, \bar{\phi}))V_{inf} = (1 + \kappa K(\phi, \bar{\phi}) + \dots)V_{inf}$$

- For canonically normalised fields, large mass for the inflaton

$$K(\phi, \bar{\phi}) = \phi\bar{\phi} \longrightarrow \eta = O(1)$$

- Generically, **scalar index**

$$|n_s - 1| \gg 1$$

Shift Symmetry

- Inflaton \longrightarrow flat direction
- Lifted by interactions
- Along the flat direction \longrightarrow **shift symmetry**

$$\phi \longrightarrow \phi + c$$

- Natural in string theory \longrightarrow motion of D3/D7

Special form of the superpotential

$$W(T, \phi) = e^{-s\phi^2/2} W\left(T - \sigma \frac{\phi^2}{2}\right)$$

After a Kahler transformation

$$K = -3 \ln\left(T + \bar{T} + \frac{\sigma}{2}(\phi - \bar{\phi})^2\right) - \frac{s}{2}(\phi - \bar{\phi})^2$$
$$W = W(T)$$

Inflation obtained by **breaking shift symmetry** in the superpotential

$$W(T) \longrightarrow W(T) + W(\phi)$$

Does not generate large values of the mass

$$m \neq O(H_{inf})$$

Mutated chaotic inflation

- Superpotential for **modulus**

$$W(T) = W_0 + Ae^{-\beta T}$$

Leads to AdS susy vacuum

Introduce a lifting term breaking susy

$$\delta V = \frac{E}{(T + \bar{T})^3} \longrightarrow V_{vac} = 0$$

- Superpotential for inflation

Chaotic inflation superpotential:

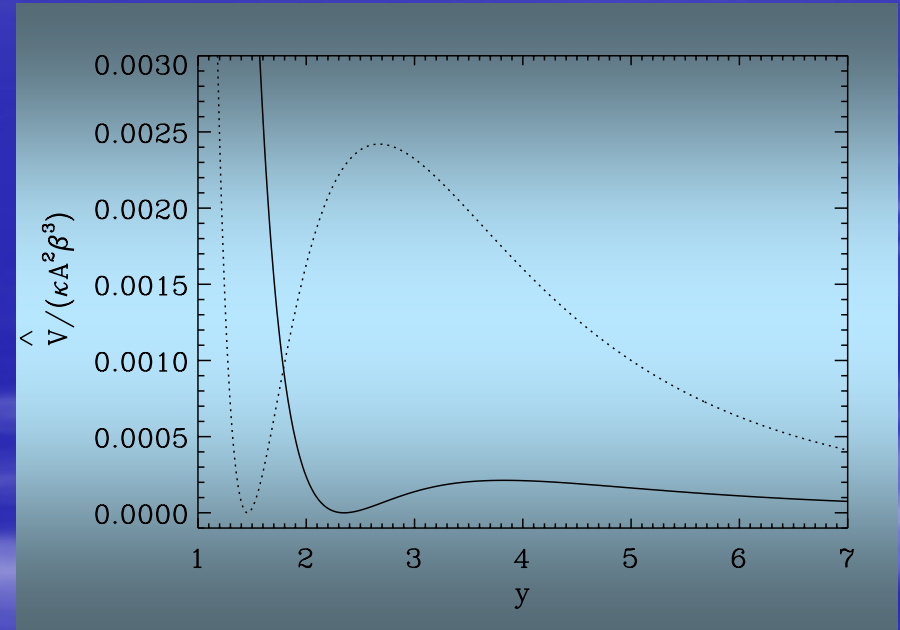
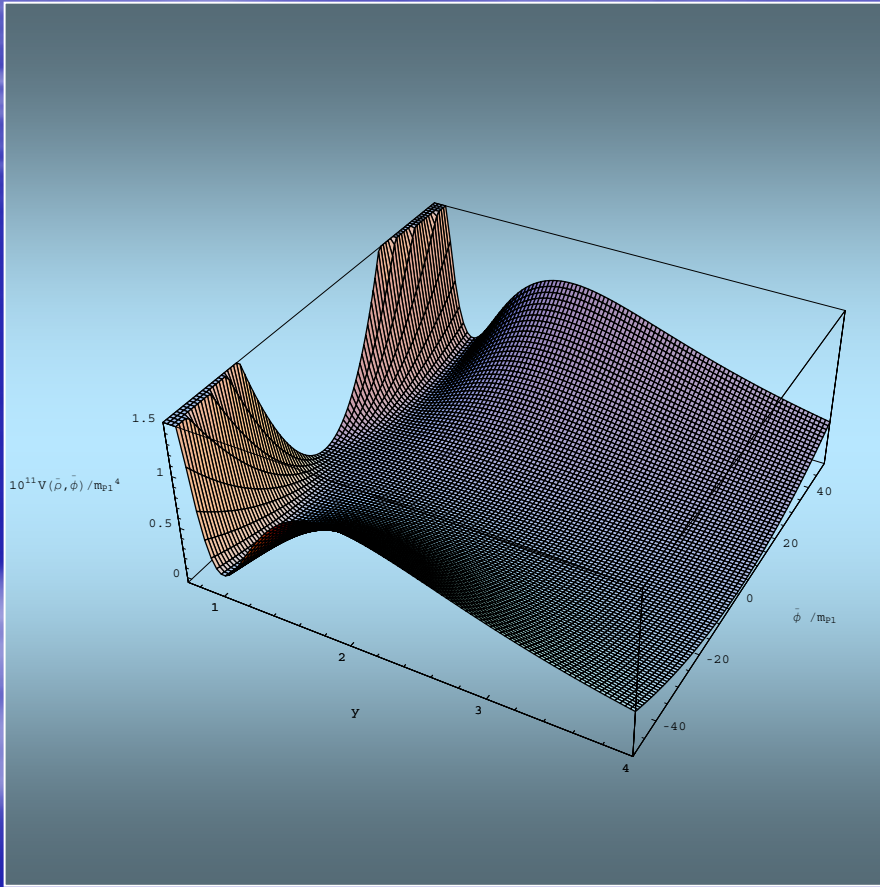
$$W(\phi) = \frac{1}{2}m\phi^2$$

Can be obtained in the condensed phase of
D3/D7 system

COBE normalisation

$$m \sim 10^{-6} m_P$$

The scalar potential



Curved inflationary trajectory

- Scalar potential

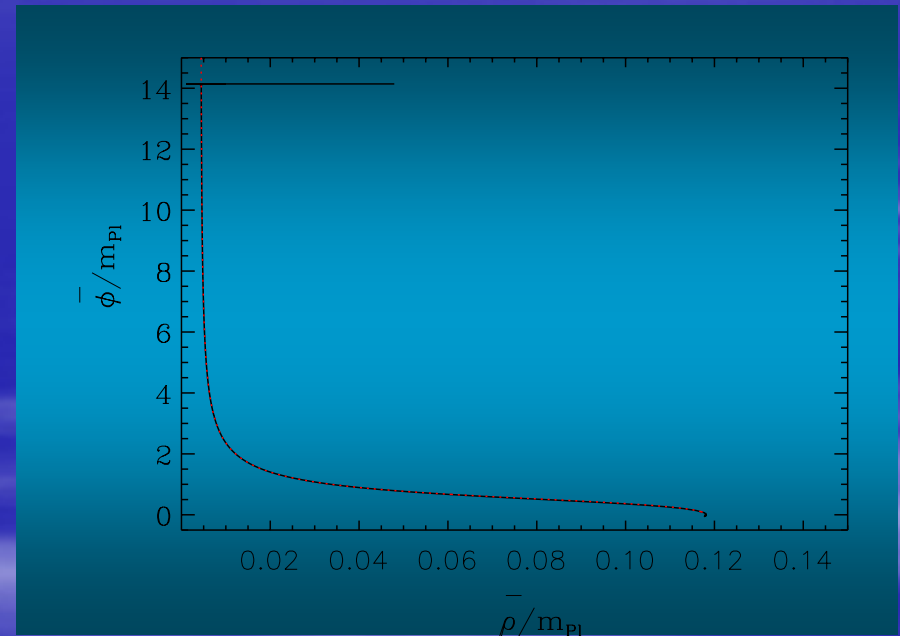
$$V(T, \phi) = v(T) + u(T)\phi^2$$

- Curved trajectory

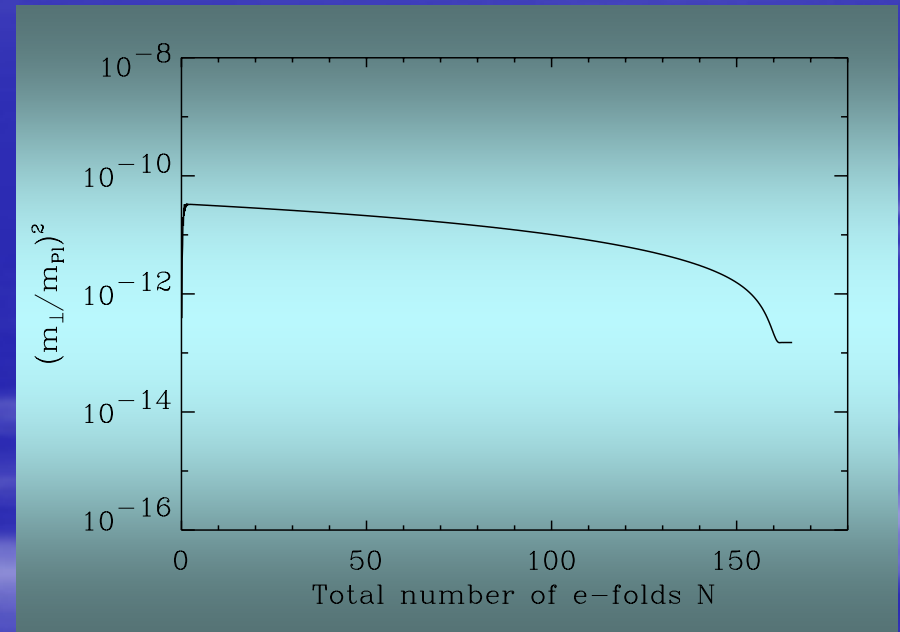
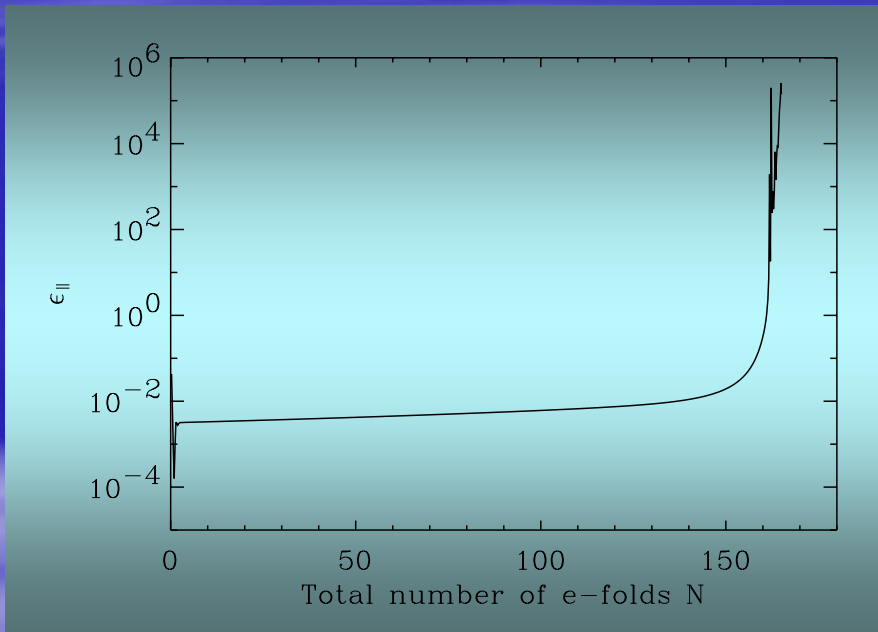
$$\phi^2 = -\frac{v'(T)}{u'(T)}$$

Inflationary valley

- End of inflation $\epsilon_{//} \sim 1$



Mutated vs Hybrid



Spectral Index

- Comparison with **chaotic** inflation

$$n_s \approx 0.967, \quad n_T \approx -0.0165$$

- **Mutated chaotic** inflation

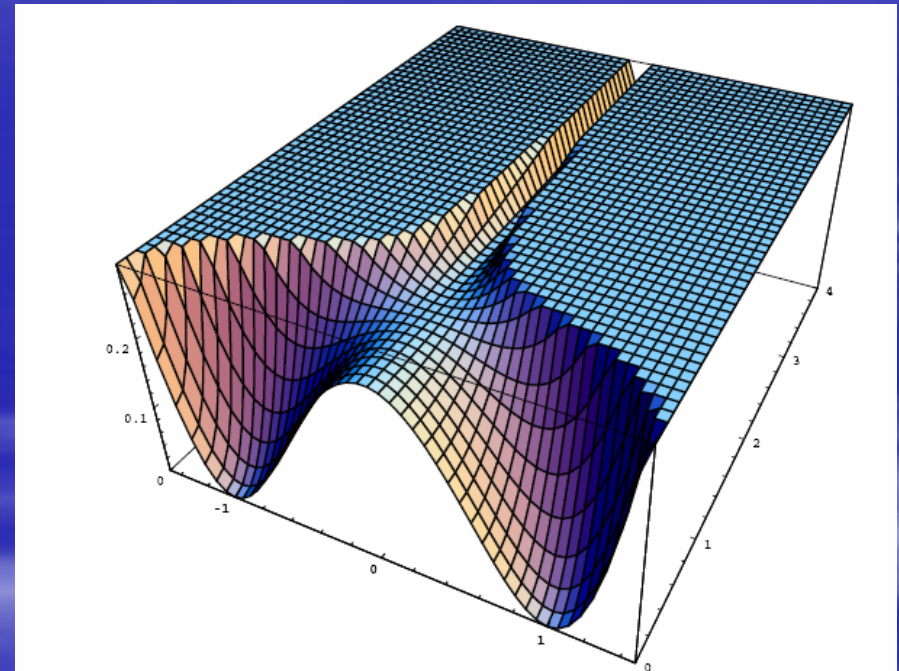
$$n_s \approx 0.979, \quad n_T \approx -0.012$$

Hybrid inflation

- Moduli can have a strong influence on inflation.
- In the **D3/D7 system**, hybrid inflation occurs where the inflaton is the interbrane distance and the waterfall fields are the open strings connecting the branes. The resulting superpotential:

$$W = \lambda \phi \phi^+ \phi^-$$

- At tree level, the potential is flat along the inflaton direction. Slope due to **one loop corrections**.
- At the end of inflation, the **waterfall fields condense** (due to a Fayet-Iliopoulos term) and cosmic strings can be formed.
- What is the influence of moduli?



F-term Hybrid Inflation

- Hybrid inflation coupled to moduli

$$W = \sqrt{2}g\phi(\phi^+\phi^- - x^2) + W_{mod}$$

Stringy origin if $x=0$,

ϕ **inflaton** (interbrane distance)

ϕ^\pm **waterfall fields** (open strings)

- Shift** symmetry

$$K = -\frac{1}{2}(\phi - \bar{\phi})^2 + (\phi^+ - \bar{\phi}^-)(\bar{\phi}^+ - \phi^-)$$

- The **potential** during **inflation** $\phi^{\pm} = 0$

$$V = V_{stab}(T) + V_0(T) - 2\sqrt{2}gx^2e^K \text{Re}(V_1)\phi + 2g^2x^4e^K V_2\phi^2$$

V_{stab} KKLT stabilisation potential

$V_0 = 2g^2x^4e^K$ Inflation scale

- **End** of inflation by instability

$$\phi_{\text{end}} = \sqrt{x^2 + \frac{V_1^2}{8g^2} - \frac{V_1}{2\sqrt{2}g}}$$

- Stability of moduli during inflation

$$m_{3/2} \geq \sqrt{\frac{N}{\delta}} V_0$$

very large gravitino mass, large sparticle masses

- Small inflation scale vs potential barrier

$$W_{mod} \geq gx^2$$

- Slow roll parameters

$$\epsilon \leq 1 \longrightarrow W_{mod} \leq gx^2$$

conflict only resolved by fine-tuning

Curved Inflation Trajectories

- Inflaton-moduli coupling leads to **curved** trajectories
- Close to the moduli minimum $\delta T(\phi)$
- **Effective potential**

$$\eta \approx -4 \frac{[(e^K V_1)']^2}{m_T^2}$$

m_T mass of the moduli field

Eta problem

- In general η large unless

$$TW'_{mod}, T^2W''_{mod} \ll (T^3W'''_{mod}W_{mod})^{1/2}$$

- Fine-tuning of

$$W_{mod} = W_0 + Ae^{-aT} + Be^{-bT}$$

may be possible.....

D-term Hybrid Inflation

- Inflation due to field dependent **Fayet-Iliopoulos** terms
- Uplifting due to a second Fayet-Iliopoulos term
- Superpotential

$$W = \lambda\phi\phi^+\phi^- + W_{mod}$$

non-analytic **superpotential** to lift an AdS vacuum

$$W_{mod} = W_0 + A\chi^{-b}e^{-aT}$$

Hybrid Inflation Coupled to Moduli

- Inflation can **destabilise** moduli if the inflation scale is much higher than the barrier scale of the moduli potential

$$\alpha \sim \frac{m_{3/2}^2}{g^2 \xi^2}, \quad \alpha \geq 1$$

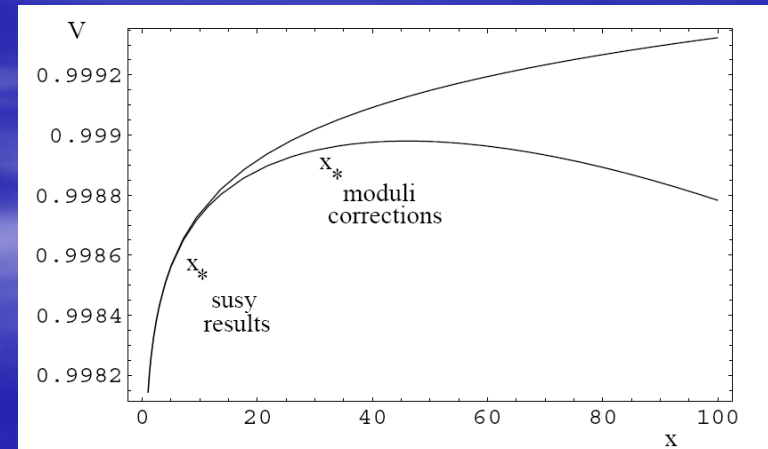
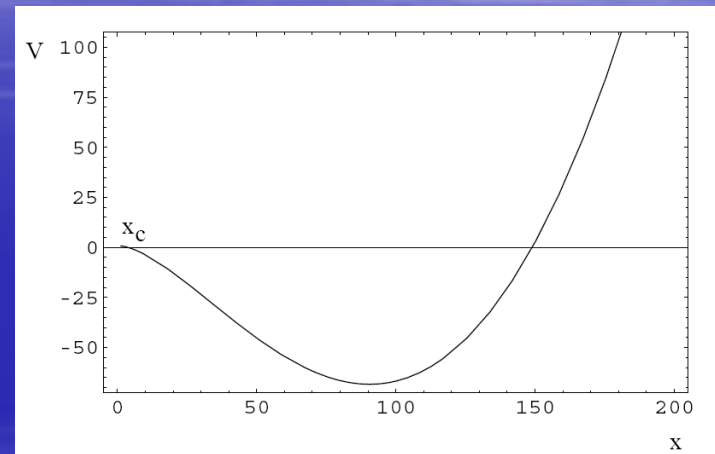
- Moduli **modify** the one loop potential

$$V = \frac{g^2 \xi^2}{2} \left(1 + \frac{g^2}{8\pi^2} \ln \frac{\lambda^2 \phi^2}{2\Lambda^2 e^{-3/2}} + \frac{\lambda^2 \phi^2}{24\pi^2} \left((\alpha+1) \ln \frac{\lambda^2 \phi^2}{2\Lambda^2 e^{-3/2}} - 1 \right) \right)$$

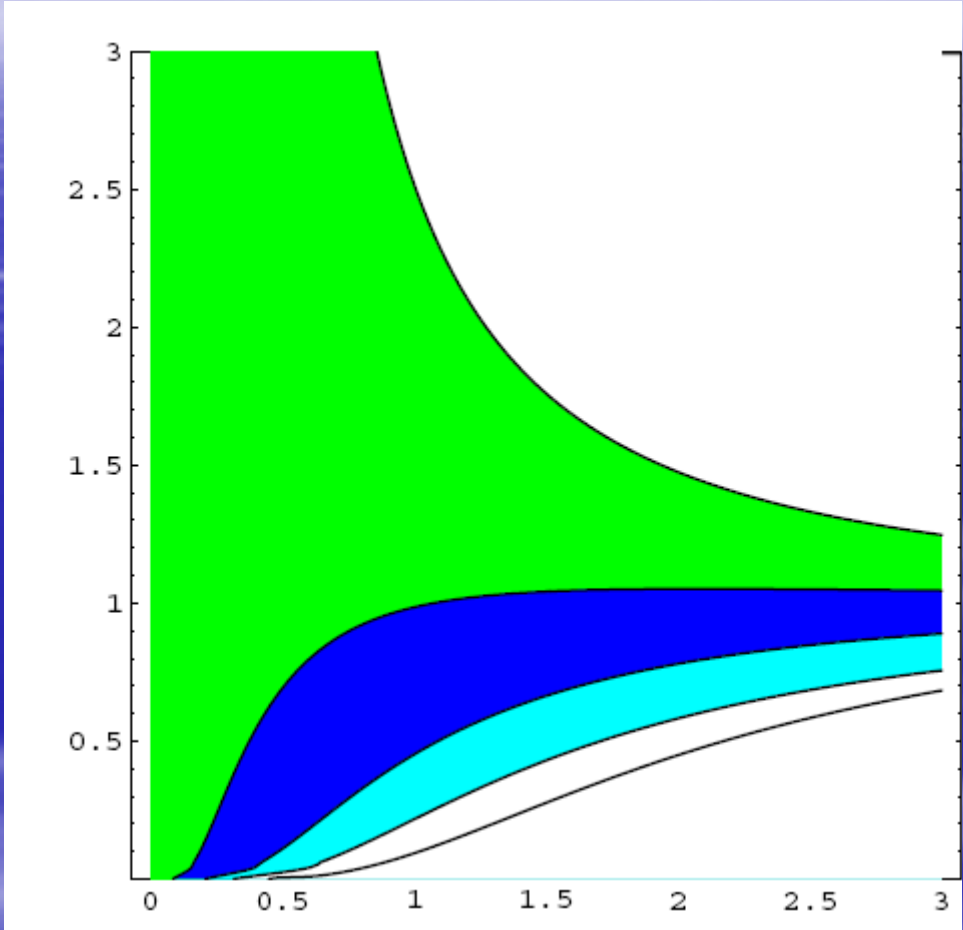
- Moduli can **prevent** graceful exit

$$\alpha \xi \leq 1$$

- Moduli can **alter** the spectral index (structure formation).



$\ln g$



$\ln \lambda$

Conclusions

- Inflationary model combining **modulus stabilisation** and **inflation**
- Curved inflationary trajectories
- Chaotic inflation preserved thanks to shift symmetry
- F-term hybrid requires fine-tuning
- D-term hybrid new features due to moduli sector