# Hierarchy Problems in String Theory: The Power of Large Volume

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This talk is based on research carried out in

hep-th/0502058 (V. Balasubramanian, P. Berglund, JC, F. Quevedo)

hep-th/0505076 (JC, F. Quevedo, K. Suruliz)

hep-th/0602233 (JC)

hep-th/0609180 (JC, D.Cremades, F. Quevedo)

hep-th/0610129 (JC, S. Abdussalam, F. Quevedo, K. Suruliz)

hep-ph/0611144 (JC, D. Cremades)

hep-ph/07xxxxx (JC, F. Quevedo)

# Hierarchies in Nature

Energy scales in nature are hierarchical:

- The Planck scale, $M_P = 2.4 \times 10^{18}$GeV.
- The GUT/inflation scale, $M \sim 10^{16}$GeV.
- The axion scale, $10^9$GeV $\lesssim f_a \lesssim 10^{12}$GeV.
- The scale of weak interactions: $M_W \sim 100$GeV.
- The scale of strong interactions: $\Lambda_{QCD} \sim 200$MeV.
- The scale of neutrino masses, $0.05$eV $\lesssim m_\nu \lesssim 0.3$eV.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$.

There is little understanding of these scales.

# Large-Volume Models

I here advocate

- stabilised exponentially large extra dimensions ($\mathcal{V} \sim (10^{-25}\text{m})^6 \sim 10^{15} l_s^6$ ).
- an intermediate fundamental scale $m_s \sim 10^{11}$GeV.

This will relate the axionic scale, the weak scale and the neutrino mass scale.

The different scales will come as different powers of the (large) volume.

# Large-Volume Models

- Extra dimensions are required by string theory.
- In four dimensional language, the size and shape of the extra dimensions appear as scalar particles (moduli).
- Moduli are naively massless uncharged scalar fields with gravitational-strength interactions.
- These are unphysical: the moduli must be given masses and stabilised.
- Large-volume models are a particular moduli stabilisation scenario that appears in flux compactifications of IIB string theory.

# Large-Volume Models

The appropriate 4-dimensional supergravity theory is

$$K = -2\ln\left(\mathcal{V}+\frac{\xi}{g_s^{3/2}}\right) - \ln\left(i\int\Omega\wedge\bar{\Omega}\right) - \ln\left(S+\bar{S}\right),$$

$$W = \int G_3\wedge\Omega + \sum_i A_i e^{-a_i T_i}.$$

We include the leading stringy $\alpha'$ corrections to the Kähler potential.

This leads to dramatic changes in the structure of the potential.

# Large-Volume Models

The simplest model (the Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$) has two moduli.

$$\mathcal{V} = \left( \left( \frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left( \frac{T_s + \bar{T}_s}{2} \right)^{3/2} \right) \equiv \left( \tau_b^{3/2} - \tau_s^{3/2} \right).$$

Computing the moduli scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi |W|^2}{g_s^{3/2} \mathcal{V}^3}.$$

The minimum of this potential can be found analytically.

# Large-Volume Models

The locus of the minimum satisfies

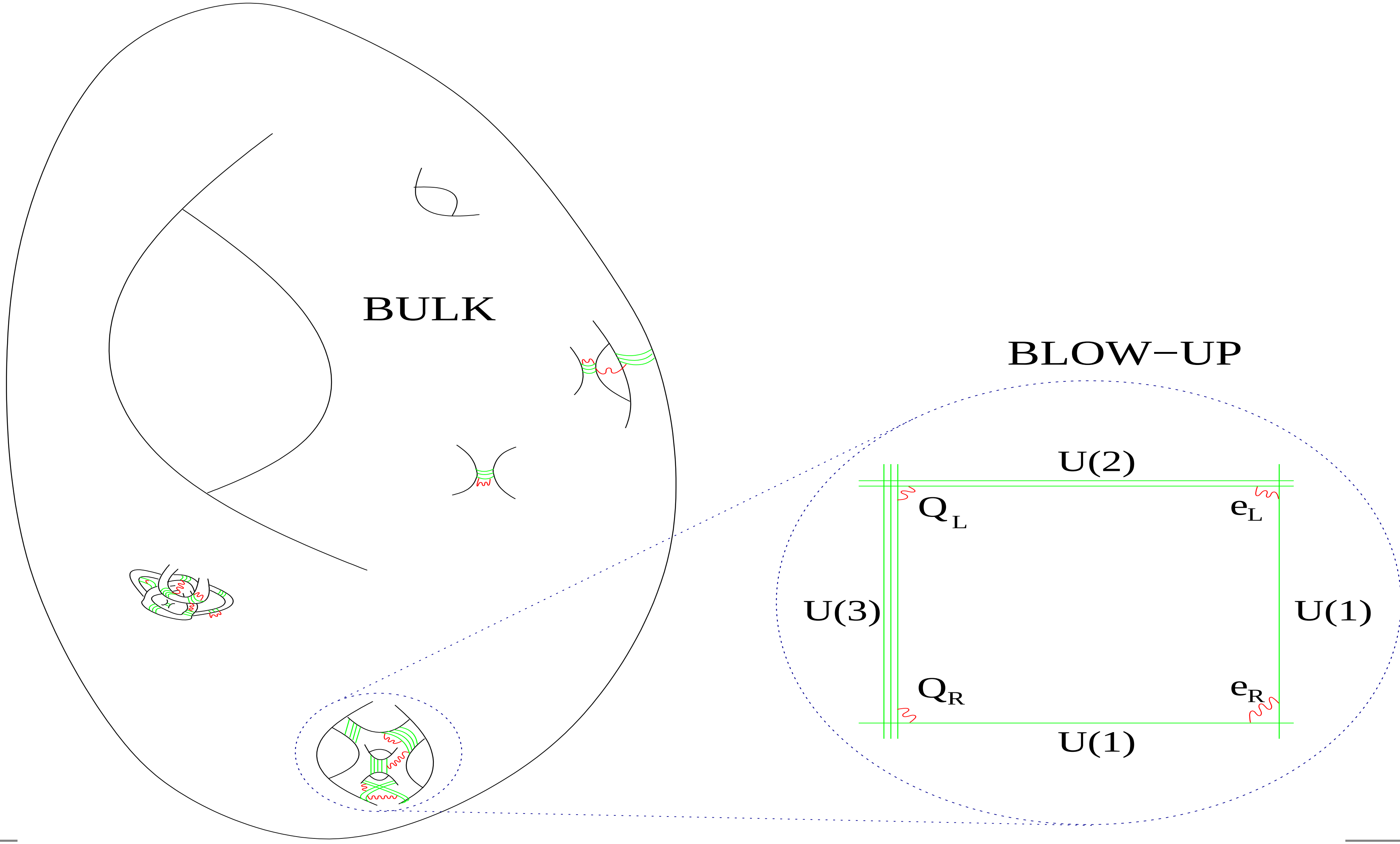
$$\mathcal{V} \sim |W| e^{c/g_s}, \qquad \tau_s \sim \ln \mathcal{V}.$$

The minimum is non-supersymmetric and at exponentially large volume.

The large volume lowers the fundamental (string) scale through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}.$$

# Large-Volume Models



# The Weak Scale

The weak scale $M_W \sim 100$ GeV can be explained by supersymmetry.

- Supersymmetry broken at $1$TeV stabilises the Higgs mass against radiative corrections.
- Supersymmetry broken at $1$TeV generates the weak scale through dynamical electroweak symmetry breaking.

But what sets the scale of supersymmetry breaking?

# The Weak Scale

- In supergravity models, the scale of supersymmetry breaking is set by the gravitino mass, $m_{3/2}$:

$$m_{3/2} = (e^{K/2} W) M_P.$$

- $K = -2 \ln \mathcal{V}$, and so

$$m_{3/2} = \left( \frac{W}{\mathcal{V}} \right) M_P.$$

- An exponentially large volume generates an exponentially small gravitino mass.
- A volume $\mathcal{V} = 10^{15} l_s^6$ gives TeV-scale supersymmetry breaking.

# The Weak Scale

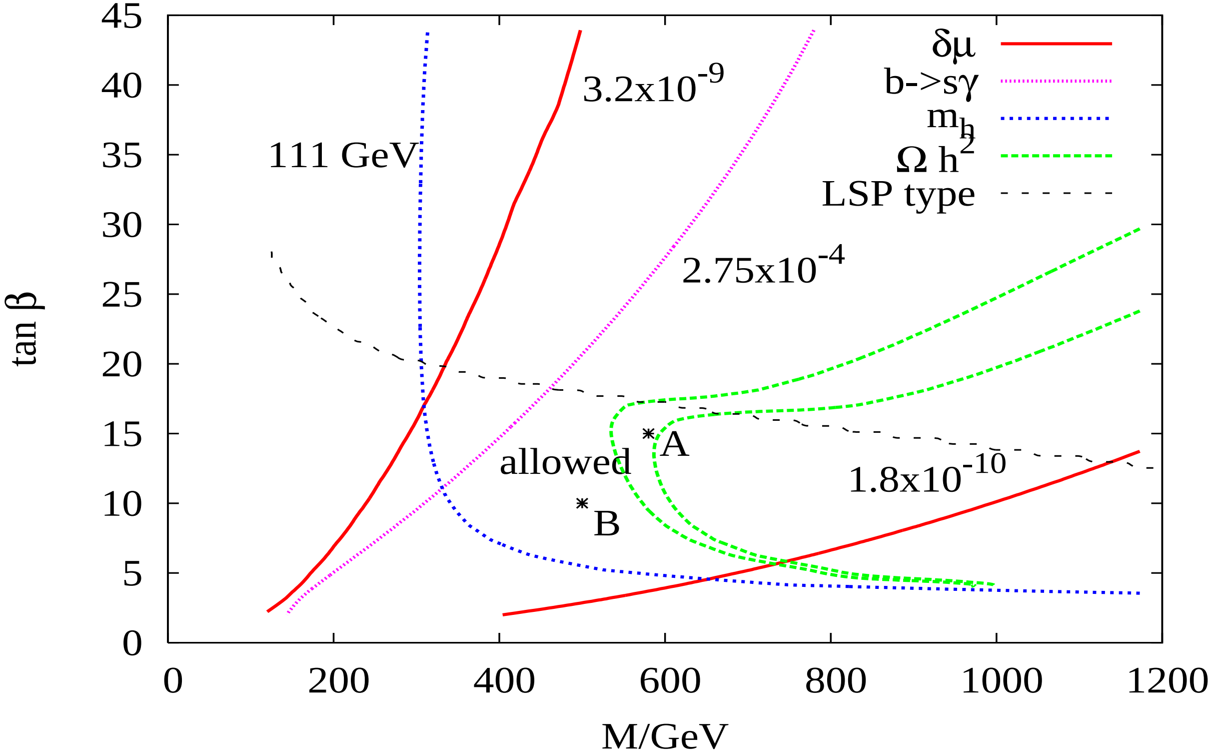
- Using the scalar potential, we can compute the soft terms that describe supersymmetry breaking.

  We get

$$
\begin{aligned}
M_i &= \frac{F^s}{2\tau_s} \equiv M, \\
m_{\alpha\bar{\beta}} &= \frac{M}{\sqrt{3}} \tilde{K}_{\alpha\bar{\beta}}, \\
A_{\alpha\beta\gamma} &= -M\,\hat{Y}_{\alpha\beta\gamma}, \\
B &= -\frac{4M}{3}.
\end{aligned}
$$

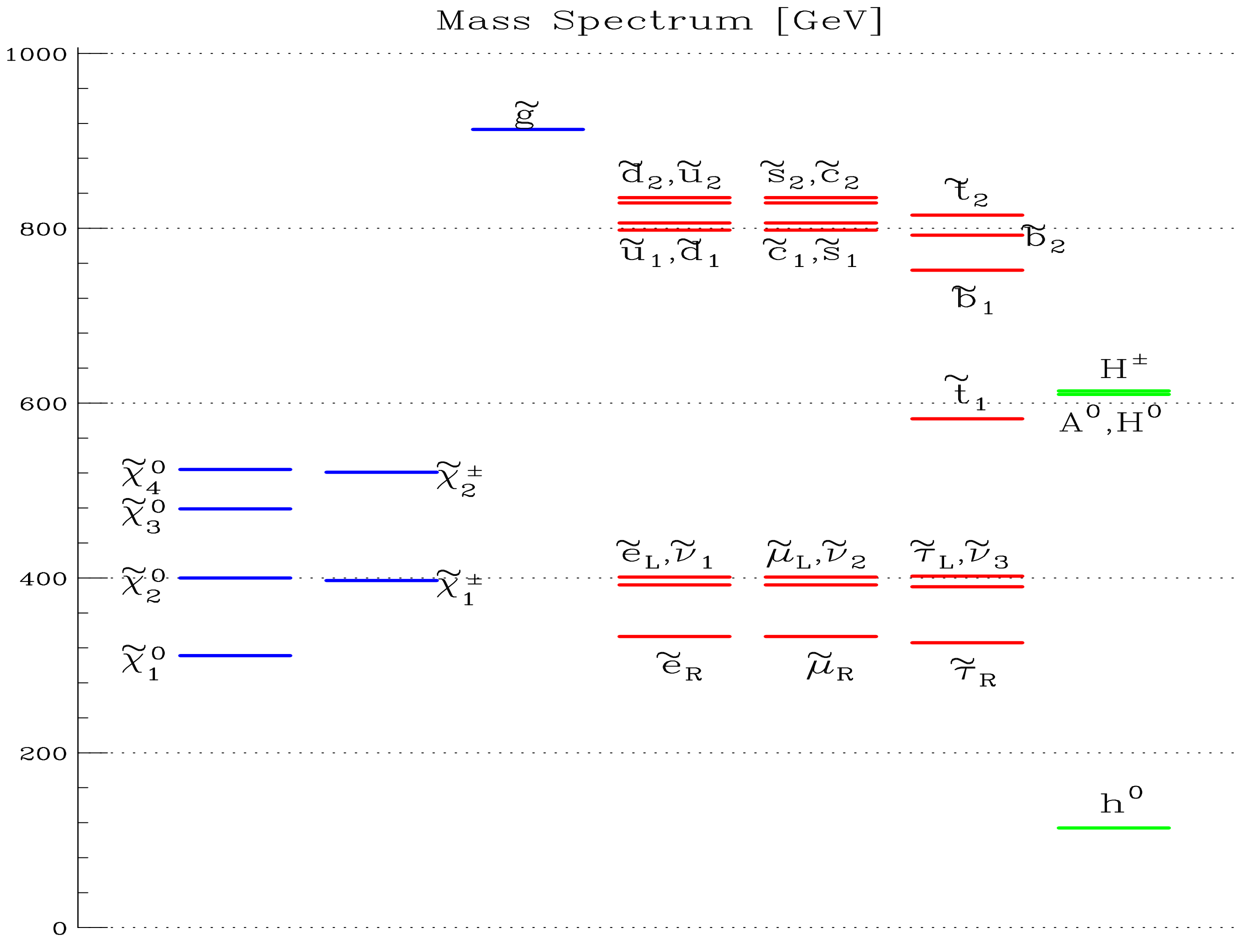
# The Weak Scale

- We run these soft terms to low energy using SoftSUSY.
- We scan over $M$ and $\tan\beta$ and impose constraints from $\Omega h^2$, $b \to s\gamma$, $m_H$, $g_\mu - 2$ and LSP type.



# The Weak Scale

A typical MSSM Spectrum (point B):



# Axions

- Axions are a well-motivated solution to the strong CP problem: why does the neutron have no electric dipole moment?
- Quantitatively, the QCD Lagrangian is

$$\mathcal{L}_{QCD} = \frac{1}{g^2} \int d^4x F^a_{\mu\nu} F^{a,\mu\nu} + \theta \int F^a \wedge F^a.$$

The strong CP problem is that naively

$$\theta \in (-\pi, \pi),$$

while experimentally

$$|\theta| \lesssim 10^{-10}.$$

# Axions

- The axionic solution promotes $\theta$ to a dynamical axion field, $\theta(x)$.
- The canonical Lagrangian for $\theta$ is

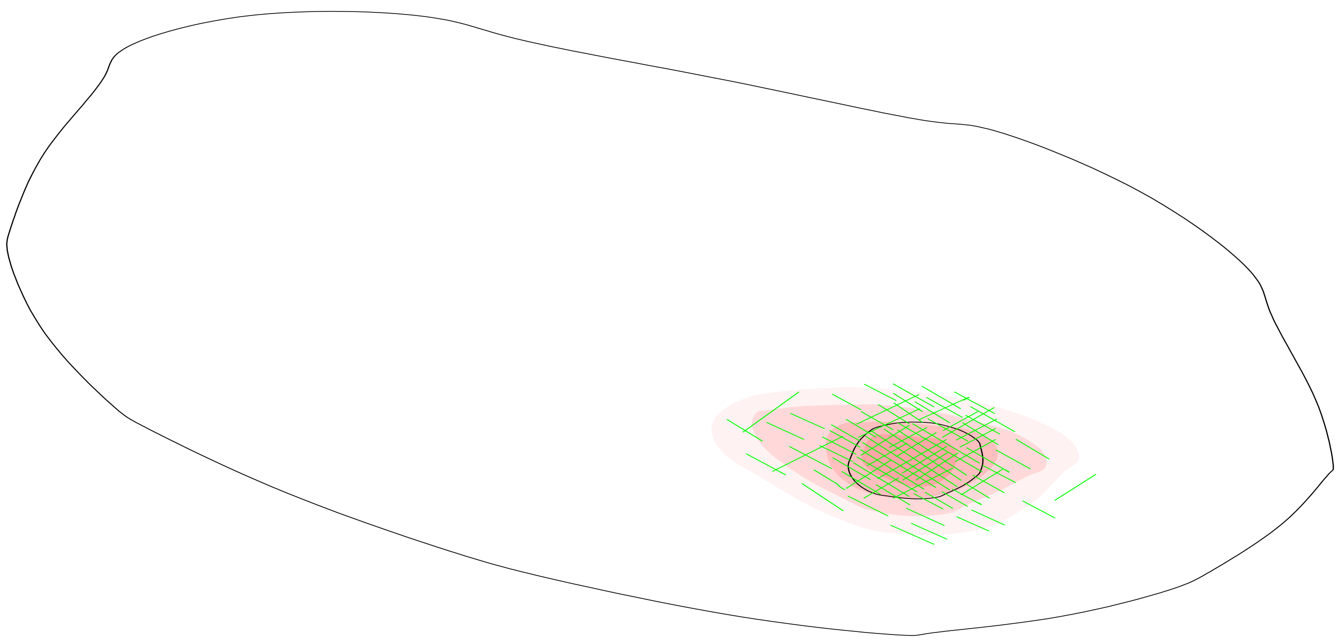
$$\mathcal{L} = \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{f_a} F^a \wedge F^a.$$

- Instanton effects generate a potential for $\theta$ with a minimum at zero.
- Bounds from stellar cooling and cosmological overproduction constrain the decay constant $f_a$.
- There exists an axion 'allowed window',

$$10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}.$$

# Axions

- In string theory, axion fields arise from dimensionally reducing higher dimensional p-form fields.
- In the large volume scenario, a QCD axion would come from the Ramond-Ramond 4-form reduced on the small cycle where the Standard Model lives.



# Axions

- Physically, the axion decay constant $f_a$ measures the suppression of the axionic coupling to matter.
- This coupling is a local coupling and thus only sees the fundamental (string) scale:

$$f_a \sim m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}.$$

(This is confirmed by a full analysis)

- A volume $\mathcal{V} = 10^{15} l_s^6$ generates an axionic scale,

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}\text{GeV}.$$

# Neutrino Masses

- Neutrino masses exist:

$$0.05\text{eV} \lesssim m_\nu^H \lesssim 0.3\text{eV}.$$

- In the seesaw mechanism, this corresponds to a Majorana mass scale for right-handed neutrinos

$$M_{\nu_R} \sim 3 \times 10^{14}\text{GeV}.$$

- Equivalently, this is the suppression scale $\Lambda$ of the dimension five Standard Model operator

$$\mathcal{O} = \frac{1}{\Lambda} HHLL.$$

# Neutrino Masses

- In supergravity, neutrino masses are generated by the superpotential operator

$$\mathcal{O}_W = \frac{\lambda}{M_P} H_2 H_2 L L \in W,$$

where $\lambda$ is dimensionless.

- This corresponds to a *physical* coupling

$$\mathcal{O}_{phys} = e^{\hat{K}/2} \frac{\lambda}{M_P} \frac{\langle H_2 H_2 \rangle L L}{(\tilde{K}_{H_2} \tilde{K}_{H_2} \tilde{K}_L \tilde{K}_L)^{\frac{1}{2}}}.$$

- This generates neutrino masses after electroweak symmetry breaking.

# Neutrino Masses

- In the large-volume scenario $\tilde{K}_\Phi \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}$, and

$$\mathcal{O}_{phys} = \frac{\lambda \mathcal{V}^{1/3}}{\tau_s^{2/3} M_P} \langle H_2 H_2 \rangle LL.$$

- Use $\mathcal{V} \sim 10^{15} l_s^6$:

$$\mathcal{O}_{phys} = \frac{\lambda}{10^{14}\text{GeV}} \langle H_2 H_2 \rangle LL,$$

giving

$$m_\nu = \lambda (0.3\,\text{eV}).$$

# Large Volumes are Power-ful

In large-volume models, an exponentially large volume naturally appears ($\mathcal{V} \sim e^{\frac{c}{g_s}}$) and generates hierarchies. Physical scales that appear are

- Supersymmetry: (fix) $m_{soft} \sim \frac{M_P}{\mathcal{V}} \sim 10^3$GeV
- Axions: $f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV
- Neutrinos/dim-5 operators: $\Lambda \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14}$GeV

All three scales are yoked in an attractive fashion.

The origin of all three hierarchies is the exponentially large volume.

# Moduli Dark Matter

- Large-volume models also predict a new scale.
- The volume modulus has a generic mass

$$M \sim \frac{m_{3/2}^{3/2}}{M_P^{\frac{1}{2}}} \sim \frac{M_P}{\mathcal{V}^{3/2}}.$$

- If we require $M_{susy} \sim 1$TeV, we need $m_{3/2} \sim 30$TeV, giving

$$M \sim 1\text{MeV}.$$

- Large-volume models predict the existence of a gravitationally coupled scalar at a scale $M \sim 1$MeV.

# Moduli Dark Matter

- Such a scalar is long-lived ($\tau \sim 10^{24} s$) and could form part of the dark matter.
- It can potentially be observed through its decays to $\gamma\gamma$ or $e^+e^-$.
- The former decay is suppressed but could generate a line in the cosmic photon background.
- The latter decay is of interest due to the unexplained 511 keV line from the galactic center.

# Summary

- Large-volume models can generate hierarchies through a stabilised exponentially large volume.
- If $m_s \sim 10^{11}$GeV, these can give the correct weak, axionic and neutrino mass scales.
- The different hierarchies come as different powers of the extra-dimensional volume.
- They also predict the existence of a gravitationally coupled scalar with mass $\sim 1$MeV.
- This may have cosmological consequences.