Massless 4D Gravitons from Asymptotically AdS_5 Spacetimes

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CPhT - Ecole Polytechnique

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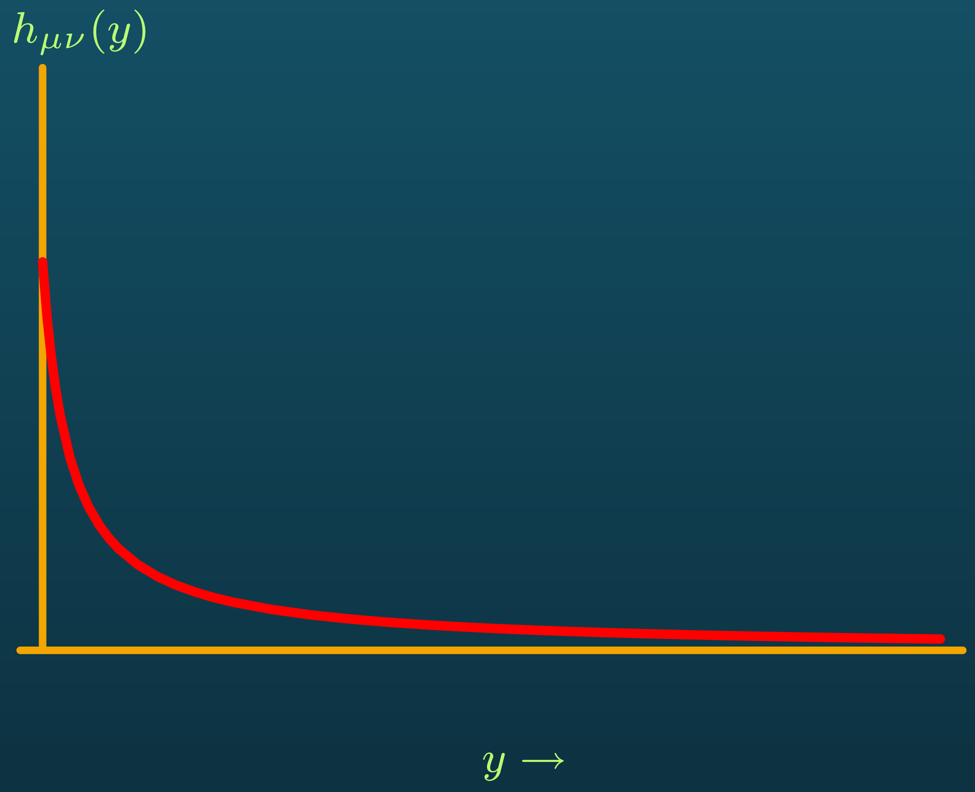
Based on hep-th/0611344 with E. Kiritsis

Prologue

RS Model:



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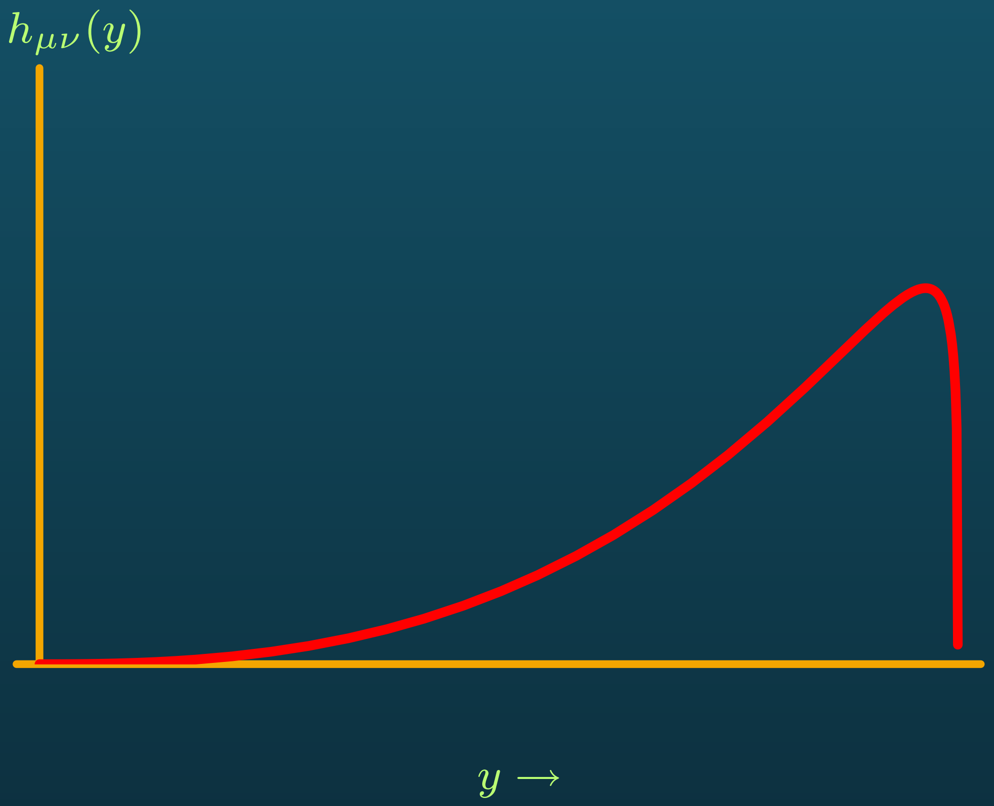

Motivation - Holographic Correspondence

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5D theory with gravity in Asymptotically AdS_5 spacetime (times some compact factor)

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- Excitations near the boundary of AdS_5 $\Leftrightarrow$ high energy modes in the FT.
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4D field theory without gravity.

- Excitations near the boundary of AdS_5 $\Leftrightarrow$ high energy modes in the FT.
- Radial Evolution away from the boundary $\Leftrightarrow$ RG flow to the IR
- Spectrum of 4D field theory particles = spectrum of normalizable fluctuations around the dual 5D geometry.

  Witten, ’97

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  $\Leftrightarrow$ graviton is a fundamental degree of freedom coupled to the 4D FT at the cut-off

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Can we have 4D graviton localized far from the boundary? (this would be emergent, rather than fundamental, in the dual FT).

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- RS-like Models

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- Einstein-Dilaton Models

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    - Linear Fluctuation Analysis
    - Hunting for Zero-Modes

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- Conclusion and Perspectives

4D Spectrum from 5D

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5D fluctuations such that:

- they have a fixed 4D mass: $\Box_4 \Phi(x,y) = m^2 \Phi(x,y)$
- are normalizable w.r.t. to the radial direction y, i.e. they have a finite 4D kinetic term.

correspond to 4D states with mass m^2 in the dual FT.

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We are interested in 4D-massless, y-normalizable fluctuations of the (tensor part of) the 5D metric component, $h_{\mu\nu}(x, y)$

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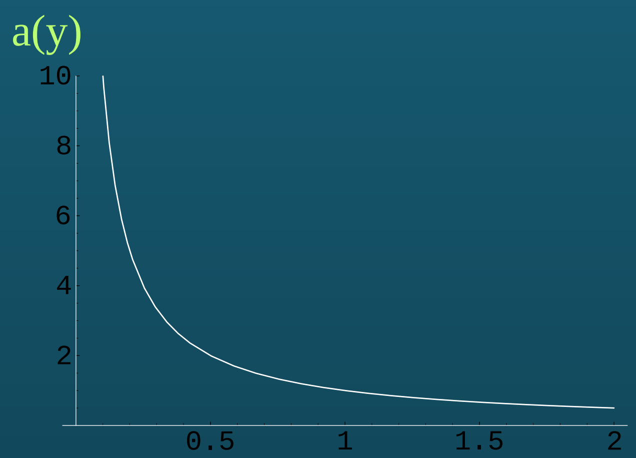
For tensor spin-2:

$$h_{\mu\nu}(x,y) \sim h^{(0)}_{\mu\nu}(x) + y^4 h^{(4)}_{\mu\nu}(x) + \ldots \qquad y \to 0$$

AdS_5 vs. RSII



AdS_5:

$$ds^2 = \frac{1}{(ky)^2} \left(dy^2 + dx_\mu^2\right) \quad 0 < y < \infty$$

$$S_{kin}[h^{(0)}] = \int_0 dy \; \frac{1}{(ky)^3} \int d^4x \left(\partial h^{(0)}\right)^2 = \infty$$

- In AdS_5 $h_{\mu\nu}^{(0)}$ is not normalizable $\Rightarrow$ not a state in the 4D FT, rather an external source added to the UV theory.

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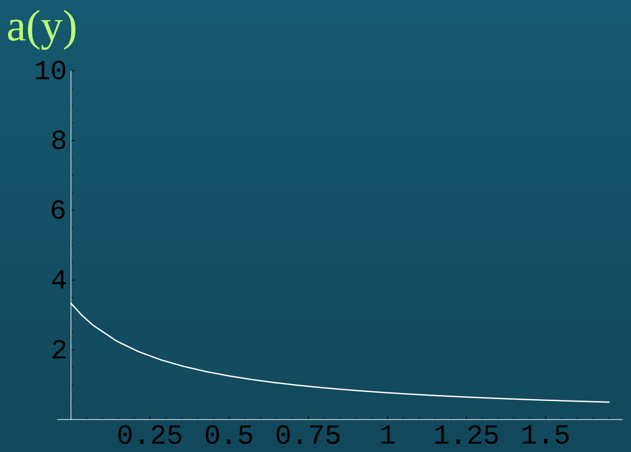

Slice of AdS_5:

$$ds^2 = \frac{1}{(1+ky)^2}\left(dy^2 + dx_\mu^2\right) \qquad 0 < y < \infty$$

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- In RSII it becomes normalizable $\Rightarrow$ the source gets a kinetic term and becomes dynamical. it is promoted to a fundamental d.o.f of the UV theory.

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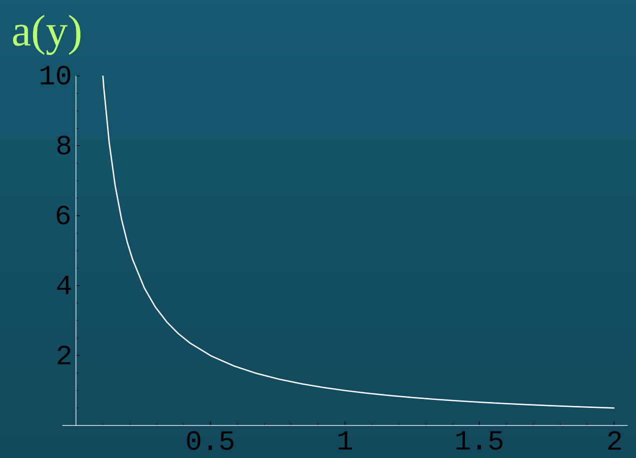

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- A normalizable $h^{(4)}_{\mu\nu}(x)$ would correspond to a low-energy excitation ("glueball"), since it has suppport in the IR (large y) region.

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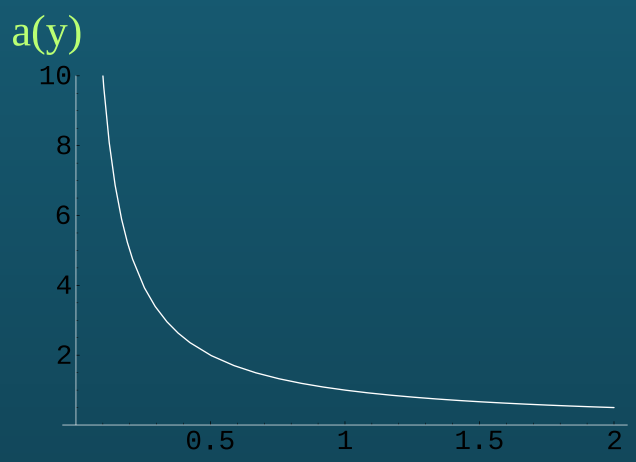

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- A normalizable $h_{\mu\nu}^{(4)}(x)$ would correspond to a low-energy excitation ("glueball"), since it has suppport in the IR (large y) region. Can we realize a set-up in which the only normalizable massless tensor mode is $h_{\mu\nu}^{(4)}(x)$?

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- theory does not "run": it is conformal all the way to the boundary
- does 5D massive gravity have a holographic interpretation anyway?

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Take solution asymptotically AdS_5 in the UV $(y \to 0)$:

$$a(y) \sim \frac{1}{ky}; \quad \Phi_0(y) \sim const; \quad V(\Phi_0(y)) \sim 2\Lambda$$

Spin-2 Fluctuations

$$ds^2 = a^2(y)\left[dy^2 + (\eta_{\mu\nu} + h_{\mu\nu}(x,y))\,dx^\mu dx^\nu\right] \qquad h^\mu_\mu = \partial^\mu h_{\mu\nu} = 0$$

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Look for normalizable solution with given 4D mass m:

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Get equation for the profile $h(y)$:

$$h''(y) + 3\frac{a'}{a}h'(y) + m^2 h(y) = 0$$

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What kinds of $B(y)$ provide a zero-energy state?

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Spin-2 Zero-Modes

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$$\Rightarrow \qquad \psi^{UV}(y) \sim y^{-3/2}, \qquad \psi^{IR}(y) \sim y^{5/2}.$$

ψ^{IR} is normalizable, $\psi^{UV}(y)$ is not.

(Notice: both are normalizable in RS, where $y > 1/\Lambda$)

IR Asymptotics

We have one candidate Zero-Mode, normalizable around $y = 0$:

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Two distinct cases:

- y range extends to $+\infty \Rightarrow \Psi_{IR} \to \infty$ as $y \to \infty$,
  not normalizable
- spacetime ends at $y = y_0$ (singularity)

Singular Case

We have one candidate Zero-Mode:

$$\Psi^{IR}(y) = e^{-B(y)} \int_0^y dy' \, e^{2B(y')}$$

example with a singularity:

$$B(y) \sim -\alpha \log(y_0 - y) \qquad a(y) \sim (y_0 - y)^{2\alpha/3} \qquad y \sim y_0$$

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Massless Spectrum

	$y \in (0, y_0)$		
$B(y_0)$	$-\alpha\, log(y_0 - y)$		
	$0 < \alpha < 1/2$	$1/2 < \alpha < 1$	$1 < \alpha < 3/2$
Spin 2	○	○	○
Spin 1	○	—	—
Spin 0	—	—	—

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- look for light, long lived, spin-2 resonance.

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- We found cases with no other scalar or vector massless degrees of freedom. This is an advantage over previous attempts.
- Our analysis indicates how one can relax the requirement of an exactly massless, strictly 4D state, to try to overcome the problems with the singularity and/or the boundary conditions in the IR.

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Concrete Example:

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$\Rightarrow$ this is the boundary conditions we need to impose on the fluctuations to keep zero-mode is in the spectrum

Logarithmic $B(y)$ - 4D Planck Scale

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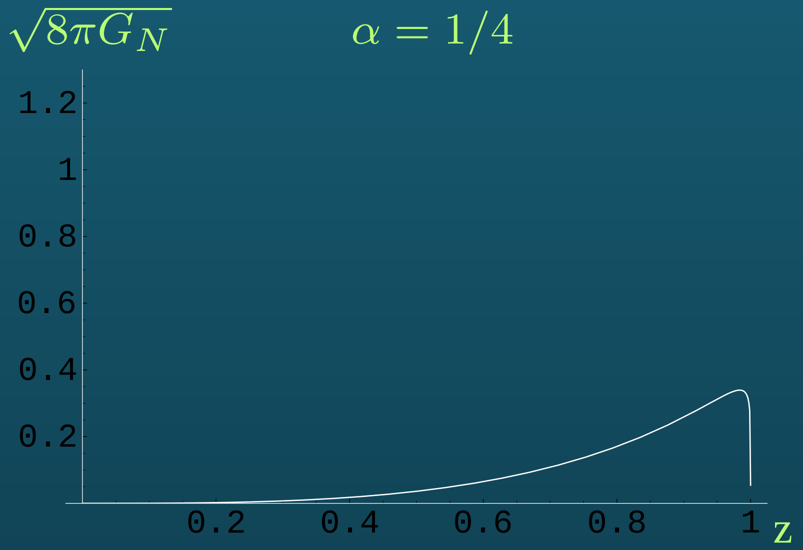
Logarithmic $B(y)$ - 4D Planck Scale II



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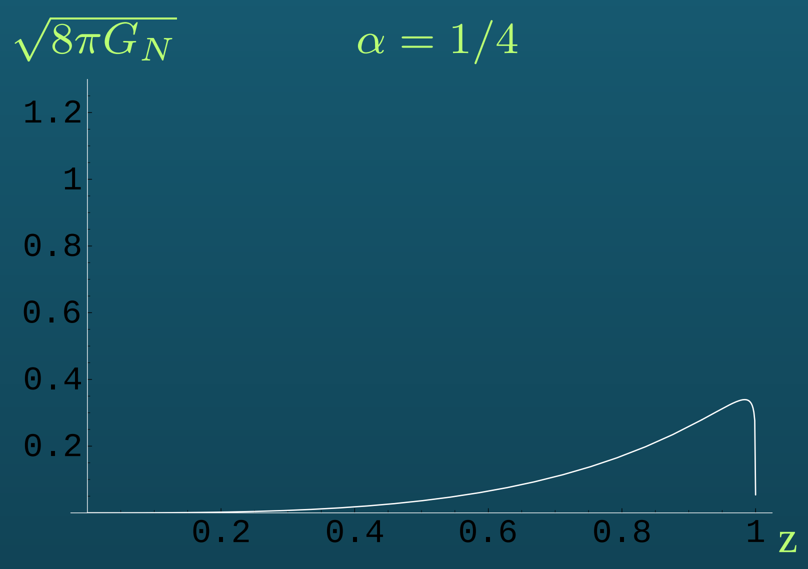

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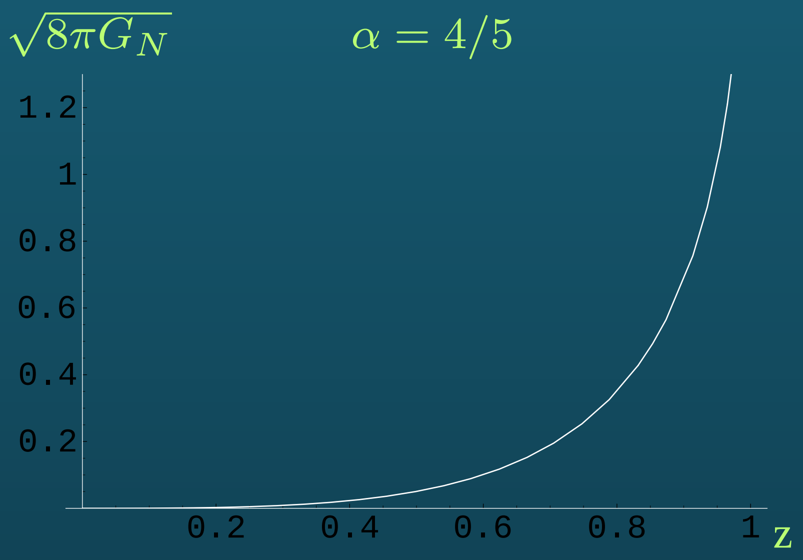

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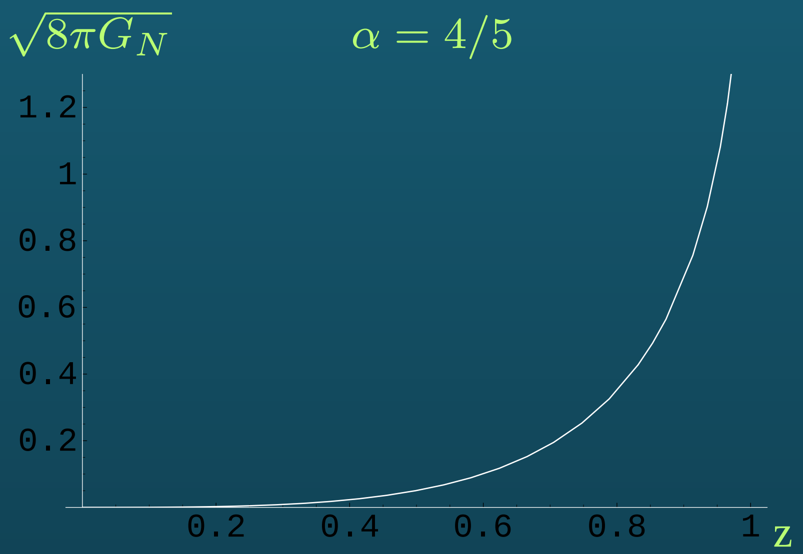

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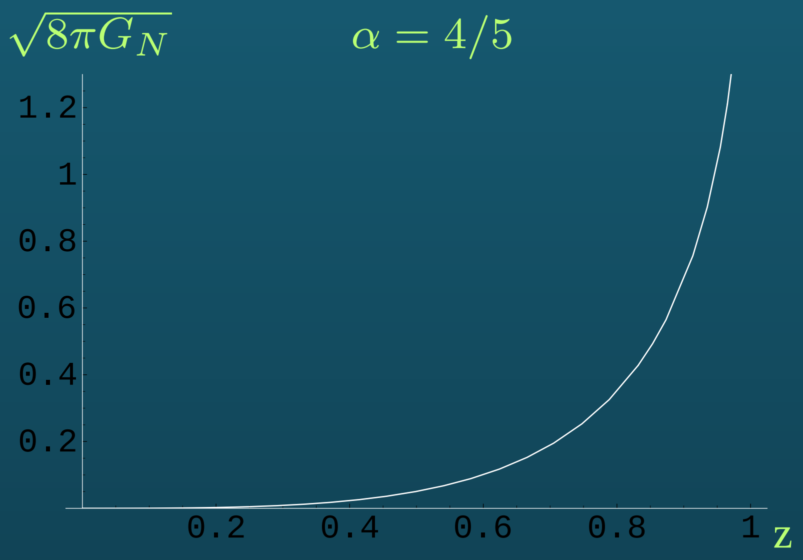

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