Warsaw, 4-7 February '09

Models of Neutrino Masses and Mixings

1

G. Altarelli
Universita' di Roma Tre/CERN

In the last decade data on ν oscillations have added some (badly needed) fresh experimental input to particle physics

ν masses are not all vanishing but they are very small

This suggests that ν's are Majorana particles and L is not conserved

ν mixing angles follow a different pattern from quark mixings

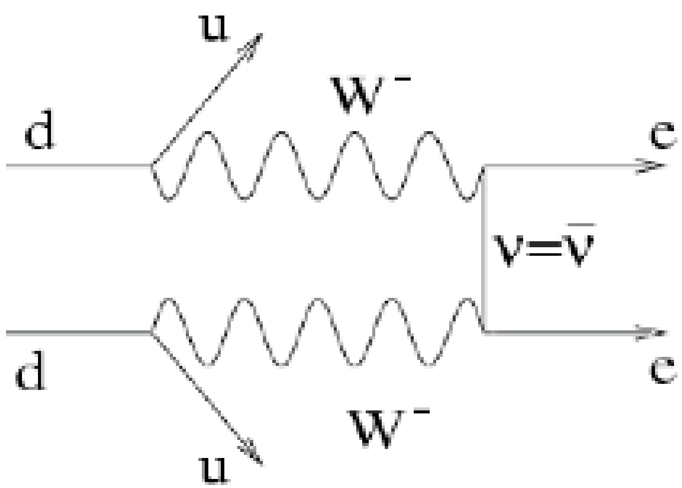
For ν masses and mixings we do not have so far a "Standard Model": many possibilities are still open.

In fact, this is also the case for quarks and charged leptons: we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

Thus ν's are interesting because they can provide new clues on this important problem

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

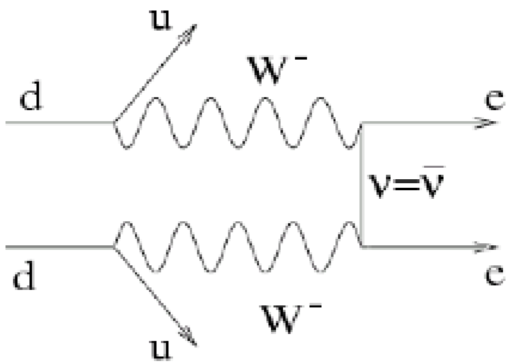
Detection of $0\nu\beta\beta$ would be a proof of L non conservation. Thus a big effort is devoted to improving present limits and possibly to find a signal.



Heidelberg-Moscow
IGEX
Cuoricino
Nemo
Sokotvina
DAMA
•••••

$0\nu\beta\beta$ = dd -> uue$^-$e$^-$

$0\nu\beta\beta$ experiments



$$\langle m_\nu \rangle^2 = \frac{1}{G(Q,Z)\,|M_{nucl}|^2\,\tau}$$

phase space

matrix elmnt
large uncrtnts

Pavan

Experiment	Isotope	$\tau_{1/2}^{0\nu}$ > [y]	range $\langle m_\nu \rangle$ [eV]
Heidelberg Moscow 2001	^{76}Ge	$1.9\ 10^{25}$	0.3-2.5
IGEX 2002	^{76}Ge	$1.57\ 10^{25}$	0.3-2.5
Cuoricino 2005	^{130}Te	$2\ 10^{24}$	0.3-0.7
NEMO 2005	^{100}Mo	$4.6\ 10^{23}$	0.6-1.0

claimed evidence only by a part of the collaboration

started in 2003

$$m_{ee} = \langle m_\nu \rangle = \left|\sum U_{ej}^2\, m_j\, e^{i\alpha_j}\right|$$

Future: a factor ~ 10 improvement in next decade

$0\nu\beta\beta$ Decay Measurements

Survey of some past and present experiments

upper limit

isotope	experiment	latest result	$Q_{\beta\beta}$ [keV]	i.a. nat.	i.a. enrich.	exposure [kg×y]	technique	material	$\tau_{1/2}^{0\nu}$ [10^{23} y]	$\langle m_\nu \rangle$ [eV]
^{48}Ca	Elegant VI	2004[11]	4271	0.19	–	4.2	scintillator	CaF_2	0.14	7.2÷44.70
^{76}Ge	Heidelberg/Moscow	2004[17]	2039	7.8	87	71.7	ionization	Ge	120.0	0.44
^{82}Se	NEMO-3	2007[22]	2995	9.2	97	1.8	tracking	Se	1.2	1.60÷4.50
^{100}Mo	NEMO-3	2007[22]	3034	9.6	95÷99	13.1	tracking	Mo	5.8	0.60÷2.40
^{116}Cd	Solotvina	2003[12]	2805	7.5	83	0.5	scintillator	$CdWO_4$	1.7	1.70
^{130}Te	Cuoricino	2007[20]	2529	33.8	–	11.8	bolometer	TeO_2	30.0	0.16÷0.84
^{136}Xe	DAMA	2002[23]	2476	8.9	69	4.5	scintillator	Xe	12.0	1.10÷2.90
^{150}Nd	Irvine TPC	1997[14]	3367	5.6	91	0.01	tracking	Nd_2O_3	0.012	3.00
^{160}Gd	Solotvina	2001[13]	1791	21.8	–	1.0	scintillator	Gd_2SiO_5	0.013	26.00

A. Nucciotti arXiv:0707.2216 [nucl-ex]

$$0.16 < m_{\beta\beta}/\mathrm{eV} < 0.52 \qquad \text{(HM claim)}\ ,$$
$$0 \le m_{\beta\beta}/\mathrm{eV} < 0.23 \qquad \text{(Cuoricino, “favorable” NME)}\ ,$$
$$0 \le m_{\beta\beta}/\mathrm{eV} < 0.85 \qquad \text{(Cuoricino, “unfavorable” NME)}$$

Arnaboldi et al

The Heidelberg-Moscow claim not disproved by Cuoricino depending on nuclear matrix elements

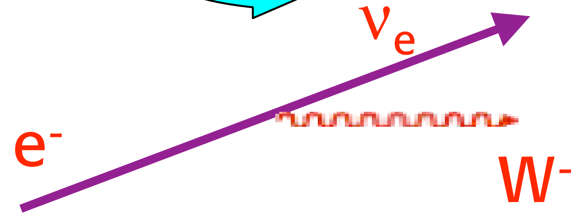
ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-

flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$$
$$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$$\nu_a(x) = e^{ip_a x}\,\nu_a \qquad p_a^2 = E^2 - m_a^2$$

$$P(\nu_e \text{<->} \nu_\mu) = |\langle \nu_\mu(L)|\,\nu_e\rangle|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Solid evidence for solar and atmosph. ν oscillations

Δm^2 values fixed:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}$ eV^2,

$\Delta m^2_{sol} \sim 8\ 10^{-5}$ eV^2

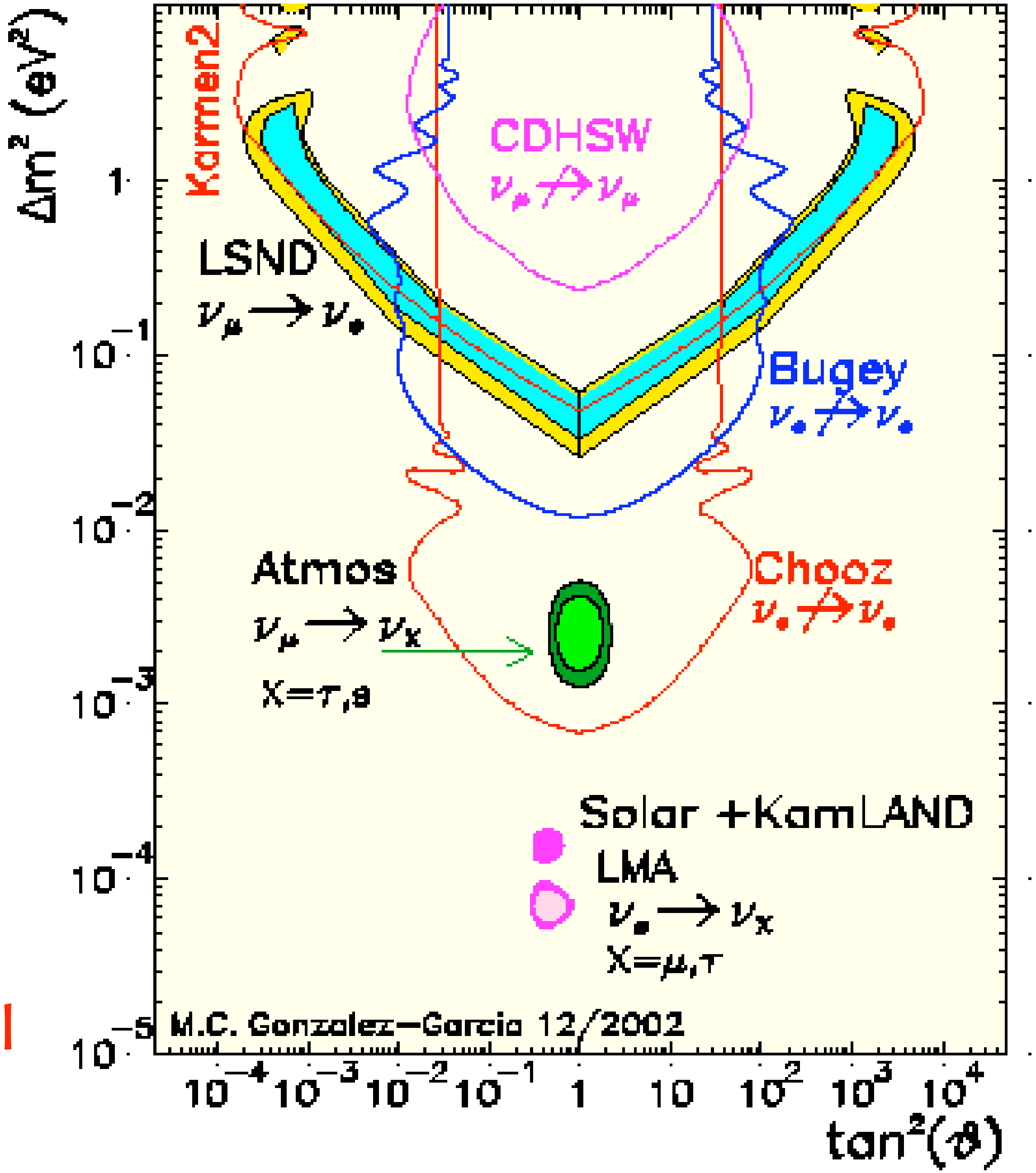
Miniboone has not confirmed LSND

mixing angles:

θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

θ_{13} (CHOOZ) small



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

End-point tritium β decay (Mainz, Troitsk)

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94\text{eV}$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.17\text{-}0.68\text{-}2.1$ eV (dep. on data&priors)

WMAP, SDSS, 2dFGRS, Ly-α

Any ν mass $< 0.06\text{-}0.23\text{-}0.7$eV

By itself CMB is only mildly sensitive to $\Sigma = \Sigma_i m_i$

Only in combination with LSS the limit becomes stronger.

And even stronger by adding the Lyman alpha forest data (but some tension among the data).

Fogli et al '08

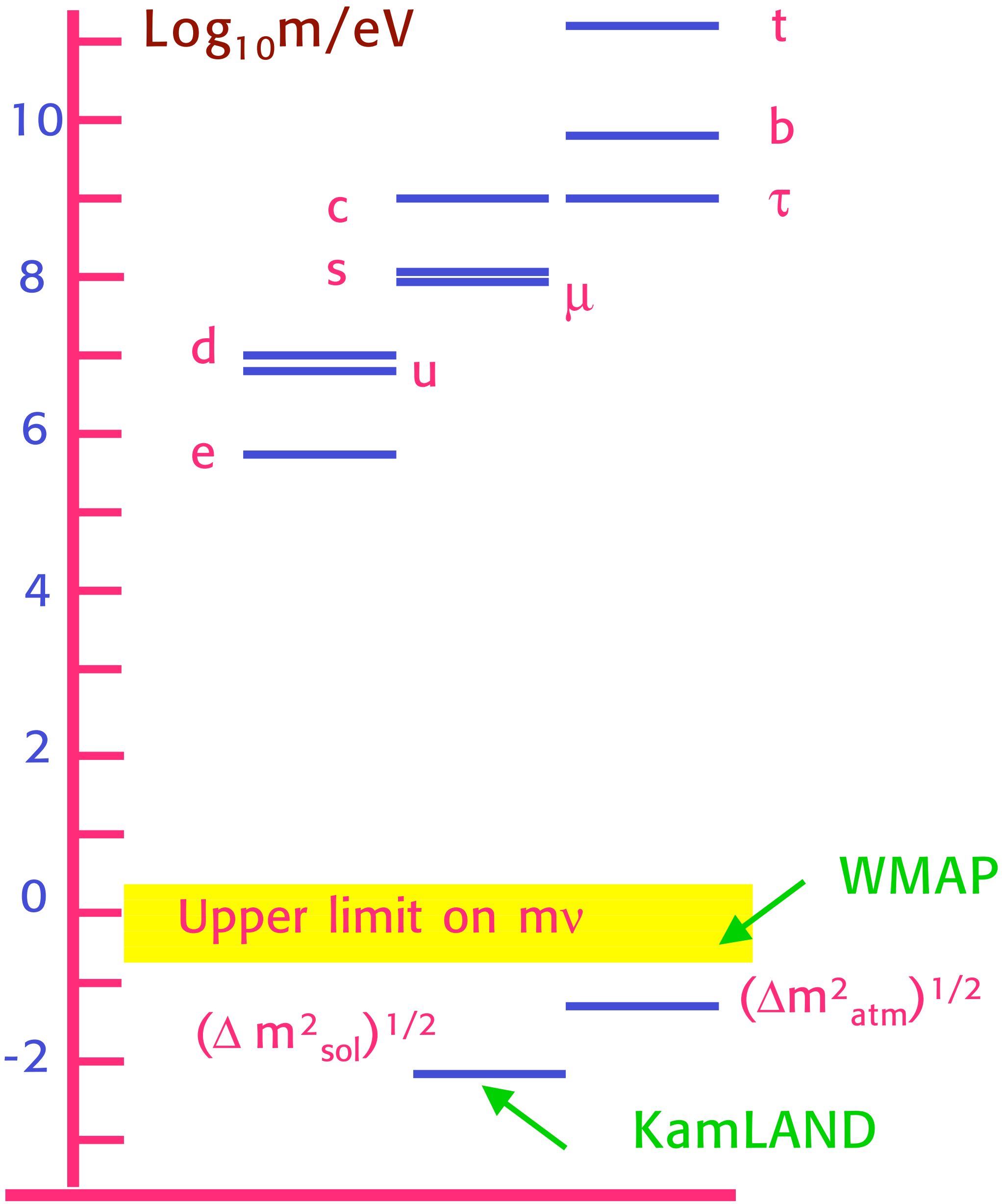
Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

CMB Cosmic Microwave Background: WMAP+ ACBAR+......

LSS Large Scale Structure (2dFGRS, SDSS)

HST +SN-Ia Hubble Space Tel. [h=0.72(7)]+ SuperNovae

BAO Baryonic Acoustic Oscillation (SDSS)



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

See-Saw Mechanism

Minkowski;
Yanagida; Gell-Mann, Ramond , Slansky;
Glashow; Mohapatra, Senjanovic…..

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\overline{\nu}_L\nu_R$ Dirac mass m from Higgs doublet(s)

$$\begin{array}{cc} & \begin{array}{cc}\nu_L & \nu_R\end{array} \\ \begin{array}{c}\nu_L \\ \nu_R\end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix}\end{array}$$

$M >> m_D$

Eigenvalues

$$|\nu_{light}| = \frac{m_D{}^2}{M} \quad , \quad \nu_{heavy} = M$$

In general ν mass terms are:

$$L_\nu = \bar{\nu}_L y \nu_R H + h.c. + \nu_R{}^T M_R \nu_R + \nu_L{}^T \frac{\lambda}{M_L} \nu_L HH$$

Dirac
$m_D = yv$
$v = \langle 0|H|0\rangle$

Majorana

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{bmatrix} \lambda v^2/M_L & m_D \\ m_D & M_R \end{bmatrix} \end{array}$$

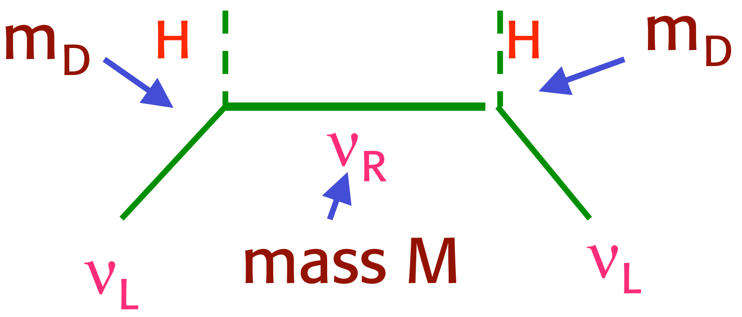
$m_{light} \sim \dfrac{m_D{}^2}{M_R}$ and/or $\dfrac{\lambda v^2}{M_L}$

$m_{heavy} \sim M_R$

$$m_{eff} = \nu^T{}_L m_{light} \nu_L$$

Neutrinos are (probably) Majorana particles: $\nu_L^T m_\nu \nu_L$

See-saw



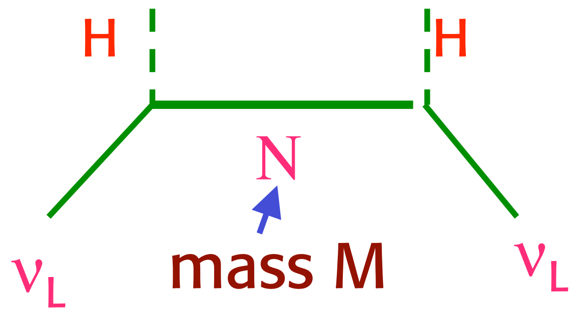
$m_\nu = m_D^T M^{-1} m_D$

connection with m_D

More in general: non ren. O_5 operator

$$O_5 = \ell^T \frac{\lambda}{M_L} \ell H H$$

e.g from



N: new particle $I_W=0,1$

Whatever the underlying dynamics O_5 is a more general effective description of light Majorana neutrino masses

ν oscillations point to very large values of M

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

Neutrinos favour SO(10) over SU(5)

ν_R completes the 16 of SO(10)

$$16_{SO(10)} = (10 + \bar{5} + 1)_{SU(5)}$$

The Majorana term $M\nu_R{}^T\nu_R$ is SU(5) but not SO(10) invariant.

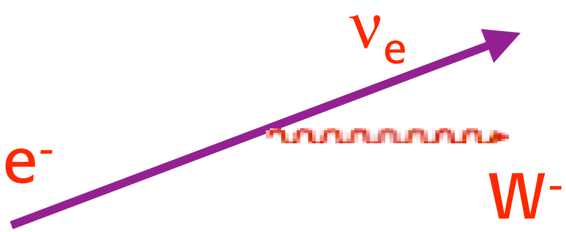
From the values of ν masses $M \sim M_{GUT}$

M could be larger than the scale where SU(5) is broken, while, in SO(10), M should be of order of the scale where B-L is broken [SO(10) contains B-L]

3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\,s_{23} \\ \dots & \dots & c_{13}\,c_{23} \end{bmatrix}$$

CHOOZ: $|s_{13}| <\sim 0.2$

atm.: ~ max

(some signs are conventional)

In general: $U = U^+_e U_\nu$

$$m_\nu \sim U^* \begin{bmatrix} e^{i\alpha_1} m_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

In general 9 parameters:
3 masses, 3 angles,
3 phases

$L^T m_\nu L$

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2) cs/\sqrt{2} & (m_1 - m_2) cs/\sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 & (m_1 s^2 + m_2 c^2 - m_3)/2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 \end{bmatrix}$$

Note:

- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e <-> \nu_\mu) = P(\nu_e <-> \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu <-> \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \qquad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

Defining:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} - \frac{2}{3}\Delta m^2_{sol}$$

and

$\overline{m^2} >> \left|\Delta m^2_{atm}\right| > \Delta m^2_{sol}$ degenerate

$\Delta m^2_{atm} < 0$ inverse hierarchy

$\Delta m^2_{atm} > 0$ normal hierarchy

Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

Schwetz et al '08

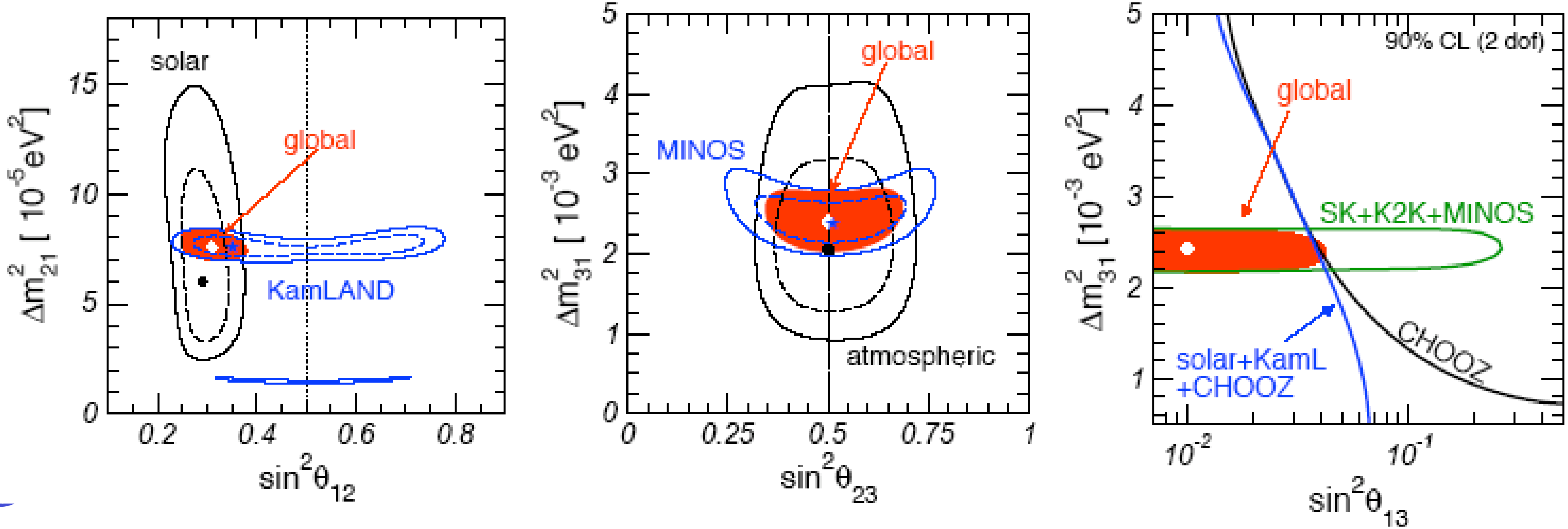
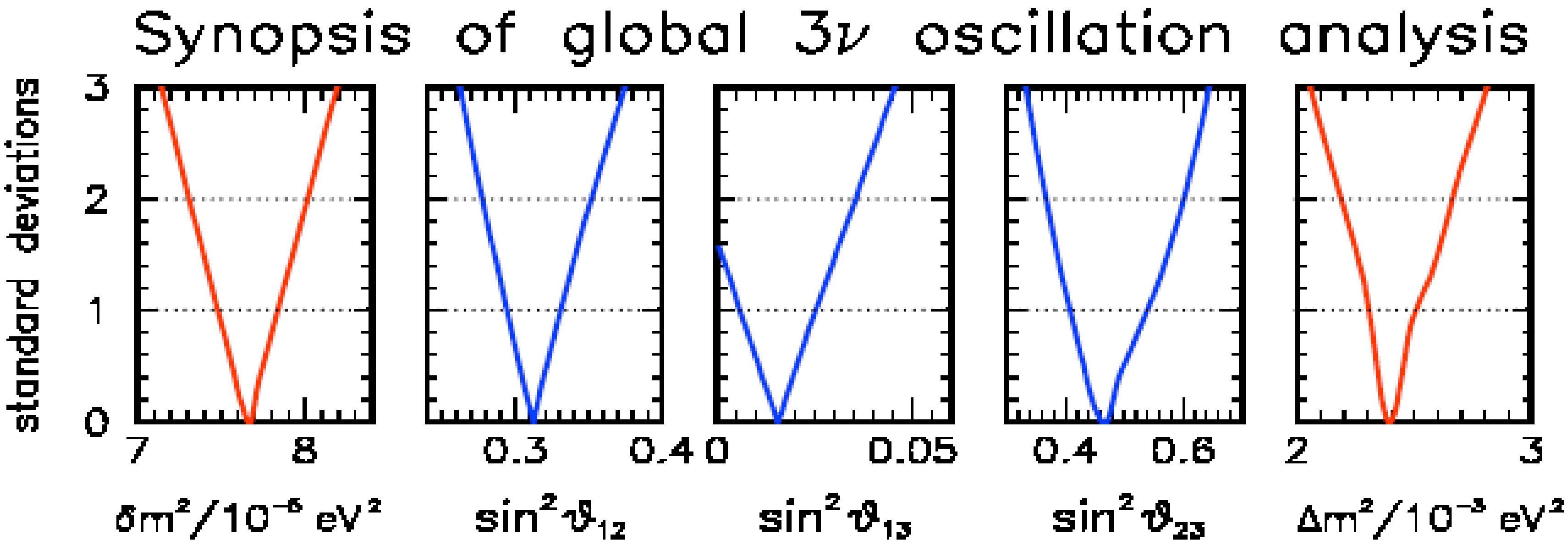


Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

Fogli et al '08

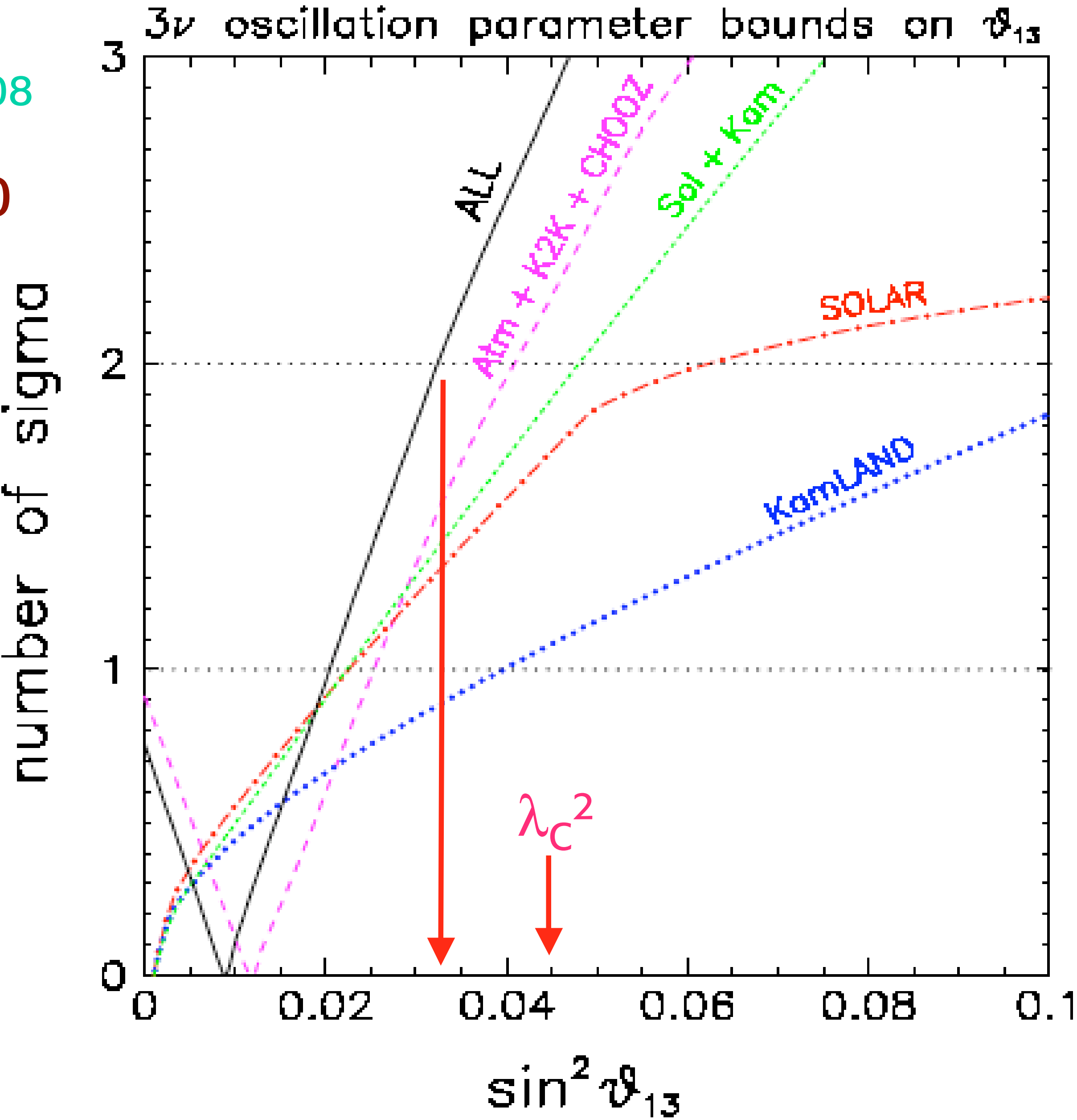


θ_{13} bounds

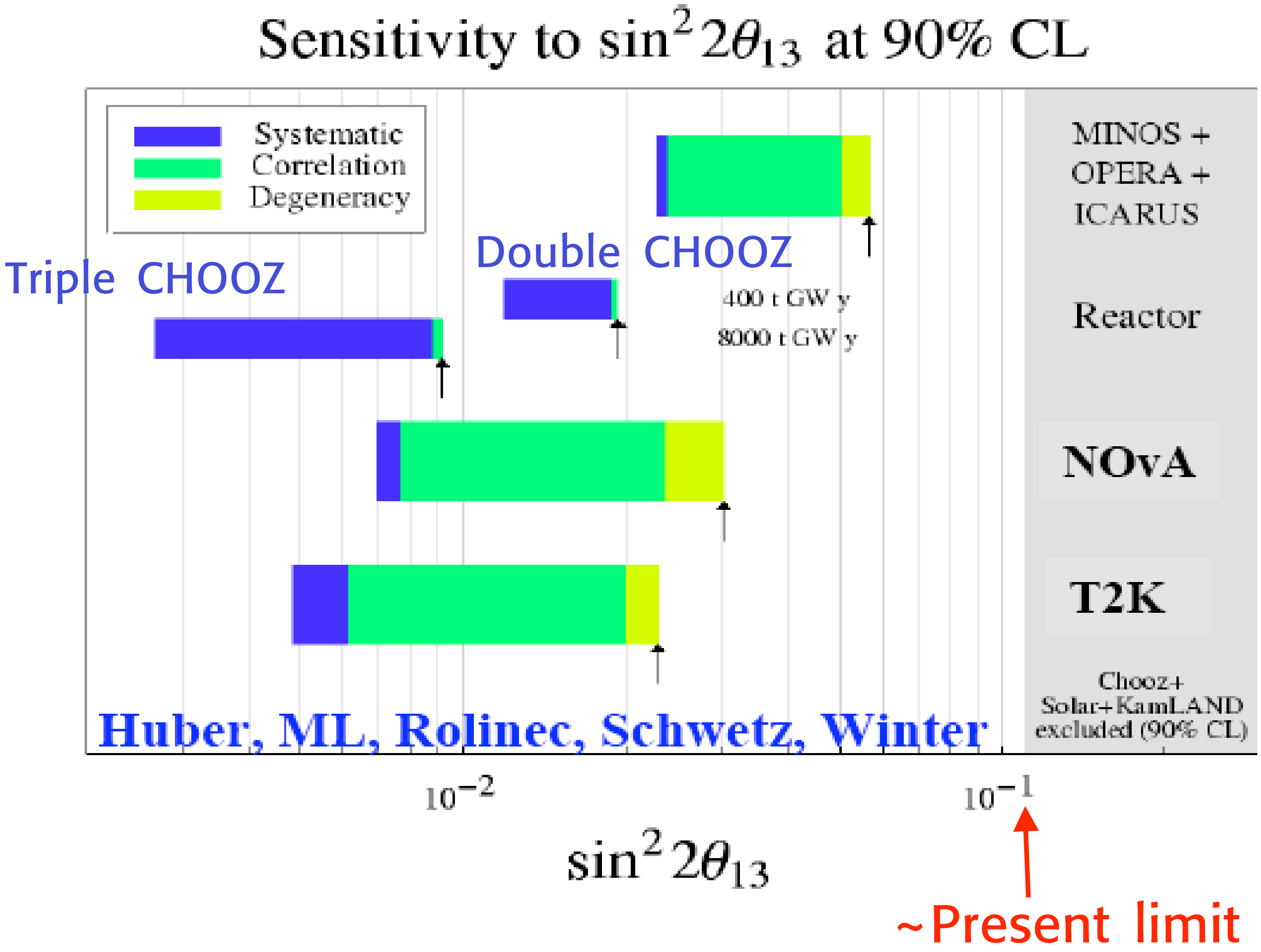
Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)

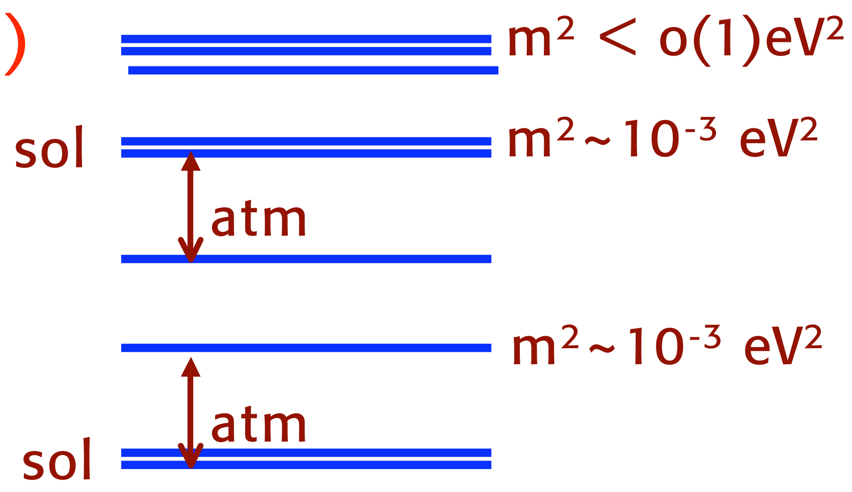


The current experimental situation on ν masses and mixings has much improved but is still incomplete

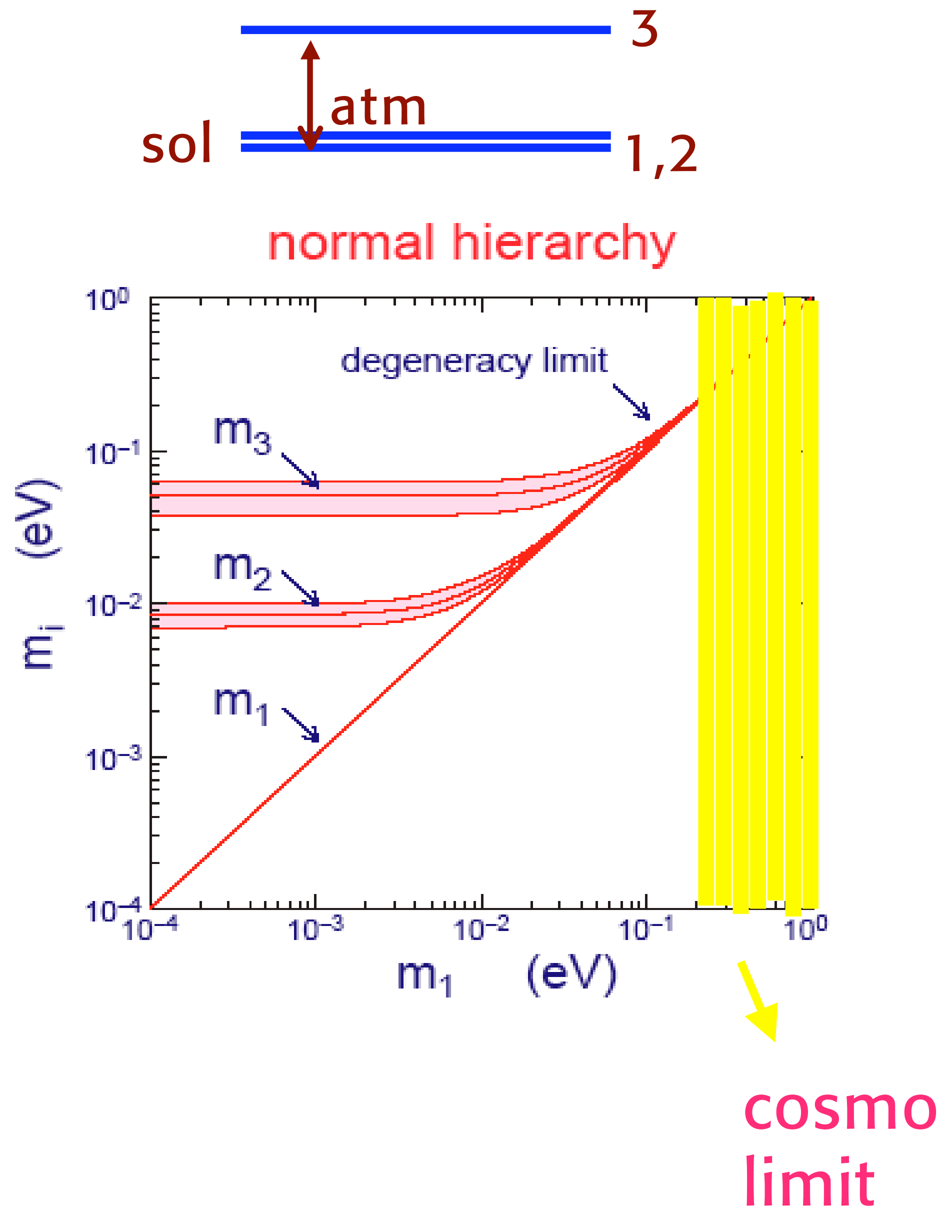
- what is the absolute scale of ν masses?
- value of θ_{13}......
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana)
- pattern of spectrum

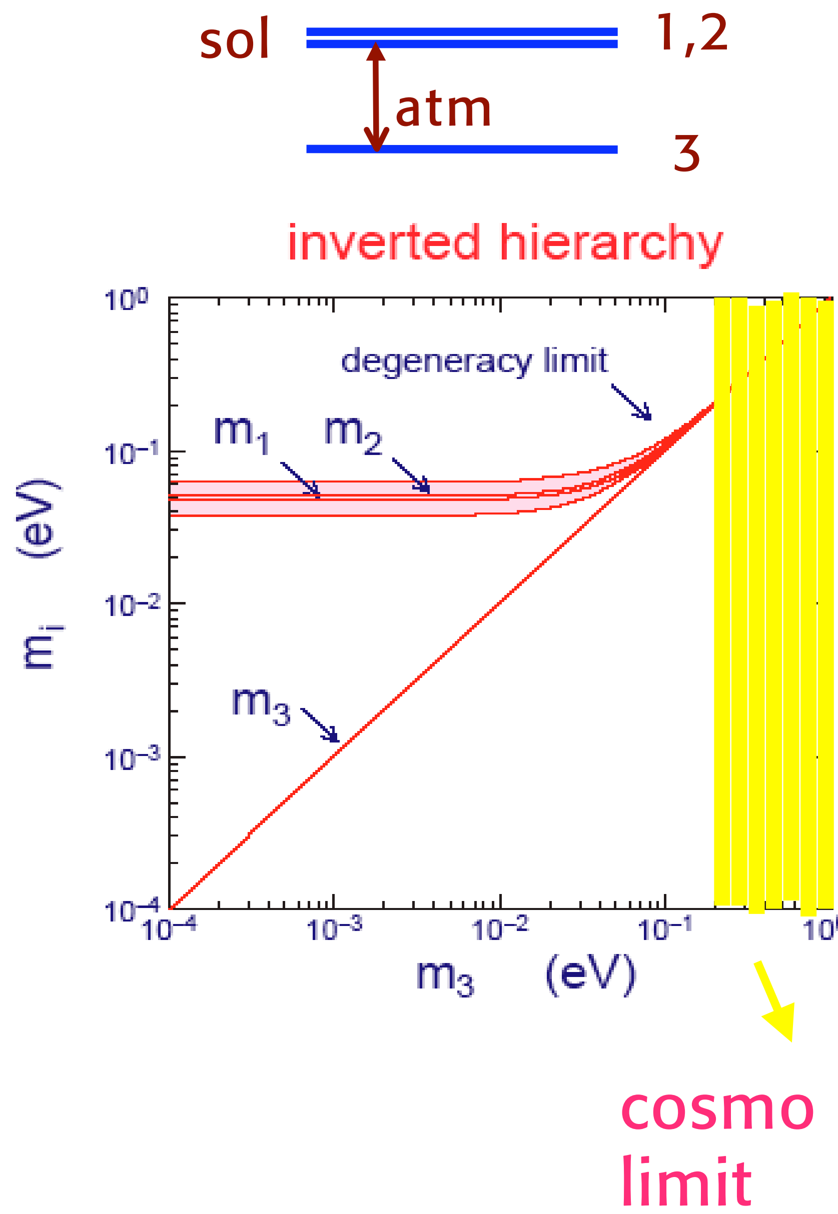
3 light ν's are OK (MiniBoone)

- Degenerate ($m^2 \gg \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



→ Different classes of models are still possible





Only moderate degeneracy allowed

0νββ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}| = c_{13}^2\, [m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

Degenerate: $\sim |m||c_{12}^2 + e^{i\alpha} s_{12}^2| \sim |m|(0.3\text{-}1)$

$|m_{ee}| \sim |m|\ (0.3\ \text{-}1) \leq 0.23\text{-}1\ \text{eV}$

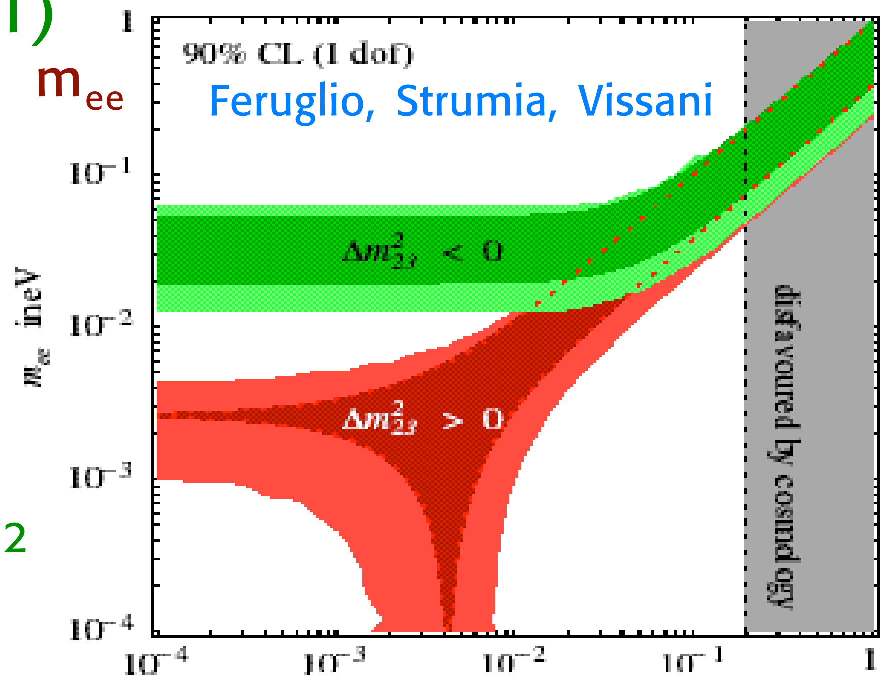
IH: $\sim (\Delta m^2_{atm})^{1/2} |c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim (1.6\text{-}5)\ 10^{-2}\ \text{eV}$

NH: $\sim (\Delta m^2_{sol})^{1/2} s_{12}^2 + (\Delta m^2_{atm})^{1/2} e^{i\beta} s_{13}^2$

$|m_{ee}| \sim (\text{few})\ 10^{-3}\ \text{eV}$

Full dependence on min m_ν



Present exp. limit: $m_{ee} < 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Baryogenesis

$n_B/n_\gamma \sim 10^{-10}$, $n_B >> n_{Bbar}$

Conditions for baryogenesis: (Sacharov '67)

- B non conservation (obvious)
- C, CP non conserv'n (B-B^{bar} odd under C, CP)
- No thermal equilib'm ($n = \exp[\mu - E/kT]$; $\mu_B = \mu_{Bbar}$, $m_B = m_{Bbar}$ by CPT

If several phases of BG exist at different scales the asymm. created by one out-of-equilib'm phase could be erased in later equilib'm phases: BG at lowest scale best

Possible epochs and mechanisms for BG:

- At the weak scale in the SM Excluded
- At the weak scale in the MSSM Disfavoured
- Near the GUT scale via Leptogenesis Very attractive

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al ….

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

I now review some ideas on model building

Old models are more generic and qualitative than present models

Anarchy
Semianarchy
Lopsided models
$U(1)_{FN}$
••••••

With better data the range for each mixing angle has narrowed and models have become more quantitative

e.g Tribimaximal mixing, A4, S4

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

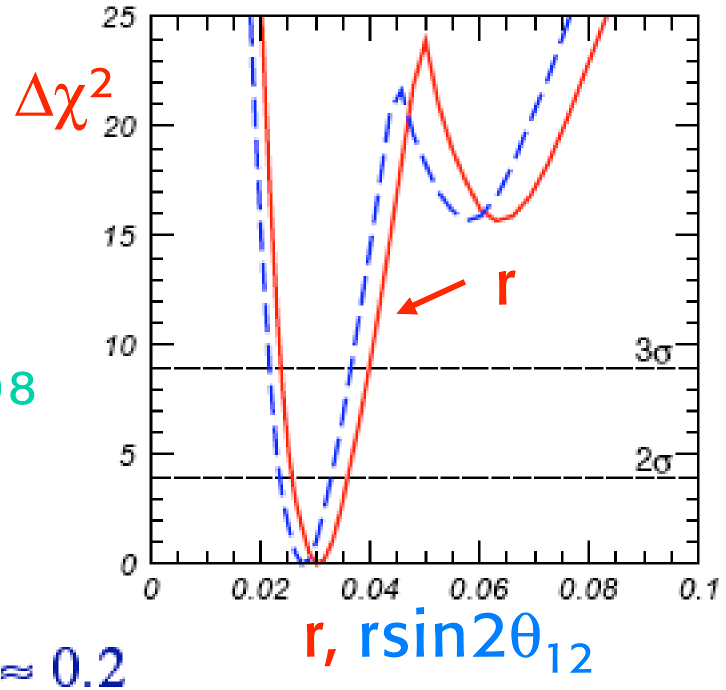
Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08

or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV



For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) $\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard

Naively large mixing --> nearly degenerate masses

$$m_i^2 >> \Delta m^2_{ij}$$

Degenerate models are less favoured by now because of:

- No clear physical motivation: after all quark and charged lepton masses are very non degenerate

- Upper bounds on m^2 that limit $m^2/\Delta m^2_{atm}$
At present, no significant amount of hot dark matter is indicated by cosmology
Only a moderate degeneracy is allowed

- Disfavoured by see-saw

- Possible renormalization group instability

It is difficult to marry degenerate models with see-saw

$$m_\nu \sim m_D^T M^{-1} m_D$$

(needs all degenerate or a sort of conspiracy between M and m_D)

So most degenerate models deny all relation to m_D and directly work with effective operators

$$O_5 = \ell^T \frac{\lambda}{M_L} \ell H H$$

Even if a symmetry guarantees degeneracy at the GUT scale it is difficult to protect it from corrections, e.g. from renormalisation group running

For degenerate models there can be large ren. group corrections to mixing angles and masses in the running from M_{GUT} dow to m_W

In fact the running rate is inv. prop. to mass differences

For a 2x2 case: $U^{Aa} = \begin{pmatrix} c_\vartheta & -s_\vartheta \\ s_\vartheta & c_\vartheta \end{pmatrix}$ $t = \frac{1}{16\pi^2} \log \frac{m}{m_Z}$

$$\frac{ds_\vartheta}{dt} = \kappa A_{21} (y_{e_2}^2 - y_{e_1}^2) s_\vartheta c_\vartheta^2 \quad \frac{dc_\vartheta}{dt} = -\kappa A_{21} (y_{e_2}^2 - y_{e_1}^2) s_\vartheta^2 c_\vartheta,$$

with $A_{21} = \frac{m_2 + m_1}{m_2 - m_1}$

k = -3/2 (SM), 1 (MSSM)

$y_e = m_e/v$ (SM), $m_e/v\cos\beta$ (MSSM)

RG corrections are generally negligible and can only be large for degenerate models especially at large $\tan\beta$

The observed mixings and splitting do not fit the typical result from pure evolution.

See, for example, Chankowski, Pokorski '01

Large neutrino mixings can induce observable $\tau \to \mu\gamma$ and $\mu \to e\gamma$ transitions

In fact, in SUSY models large lepton mixings induce large s-lepton mixings via RG effects (boosted by the large Yukawas of the 3rd family)

Detailed predictions depend on the model structure and the SUSY parameters.
Lopsided models tend to lead to the largest rates.

Typical values: $B(\mu \to e\gamma) \sim 10^{-11} - 10^{-14}$ (now: $\sim 10^{-11}$)
MEG experiment at PSI very interesting
$B(\tau \to \mu\gamma) < \sim 10^{-7}$ (now: $\sim 10^{-7}$)

See, e.g., ••••• Lavignac, Masina, Savoy'02
Masiero, Vempati, Vives'03; Babu, Dutta, Mohapatra'03;
Babu, Pati, Rastogi'04; Blazek, King '03; Petcov et al '04; Barr '04 ••••••

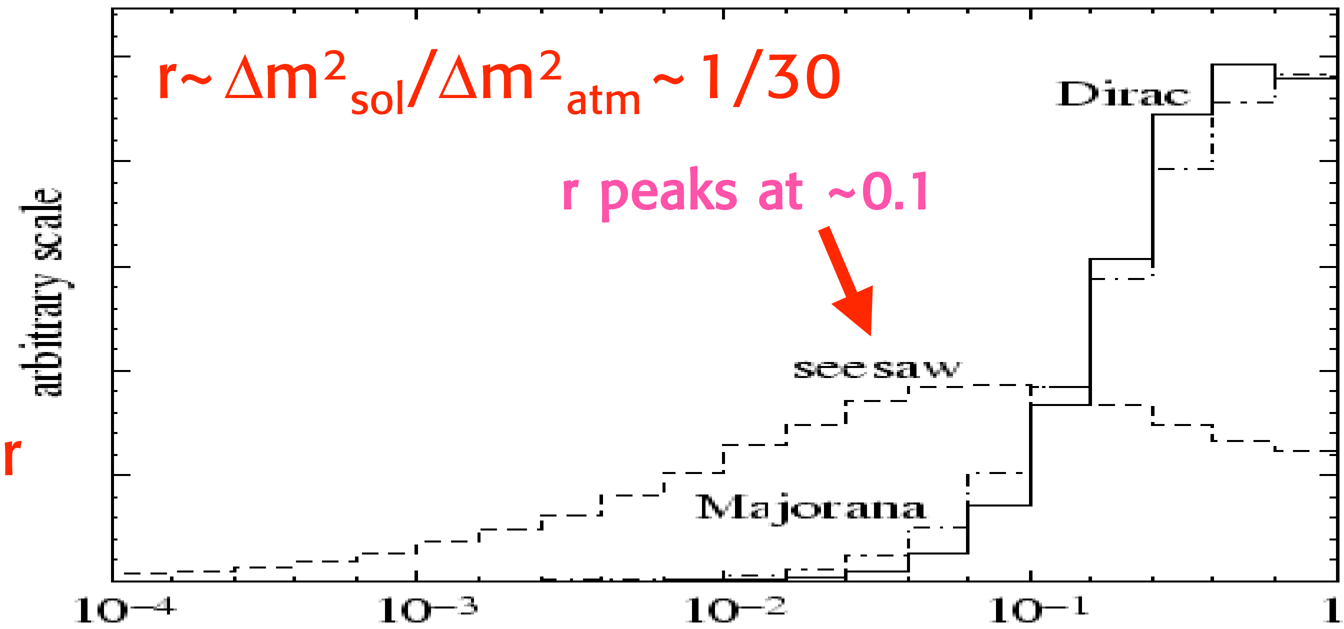
Anarchy (or accidental hierarchy):
No structure in the neutrino sector

A bit extreme!

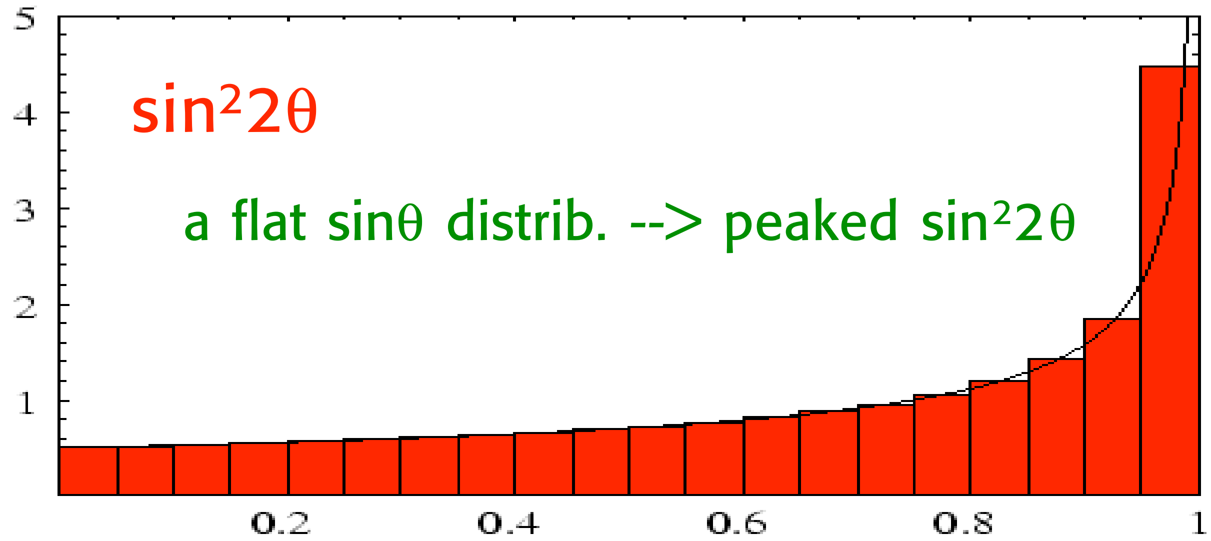
Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy
from random m,M

could fit the data on r



But: all mixing angles
should be not too large,
not too small →



Predicts θ_{13} near bound
θ_{23} sizably non maximal

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in F^{bar} ~ 5^{bar}

$m_u \sim 10 . 10$ strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar} . 10 \sim m_e^T$ milder hierarchy $m_d : m_s : m_b$

or $m_e : m_\mu : m_\tau$

$m_\nu \sim 5^T . 5$ or for see saw $(5.1)^T\,(1.1)\,(1.5)$

no hierarchy

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

$$\begin{cases} T \; : (3, 2, 0) \\ F^{bar}: (0, 0, 0) \\ 1: (0, 0, 0) \end{cases}$$

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H\, (\theta/M)^{\overbrace{q1+q2+qH}^{\Delta_{charge}}} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

$q(\bar{5})\sim(2, 0, 0)$ with no see-saw --> no structure in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$ Note: $\theta_{13}\sim\lambda^2$, $\theta_{23}\sim 1$

with coeff.s of o(1) and det23~o(1)
["semianarchy", while $\lambda\sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{bmatrix} \lambda^4 & \lambda^2 & 0 \\ \lambda^2 & \eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12}\sim\lambda^2$
But if, accidentally, $\eta\sim\lambda^2$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is naturally small, but θ_{12} large and the hierarchy $m^2_3 >> m^2_2$ are accidental

Ramond et al, Buchmuller et al

⊕ With see-saw, one can do much better (see later)

Is normal hierarchy compatible with large ν mixings?

- In the 2-3 sector we need both
  large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing
  (pert. th.: $\theta_{ij} \sim 1/|E_i-E_j|$)
  is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\quad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5):
LH for d quarks $\longleftrightarrow$ RH for l^- leptons

$$\overset{\bar{5}}{} \quad m_d \sim \bar{d}_R d_L \quad \overset{10}{}$$

$$\overset{10}{} \quad m_e \sim \bar{e}_R e_L \quad \overset{\bar{5}}{}$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models often large atmospheric mixing arises from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on $SU(5)xU(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters)

Of course, SU(5) can also be coupled with non abelian flavour symmetries, eg $O(3)_F$, $SU(3)_F$, A4, S4 (see next lecture) and become more predictive

- SO(10) models are more predictive but less flexible

Albright, Barr; Babu et al; Bajic et al; Barbieri et al; Buccella et al; King et al; Mohapatra et al; Raby et al; G. Ross et al

SU(5)xU(1)

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim\ 5^{bar}\ 10$
$m_{\nu D} \sim 5^{bar}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

With suitable charge assignments all relevant patterns can be obtained

The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

model	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Example: Normal Hierarchy

G.A., Feruglio, Masina'02

1st fam. 2nd 3rd

$$q(10):\ (5,\,3,\,0)$$
$$q(\bar 5):\ (2,\,0,\,0)$$
$$q(1):\ (1,-1,\,0)$$

Note: not all charges positive --> det23 suppression

$$q(H) = 0,\ q(\bar H) = 0$$
$$q(\theta) = -1,\ q(\theta') = +1$$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar 5_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar 5_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. O(1) omitted, only orders of magnitude predicted

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{pmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{pmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}^T M_{RR}^{-1} m_{\nu D}$

$$m_\nu \sim v_u^2/M \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix},$$

$$\det_{23} \sim \lambda^2$$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

Masses in SO(10) models

$16 \times 16 = 10 + 126 + 120$

If no non-ren mass terms are allowed a simplest model needs a 10 and a 126:

Bajc, Senjanovic, Vissani '02
Goh, Mohapatra, Ng '03

$$\mathcal{L}_Y = 10_H 16 y_{10} 16 + 126_H 16 y_{126} 16,$$

leading to

$$m_d = \alpha y_{10} + \beta y_{126}, \qquad m_e = \alpha y_{10} - 3\beta y_{126},$$

and $m_\nu \propto m_d - m_e \propto 126$

In the 23 sector, both m_d and m_e can be obtained (by $U(1)_F$) as:

$$m_{d,e} \sim \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}$$

Then b-τ unification forces a cancellation $1 \to \lambda^2$ in m_ν, which in turn makes a large 23 neutrino mixing.

Also predicts θ_{12} large, $r \sim \lambda^2$, θ_{13} near the bound

In other SO(10) models one avoids large Higgs represent'ns (120, 126) by relying on non ren. operators like $16_i\, 16_H\, 16_j\, 16'_H$ or $16_i\, 16_j\, 10_H 45_H$ (a lot of such terms are needed to reproduce all masses and mixings)

In the flavour-symmetric limit, the lowest dimension mass terms $16_3 16_3 10_H$ is only allowed for the 3rd family.

In particular, both lopsided and L-R symmetric models can be obtained in this way

Babu, Pati, Wilczek
Albright, Barr
Ji, Li, Mohapatra
Dermisek, Raby
....

GUT models often contain ad hoc ingredients and a lot of parameter fitting

Data have become more precise
Next lecture: models of Tri-Bimaximal mixing

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3 : 0.29\text{-}0.33$
$\sin^2\theta_{23} = 1/2 : 0.41\text{-}0.54$
$\sin^2\theta_{13} = 0 : \; < \sim 0.02$

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$