Volume modulus inflation and a low scale of SUSY breaking

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Outline

- Motivations
- Necessary conditions for inflation compatible with low energy SUSY breaking
- Model building
- Conclusions

KKLT moduli stabilization

F-term potential in 4D SUGRA:

$$V = e^K \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3 |W|^2 \right)$$

Kähler potential for the volume modulus:

$$K = -3 \ln(T + \overline{T})$$

For fixed dilaton and CSM fluxes contribute constant term to the superpotential:

$$W = A$$

Introducing non-perturbative correction (e.g. gaugino condensation) to the superpotential:

$$W = A + Ce^{-cT}$$

volume modulus can be stabilized at AdS SUSY minimum.

We live in dS space $\Rightarrow$ $\overline{D3}$-branes introduced to uplift minimum to dS space:

$$\Delta V = \frac{E}{(T + \overline{T})^2}$$

Relation between Hubble scale and gravitino mass

KKLT stabilization allow for constructing models of inflation within string theory e.g. racetrack inflation with 2 non-perturbative terms in the superpotential:

$$W = A + Ce^{-cT} + De^{-dT}$$

In generic inflationary model based on KKLT moduli stabilization Hubble scale during inflation is related to the gravitino mass (Kallosh, Linde '04):

$$H \lesssim m_{3/2}$$

If we insist on a low energy SUSY breaking, the relation $H \lesssim m_{3/2}$ forces us to construct low scale inflationary models ($H \sim \mathcal{O}(1\mathrm{TeV})$) $\Rightarrow$ very hard to find such models (no moduli inflation model of this type constructed so far)

Large $m_{3/2}$ originates from a deep SUSY AdS minimum before uplifting $\Rightarrow$ for the SUSY Minkowski minimum ($m_{3/2} = 0$) there is no relation between H and $m_{3/2}$

Non-SUSY stationary points

If we want to end inflation in the SUSY Minkowski minimum, it has to be initiated in the vicinity of the non-SUSY saddle point.

We derive necessary conditions for the existence of flat non-SUSY saddle points (suitable for inflation)

It is more convenient to work with function G which is given by:

$$G(\Phi_I, \Phi_I^\dagger) = K(\Phi_I, \Phi_I^\dagger) + \log W(\Phi_I) + \log \overline{W}(\Phi_I^\dagger)$$

In terms of G, scalar potential takes the following form:

$$V = e^G \left(G^{I\bar{J}} G_I G_{\bar{J}} - 3 \right)$$

Stationary conditions $\nabla_I V = \partial_I V = 0$ imply:

$$G_I(G^K G_K - 2) + G^K \nabla_I G_K = 0$$

The second covariant derivatives of the potential at a stationary point (equal to the ordinary ones) read:

$$V_{I\bar{J}} = e^G \left(G_{I\bar{J}}(\widehat{G}^2 - 2) - G_I G_{\bar{J}}(\widehat{G}^2 - 3) + \nabla_I G_K \nabla_{\bar{J}} G^K - R_{I\bar{J}K\bar{L}} G^K G^{\bar{L}} \right) ,$$

$$V_{IJ} = e^G \left((\nabla_I G_J + \nabla_J G_I) \frac{\widehat{G}^2 - 1}{2} - G_I G_J (\widehat{G}^2 - 3) + \frac{1}{2} G^K \{\nabla_I, \nabla_J\} G_K \right) ,$$

$\widehat{G} \equiv \sqrt{G^I G_I}$ is related to the value of the potential as follows:

$$\widehat{G}^2 = 3 + e^{-G} V .$$

Similar results have been found before in the case $\widehat{G}^2 = 3$ (Minkowski stationary points). (Gomez-Reino, Scrucca ’06)

From the inflationary point of view, dS saddle points are of the main interest, for which $\widehat{G}^2 > 3$.

Non-SUSY stationary points - one-field case

For non-canonically normalized fields physical mass matrix is given by:

$$M^2 = \begin{pmatrix} m^2_{X\overline{X}} & m^2_{XX} \\ m^2_{\overline{X}\overline{X}} & m^2_{\overline{X}X} \end{pmatrix} ,$$

with the entries

$$m^2_{X\overline{X}} = \frac{V_{X\overline{X}}}{G_{X\overline{X}}} , \qquad m^2_{XX} = \frac{V_{XX}}{G_{X\overline{X}}} ,$$

which can be written in the following form:

$$m^2_{X\overline{X}} = e^G \left(2 - \widehat{G}^2 R_X\right)$$

$$m^2_{XX} = \theta^2_X e^G \left[-2\left(\widehat{G}^4 - 3\widehat{G}^2 + 1\right) + \widehat{G}^4 A_{XXX} + 3\widehat{G}^4\left(\widehat{G}^2 - 2\right) A_{XX\overline{X}} - \widehat{G}^6 A_{XXX\overline{X}}\right]$$

where $\theta^2_X \equiv G_X / G_{\overline{X}}$, $A_{XX} \equiv G_{XX}/G_X G_X$, $A_{X\overline{X}} \equiv G_{X\overline{X}}/G_X G_{\overline{X}}$, etc. . . .
The curvature scalar of the Kähler manifold, R_X, is given by:

$$R_X = \frac{G_{XX\overline{X}\overline{X}}}{G^2_{X\overline{X}}} - \frac{G_{XX\overline{X}} G_{X\overline{X}\overline{X}}}{G^3_{X\overline{X}}} .$$

For studying inflation it is very convenient to work with the η-matrix.

The entries of the η-matrix are given by the second covariant derivatives with respect to real fields in the following way:

$$\eta^x_x = \frac{g^{xx}\nabla_x\nabla_x V}{V} \, ,$$

where $g^{xx} = G^{X\overline{X}}/2$

At the stationary points the entries of the η-matrix are proportional to the corresponding entries of the mass-matrix M^2

The necessary condition for a flat saddle point (positivity of the η-matrix trace) is relatively simple:

$$m^2_{X\overline{X}} > 0\,,$$

which implies

$$R_X < \frac{2}{\widehat{G}^2} < \frac{2}{3}$$

because $\widehat{G}^2 > 3$ for dS saddle points.

The above condition can be used to eliminate some models even without specifying the superpotential!

W-independent sufficient condition for the positivity of the η-matrix trace:

$$R_X \leqslant 0$$

Volume modulus as the inflaton

Kähler potential for the volume modulus:

$$K = -3\ln(T + \overline{T})$$

The curvature scalar takes the form:

$$R_T = \frac{2}{3}$$

The trace of the η-matrix is constant and negative:

$$\eta_t^t + \eta_\tau^\tau = -\frac{4}{3}$$

where $t = \text{Re}T$ and $\tau = \text{Im}T$.

No inflation for any superpotential $\Rightarrow$ corrections to Kähler potential required

Corrections to Kähler potential

The necessary condition for the positivity of the η-matrix trace:

$$R_T < 2/3$$

Sufficient condition:

$$R_T \leqslant 0$$

We consider Kähler potential with leading α'-correction and string loop correction:

$$K = -3\ln(T+\overline{T}) - \frac{\xi_{\alpha'}}{(T+\overline{T})^{3/2}} - \frac{\xi_{\text{loop}}}{(T+\overline{T})^{2}}$$

Curvature scalar for this setup reads:

$$R_T = \frac{2}{3} - \frac{35}{48}\frac{\xi_{\alpha'}}{(T+\overline{T})^{3/2}} - \frac{8}{3}\frac{\xi_{\text{loop}}}{(T+\overline{T})^{2}} + \ldots$$

Relatively small corrections could make trace of the η-matrix positive.

No inflation in Kallosh-Linde model

The superpotential in KL model reads:

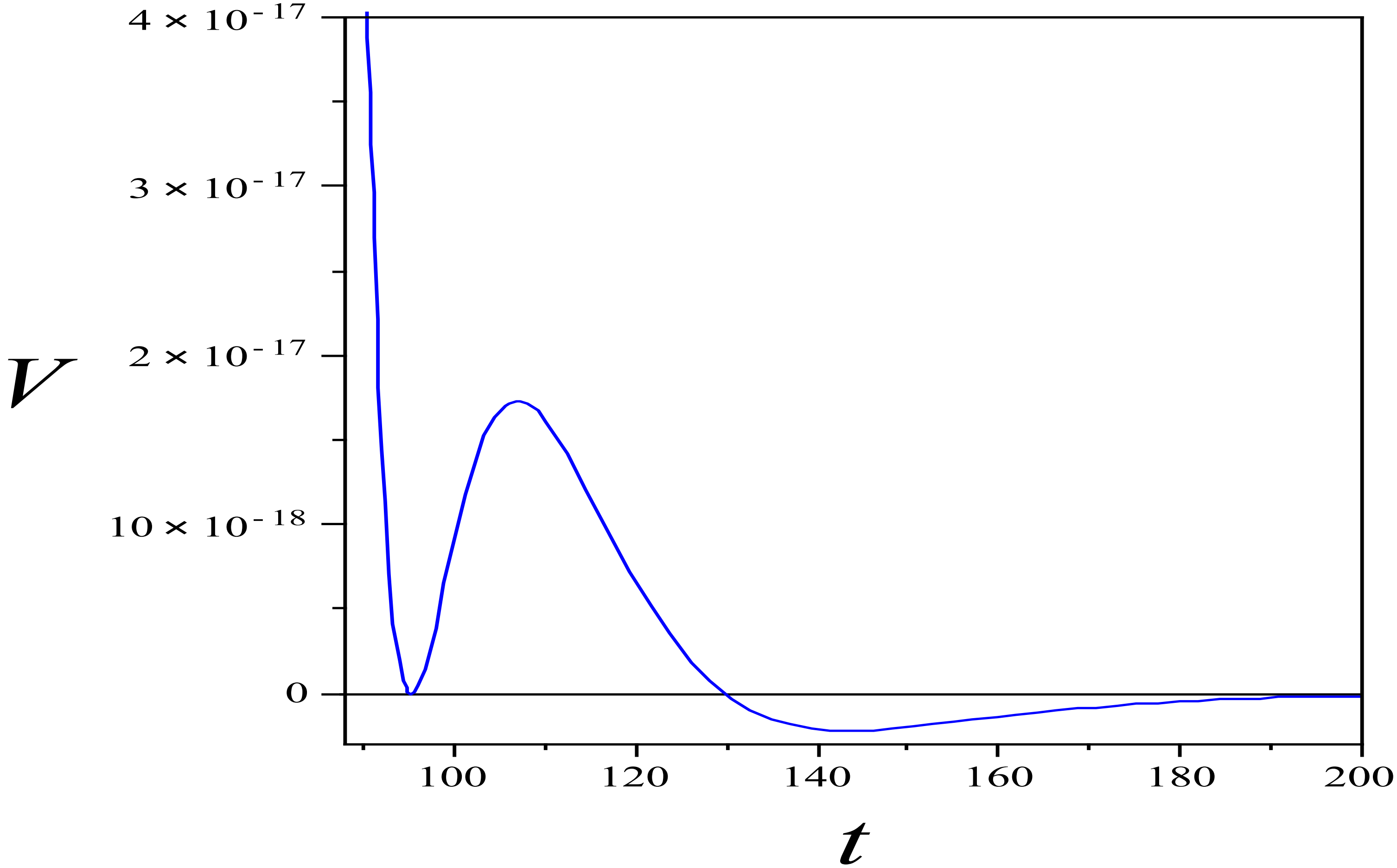
$$W = A + Ce^{-cT} + De^{-dT}$$

SUSY Minkowski minimum exists for fine-tuned value of A:

$$A = -C \left| \frac{cC}{dD} \right|^{\frac{c}{d-c}} - D \left| \frac{cC}{dD} \right|^{\frac{d}{d-c}}$$

SUSY Minkowski minimum occures at:

$$T_{\rm mink} = t_{\rm mink} = \frac{1}{c-d} \ln \left| \frac{cC}{dD} \right| , \qquad \qquad \tau_{\rm mink} = 0$$

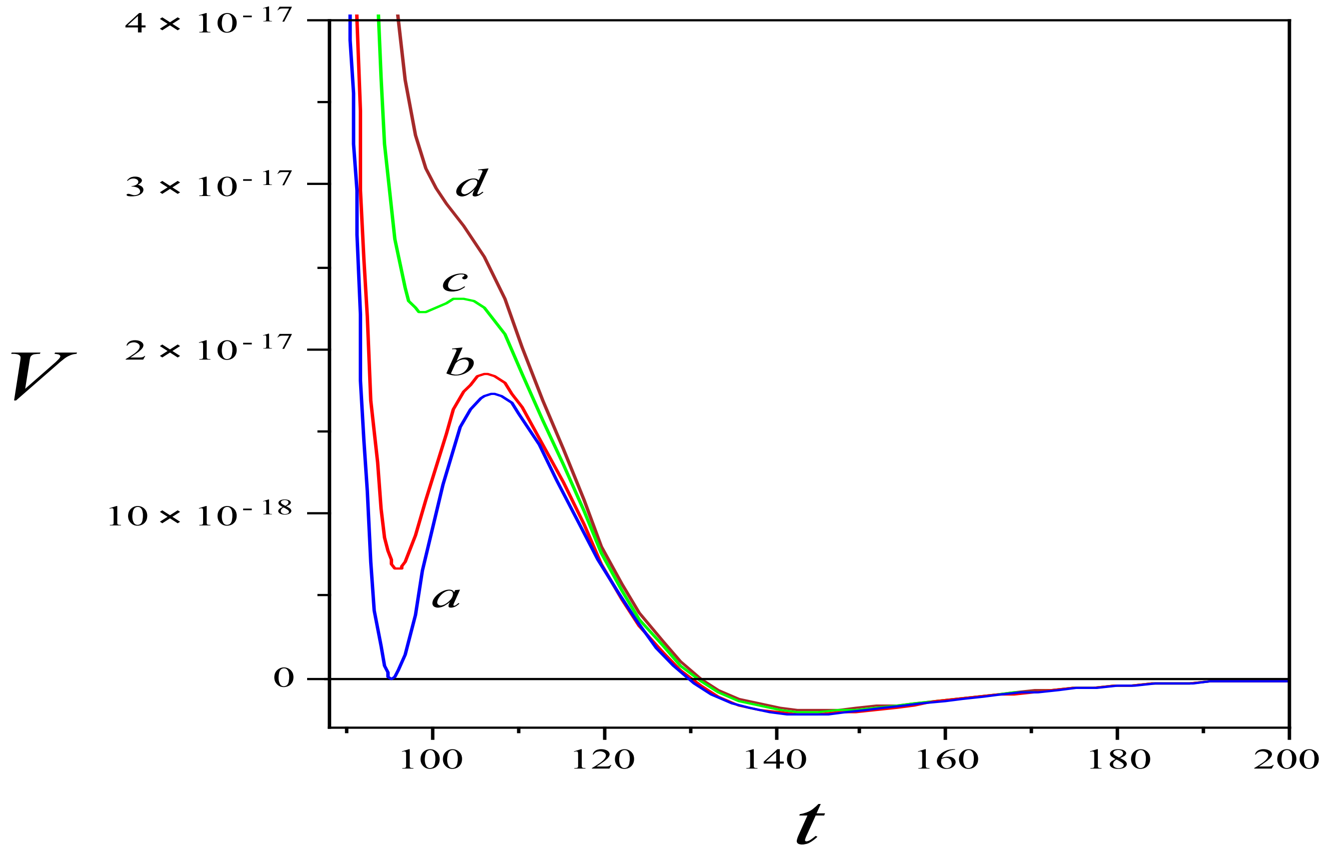


η-matrix is diagonal and parameter $\eta = \eta^t_t = \frac{2t^2}{3}\frac{V_{tt}}{V}$ in the limit $ct, dt \gg 1$ equals:

$$|\eta| \approx \frac{4cdt^2}{3} \gg 1$$

For relatively small values of τ, minimum in the t-direction disappears $\Rightarrow$ There is no saddle point with instability in the τ-direction.

Different lines correspond to different values of $c\tau$: (a) $c\tau = 0$, (b) $c\tau = 0.2$, (c) $c\tau = 0.4$, (d) $c\tau = 0.5$.



Triple gaugino condensation model

The superpotential reads:

$$W = A + Be^{-bT} + Ce^{-cT} + De^{-dT}$$

Kähler potential with leading corrections:

$$K = -3\ln(T+\overline{T}) - \frac{\xi_{\alpha'}}{(T+\overline{T})^{3/2}} - \frac{\xi_{\mathrm{loop}}}{(T+\overline{T})^{2}}$$

SUSY Minkowski conditions ($\partial_T W = W = 0$) cannot be solved analytically. Solution is not unique. There are 2 types of solutions:

- $\tau_{\mathrm{mink}} = 0$ and A real $\rightarrow$ structure of the potential as in KL model $\rightarrow$ no inflation
- $\tau_{\mathrm{mink}} \neq 0$ and A complex $\rightarrow$ inflation can be realized only if B (C or D) complex

Triple gaugino condensation model - example

Inflation can be obtained e.g. for the following set of parameters ($B = B_0 + i\beta_0$):

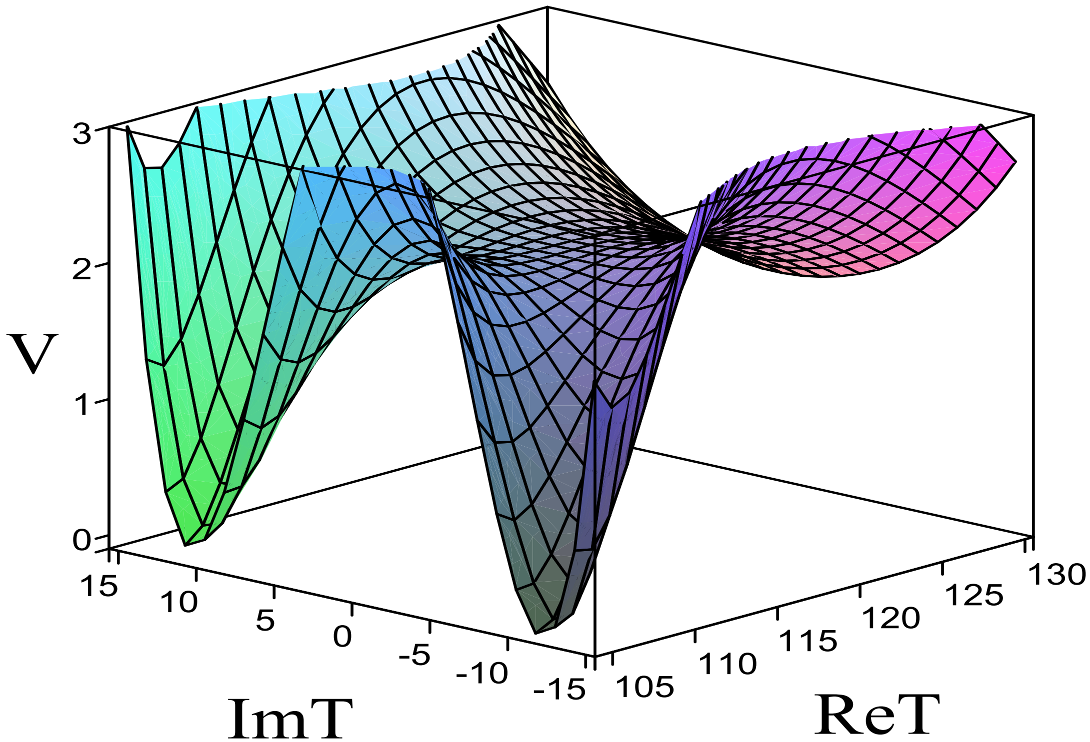
$$B_0 = -5.454364 \cdot 10^{-2}\,, \quad \beta_0 = 5.939476 \cdot 10^{-5}\,, \quad C = -\frac{1}{75}\,, \quad D = \frac{1}{30}\,,$$
$$b = \frac{2\pi}{70}\,, \quad c = \frac{2\pi}{100}\,, \quad d = \frac{2\pi}{90}\,, \quad \xi_{\alpha'} = 500\,, \quad \xi_{\mathrm{loop}} = 5000\,.$$

For SUSY Minkowski minimum to exist, $A = A_0 + i\alpha_0$ has to be fine-tuned as follows:

$$A_0 = 7.20585 \cdot 10^{-7}\,, \qquad \alpha_0 = -9.41347 \cdot 10^{-8}$$

For unbroken SUSY $W = 0$ but in our world SUSY is broken so $W \approx 0$ and the fine-tuning of A may be somewhat relaxed.

Inflationary potential



AdS minimum:	$t_{\rm AdS} = 104.646$,	$\tau_{\rm AdS} = -11.664$
SUSY Minkowski minimum:	$t_{\rm mink} = 104.473$,	$\tau_{\rm mink} = 10.885$
Inflationary saddle point:	$t_{\rm saddle} = 115.475$,	$\tau_{\rm saddle} = -0.183$

Fine-tuning

Triple gaugino condensation model is the first one that accommodates TeV-range gravitino mass and high scale of inflation but requires significant amount of fine-tuning:

Two parameters fine-tuned to (almost) cancel diagonal and off-diagonal entry of the η-matrix $\Rightarrow$ one more fine-tuning than in typical models (e.g. racetrack inflation)

Is this additional tuning necessary in models with light gravitino?

NO, if parameters of W are real and $\tau = 0$ during inflation $\Rightarrow$ off-diagonal entry of the η-matrix vanishes automatically and one tuning is enough

Is it possible to construct such models?

Inflection point inflation

Inflation in the t-direction ($\tau = 0$) can occur in the vicinity of the inflection point.

Naturally realized with positive exponents in gaugino condensation terms.

Positive exponents may occur when gauge kinetic function takes the form:

$$f = w_S S + w_T T\,.$$

Gaugino condensation generates:

$$W_{\rm np} = B e^{-\frac{2\pi}{N}(w_S S + w_T T)}\,.$$

When $w_T < 0$ and dilaton S is stabilized at higher scales, positive exponents appear in effective theory for the volume modulus:

$$W_{\rm np}^{\rm eff} = B^{\rm eff} e^{bT}\,.$$

Gauge kinetic functions of this type realized in string theory (Marchesano, Shiu '04; Cascales, Uranga '03; Lukas, Ovrut, Waldram '97)

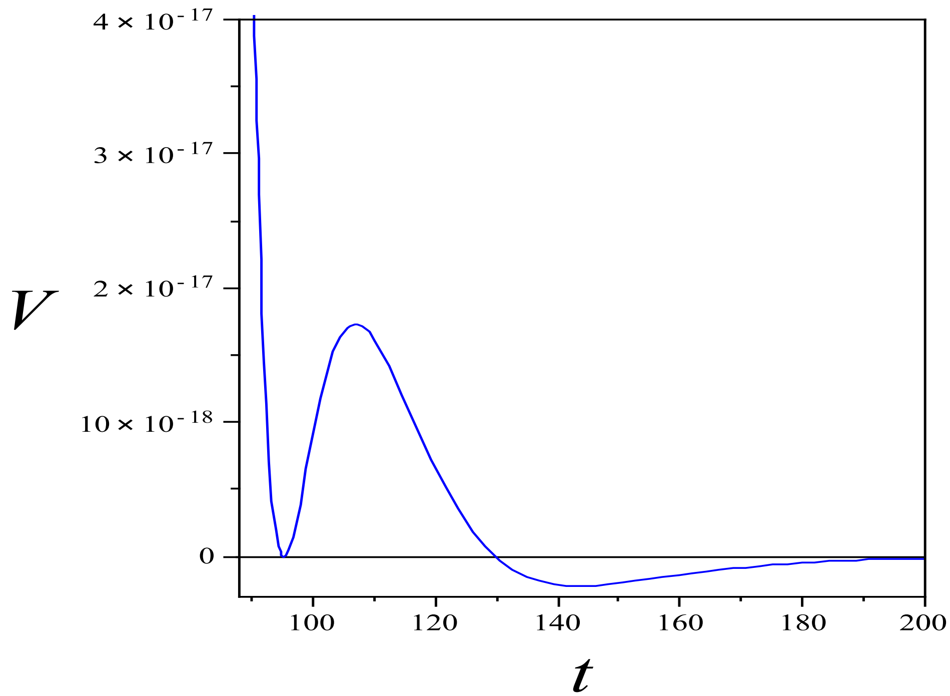
Model building with positive exponents

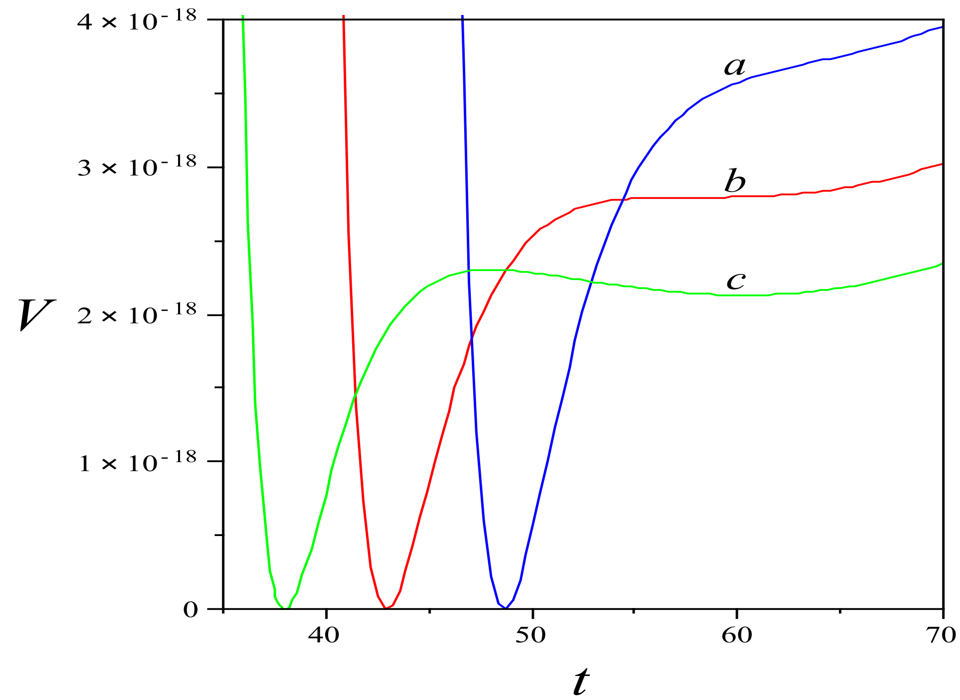
We consider superpotential with 2 gaugino condensates:

$$W = A + Ce^{cT} + De^{dT}\,.$$

$c<0$ and $d<0$ (KL model) $\Rightarrow$ SUSY AdS minimum $\Rightarrow$ no inflation with light gravitino

$\left.\begin{array}{l} c<0 \text{ and } d>0 \\ c>0 \text{ and } d>0 \end{array}\right\} \Rightarrow$ nonSUSY AdS/dS minimum $\Rightarrow$ inflection point inflation





What is the price for the working model?

Fine-tuning of one parameter (e.g. C) at the level of 10^{-5} (similar to racetrack inflation).

Stabilization of the τ-direction through string corrections to tree-level Kähler potential.

Fine-tuning of the initial conditions for t at the level of one percent.

TeV-range gravitino mass requires:

Fine-tuning of A at the level of 10^{-5}.

Threshold corrections to gauge kinetic function

We consider superpotential with 1 gaugino condensate and threshold corrections:

$$W = A + (C_0 + C_1 T)e^{cT}\,.$$

SUSY Minkowski minimum exists for fine-tuned value of A:

$$A = \frac{C_1}{c}\exp\left(-\frac{cC_0}{C_1} - 1\right)\,.$$

SUSY Minkowski minimum occures at:

$$T_{\rm Mink} = -\frac{1}{c} - \frac{C_0}{C_1}\,.$$

For positive c inflection point inflation with light gravitino can be realized

The same conditions for successful inflation as in double gaugino condensation model

Very modest model $\rightarrow$ only 4 parameters in W

$\rightarrow$ 1 parameter in K (to stabilize the τ direction)

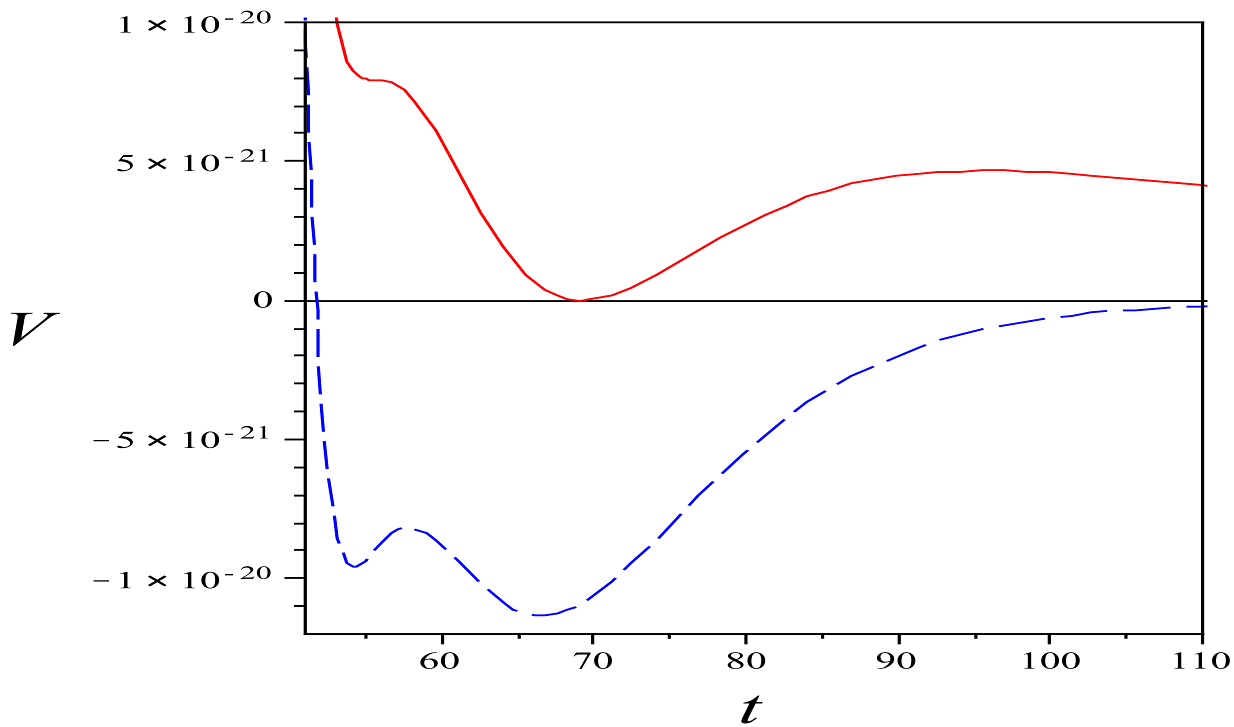
Negative exponents and inflation with heavy gravitino

Without positive exponents inflation possible only with uplifting (heavy gravitino)

$$\Delta V = \frac{E}{t^2}\,.$$

Examples: $\longrightarrow$ KL model (Linde, Westphal '07)

$\longrightarrow$ Single gaugino condensation with threshold corrections



Fine-tuning and the overshooting problem

Fine-tuning of the potential and the initial conditions is inevitably related to the height of the barrier that protects the inflaton from overshooting Minkowski vacuum.

Uplifting is decreasing function of $t \Rightarrow$ maximum (before uplifting) necessarily much below 0 if the barrier is to be high (after uplifting)

Fine-tuning of the parameters in models with negative exponents (heavy gravitino) at least $10^{-8} \Rightarrow$ substantially bigger than in models with positive exponents (light gravitino)

Conclusions

- For the volume modulus, parameter η necessarily smaller than $-2/3$ and inflation with TeV-range gravitino mass cannot be realized unless corrections to the leading Kähler potential are included.

- Even for corrected Kähler potential inflation cannot be realized in KL model with only two non-perturbative terms in the superpotential.

- Adding third non-perturbative term to the superpotential makes inflation possible but significant amount of fine-tuning is necessary.

- Positive exponents in non-perturbative terms help in realizing inflection point inflation with light gravitino. Double gaugino condensation or single one with threshold corrections are enough to realize successful models.

- Inflection point models with all exponents negative can be realized only with uplifting (heavy gravitino) and suffer from overshooting problem.

- To overcome overshooting problem in models with all exponents negative parameters has to be much more fine-tuned than in models with positive exponents where overshooting problem is absent.