HIERARCHICAL NEUTRINO MASSES AND MIXING IN GUTS

THE CASE OF FLIPPED $SU(5)$[1]

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[1] J. Rizos and K.T. arXiv:0912.3997 to appear in PLB

The wealth of Experimental Data on **Neutrino Masses** and **Mixing** have motivated the study of possible relevant mechanisms. These attempts are more appealing if they are developed within the existing frameworks of **Unification** and **Supersymmetry**.

Among existing proposals the ***most appealing*** is the so-called

Seesaw Mechanism

giving an elegant answer to the smallness of the neutrino mass.

$$\mathbf{m}\,\nu\,N^c + \mathbf{M}\,N^c\,N^c = [\,\nu,\ N^c\,]\begin{bmatrix} 0 & \mathbf{m} \\ \mathbf{m} & \mathbf{M} \end{bmatrix}\begin{bmatrix} \nu \\ N^c \end{bmatrix} \Longrightarrow \begin{cases} \frac{\mathbf{m^2}}{\mathbf{M}} \\ \mathbf{M} \end{cases}$$

The seesaw-GUT scenario does not lead by default to an understanding of the neutrino mass, and extra ingredients are required provided by the specific model. The simplest choice, $\mathbf{SU(5)}$, is not appealing since the right-handed neutrino is a gauge singlet and the scale M is arbitrary

$$Y^{(\nu)}\,\mathcal{F}_{\overline{5}}\,N^c\,\mathcal{H}_5 \,+\, M\,N^c\,N^c \;=\; Y^{(\nu)}\,\langle H^c\rangle\,\nu\,N^c \,+\, M\,N^c N^c \,+\, \ldots .$$

In contrast, in $\mathbf{SO(10)}$, the right-handed neutrino is unified in the $\underline{\mathbf{16}}$ but its Majorana mass cannot easily arise in a minimal context. Effectively, we have

$$Y\,\Psi_{16}\,\Psi_{16}\,\mathcal{H}_{10} \,+\, \Psi_{16}\cdot\frac{\langle\overline{16}_H \times \overline{16}_H\rangle}{M}\cdot\Psi_{16}$$

In the model based on the $\mathbf{SU(5)} \times \mathbf{U(1)}$ group, the so-called ***flipped SU(5) GUT*** the right-handed neutrino is part of the $(\underline{\mathbf{10}},\ \mathbf{1})$ rep., thus, realizing a GUT-coupling that gives rise to the right-handed neutrino mass. The matter ($\mathcal{F},\ f^c,\ \ell^c$) and Higgs ($\mathcal{H},\ \mathcal{H}^c,\ h,\ h^c$) chiral superfield content of the model is

$$\mathcal{F}(\underline{10},\ 1)\ =\ (\mathcal{Q},\ \mathcal{D}^c,\ \mathcal{N}^c)\ ,\quad f^c(\underline{\overline{5}},-3)\ =\ (\mathcal{L},\mathcal{U}^c)\ ,\ \mathcal{L}^c(\underline{1},5),$$

$$\mathcal{H}(\underline{10},\ 1)\ =\ (\mathcal{Q}_H,\ \mathcal{D}^c_H,\ \mathcal{N}^c_H)\ ,\quad \overline{\mathcal{H}}(\underline{\overline{10}},\ -1)\ =\ \left(\overline{\mathcal{Q}}_H,\ \overline{\mathcal{D}}^c_H,\ \overline{\mathcal{N}}^c_H\right)\ ,$$

$$h(\underline{5},\ -2)\ =\ (\mathcal{H}_1,\ \mathcal{D}_H)\ ,\quad h^c(\underline{\overline{5}},\ 2)\ =\ \left(\mathcal{H}_2,\ \overline{\mathcal{D}}_H\right)\ .$$

The renormalizable superpotential

$$\mathcal{W}\ =\ Y^{(d)}_{ij}\,\mathcal{F}_i\mathcal{F}_j\,h\ +\ Y^{(u)}_{ij}\,\mathcal{F}_i\,f^c_j\,h^c\ +\ Y^{(\ell)}_{ij}\,f^c_i\,\mathcal{L}^c_j\,h^c$$

$$+\,\lambda\mathcal{H}\mathcal{H}\,h\ +\ \lambda'\overline{\mathcal{H}\mathcal{H}}\,h^c\ +\ \mu\,h\,h^c$$

is the most general under R-parity and the discrete $\mathcal{Z}_2$ symmetry that changes the sign of $\mathcal{H}\ \rightarrow\ -\mathcal{H}$, while all other fields remain unchanged.

F and D-flatness are satisfied with non-zero vevs of $\mathcal{H} = (\mathcal{Q}_H, \mathcal{D}^c_H, \mathcal{N}^c_H)$ and $\overline{\mathcal{H}} = \left(\overline{\mathcal{Q}}_H, \overline{\mathcal{D}}^c_H, \overline{\mathcal{N}}^c_H\right)$ in the direction

$$\langle N^c_H \rangle = \langle \overline{N}^c_H \rangle \equiv M_X$$

that affects the breaking

$$\mathbf{SU(5)} \times \mathbf{U(1)} \rightarrow \mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)} .$$

The fields $\mathcal{Q}_H$, $\overline{\mathcal{Q}}_H$ and a combination of $\mathcal{N}^c_H$, $\overline{\mathcal{N}}^c_H$ will be removed by the Higgs mechanism while the triplets $\mathcal{D}^c$, $\overline{\mathcal{D}}^c$, $\mathcal{D}_H$, $\overline{\mathcal{D}}_H$ will obtain large masses λM_X, $\lambda' M_X$ through the couplings $\lambda \mathcal{H}\mathcal{H}\, h$ and $\lambda' \overline{\mathcal{H}\mathcal{H}}\, h^c$. Thus, the triplets are split from the (massless) doublets. So far, the right-handed neutrino participates in the term $Y^{(u)}_{ij}\, \mathcal{F}_i\, f^c_j\, h^c$ leading to the neutrino Dirac mass

$$Y^{(u)}_{ij}\, N^c_i\, \ell_j\, H_2 + Y^{(u)}_{ij}\, Q_i\, u^c_j\, H_2 + \ldots .$$

With the given set of fields a renormalizable right-handed neutrino mass term cannot arise, although the $D = 4$ operator

$$\mathcal{F} \cdot \frac{\overline{\mathcal{H}\mathcal{H}}}{M} \cdot \mathcal{F} = N^c \frac{\left(\overline{N}_H^c\right)^2}{M} N^c + \ldots .$$

is allowed by the symmetries of the superpotential. Such a term can arise if we introduce a set of superheavy singlets whose exchange generates the above term. Denoting these fields by $\mathcal{S}_i$, we may assign to them the R-parity (or matter parity) of matter superfields. Thus, the most general renormalizable superpotential that can be added to $\mathcal{W}$ is

$$Y_{ij}^{(s)}\, \mathcal{S}_i \mathcal{F}_j \overline{\mathcal{H}} + \frac{1}{2} M_{ij}^{(s)} \mathcal{S}_i \mathcal{S}_j \,.$$

Note that the necessity of the singlet sector is anticipated by the following additional argument : *The large mixing encountered in neutrinos suggests that its origin is different from the corresponding Cabbibo mixing of quarks. Thus, a sector of the theory outside the GUT is required with a larger characteristic mass-scale, presumably of the order of the string or Planck scale.*

Neutrino masses arise from the terms

$$Y^{(u)}_{ij}\, N^c_i\, \ell_j\, H_2 \;+\; Y^{(s)}_{ij}\, S_i\, N^c_j\, \overline{H} \;+\; \frac{1}{2} M^{(s)}_{ij} S_i S_j$$

$$\Downarrow$$

$$Y^{(u)}_{ij}\, \frac{v_2}{\sqrt{2}}\, N^c_i\, \nu_j \;+\; Y^{(s)}_{ij}\, M_X\, S_i\, N^c_j \;+\; \frac{1}{2} M^{(s)}_{ij} S_i S_j$$

where $\frac{v_2}{\sqrt{2}} = \langle H_2 \rangle$. Thus, neutrinos participate in the 9×9 mass-matrix

$$\begin{pmatrix} 0 & \frac{v_2}{\sqrt{2}} Y^{(u)} & 0 \\ \frac{v_2}{\sqrt{2}} Y^{(u)} & 0 & Y^{(s)} M_X \\ 0 & Y^{(s)} M_X & M^{(s)} \end{pmatrix},$$

in a $(\nu,\, N^c,\, S)$ basis.

In the limit that the electroweak scale is neglected, the relevant part of the matrix is the 6×6 matrix

$$\begin{pmatrix} 0 & Y^{(s)} M_X \\ Y^{(s)} M_X & M^{(s)} \end{pmatrix} .$$

The natural mass scale for the singlets should be $M^{(s)} >> M_X$. Then, an effective

singlet-seesaw mechanism

operates leading to the right-handed neutrino mass

$$\mathbf{M_R} \approx \mathbf{M_X^2 \, Y^{(s)\perp} \, M^{(s)-1} \, Y^{(s)}} .$$

If we take $M_X \sim 10^{16}\, GeV$ and $M^{(S)} \sim 10^{18}\, GeV$, for a choice of the dimensionless singlet coupling $Y^{(s)} \sim O(0.1) - O(1)$, we obtain the scale of M_R to be $M_R \sim 10^{12} - 10^{14}\, GeV$. If we take the singlet mass scale to coincide with the string scale, we obtain $M_R \sim 10^{13} - 10^{15}\, GeV$.

In the limit that the three approximate mass-eigenstates with masses $O(M^{(s)})$ decouple, the neutrino mass matrix, in the ν, $N^{c\prime}$ basis of left-handed neutrinos and ``*light*" right-handed neutrino approximate mass-eigenstates, is

$$\begin{pmatrix} 0 & \frac{v_2}{\sqrt{2}} Y^{(u)} \\ \frac{v_2}{\sqrt{2}} Y^{(u)} & M_R \end{pmatrix}$$

and we have the operation of the ***standard seesaw mechanism*** leading to three light neutrinos of mass

$$\mathbf{M^{(\nu)}} \approx \frac{\mathbf{v_2^2}}{\mathbf{2}} \mathbf{Y^{(u)}} \mathbf{M_R^{-1}} \mathbf{Y^{(u)}} \approx$$

$$\frac{\mathbf{v_2^2}}{\mathbf{2M_X^2}} \mathbf{Y^{(u)}} \mathbf{Y^{(s)}}^{-1} \mathbf{M^{(s)}} \left(\mathbf{Y^{(s)}}^{-1} \right)^{\perp} \mathbf{Y^{(u)}}$$

leading to $m_t^2/M_R \sim 10^{-1}\, eV$.

Apart from family structure, the scale of the neutrino masses is

$$[M^{(\nu)}] \sim \left[\frac{(m^{(u)})^2}{M_R}\right] \Longrightarrow [M^{(\nu)}]_{33} \sim \frac{m_t^2}{[M_R]} \sim 10^{-1}\, eV\,.$$

In the light neutrino mass formula we may factor out the mass scale

$$m_\nu = \frac{v_2^2 [M^{(s)}]}{M_X^2}$$

and replace $M^{(\nu)} = m_\nu \, \hat{M}^{(\nu)}$ with the dimensionless matrix

$$\hat{\mathbf{M}}^{(\nu)} = \mathbf{Y}^{(\mathbf{u})}\, \mathbf{Y}^{(\mathbf{s})^{-1}}\, \hat{\mathbf{M}}^{(\mathbf{s})} \left(\mathbf{Y}^{(\mathbf{s})^{-1}}\right)^{\perp} \mathbf{Y}^{(\mathbf{u})}$$

where $\hat{M}^{(s)}$ is dimensionless.

It should be noted that the right-handed neutrino mass scale was generated naturally through a **seesaw mechanism** in terms of the **unification scale**, related to the unification of gauge couplings and the **singlet sector mass scale**.

Independently of the natural determination of neutrino scales, the neutrino mass formula

$$M^{(\nu)} \approx \frac{v_2^2[M^{(s)}]}{2M_X^2} Y^{(u)} \left(Y^{(s)} \left(M^{(s)} \right)^{-1} Y^{(s)} \right)^{-1} Y^{(u)} .$$

combines two sources of family structure:

One of them, represented by the up-quark Yukawa coupling matrix, will impart to the neutrino masses the hierarchical structure existing in the quark sector.

The other, represented by $Y^{(s)}$ and $M^{(s)}$ endows neutrinos with an extra component of mixing.

The existing experimental data

$$\Delta m_{32}^2 = |m_3^2 - m_2^2| \approx 2.5 \times 10^{-3}\, eV^2,$$

$$\Delta m_{21}^2 = |m_2^2 - m_1^2| \approx 7.3 \times 10^{-5}\, eV^2$$

can be expressed in terms of $\lambda = 0.22$ as

$$\frac{\Delta m_{32}}{\Delta m_{21}} = \sqrt{\frac{7.3}{2.5}} \times 10^{-1} \approx 0.171 \approx 3.53\, \lambda^2 .$$

This ratio can be interpreted in various ways.The most straightforward interpretation is in terms of **a hierarchical pattern**.

For example

$$m_3 \approx 5 \times 10^{-2}\, eV, \quad m_2 \approx 8.5 \times 10^{-3}\, eV, \quad (m_1 << m_2, m_3)$$

The hierarchical structure of the up-quark Yukawa matrix, expressed in terms of the Cabbibo parameter $\lambda \approx 0.22$, will be inherited to the neutrino mass matrix. The structure of the latter will depend on the dependence of the singlet sector parameters on λ.

Can we employ Ansätze for the couplings and mass of the singlet sector that lead to hierarchical neutrino masses?

We start by employing an Ansatz for $Y^{(u)}$, although the precise choice is not crucial.

Ansatz-I

$$Y^{(u)} = \begin{pmatrix} 0 & e_1\lambda^6 & 0 \\ e_1\lambda^6 & 0 & e_2\lambda^2 \\ 0 & e_2\lambda^2 & e_3 \end{pmatrix}$$

and

$$Y^{(s)} = \begin{pmatrix} c_1\lambda^5 & 0 & 0 \\ 0 & c_2\lambda^2 & 0 \\ 0 & 0 & c_3 \end{pmatrix}.$$

$Y^{(s)}$ has been chosen diagonal for simplicity.

The singlet mass-matrix $\left(\hat{M}^{(s)}\right)_{ij} = \hat{M}_{ij}$ will be chosen to be an entirely generic symmetric matrix.

This Ansatz leads to the neutrino mass hierarchy

$$\lambda^{\mathbf{8}} : \lambda^{\mathbf{2}} : \mathbf{1}$$

namely

$$M_3 \approx M_3^{(0)} + \lambda^2 M_3^{(1)} + \lambda^3 M_3^{(2)} + \ldots$$

$$M_2 \approx \lambda^2 M_2^{(0)} + \lambda^3 M_2^{(1)} + \lambda^4 M_2^{(2)} + \ldots, \quad .$$

$$M^{(1)} \approx \lambda^8 M_1^{(0)} + \lambda^9 M_1^{(1)} + \ldots, \quad .$$

The corresponding eigenvectors determine the associated neutrino mass-diagonalization matrix is

$$\mathbf{U}^{(\nu)} = \begin{pmatrix} 1 & b\lambda^3 & (c-ab)\lambda^4 \\ -b\lambda^3 & 1-\frac{\lambda^2}{2}a^2 & -\lambda a - \lambda^2\overline{a} \\ -c\lambda^4 & \lambda a + \lambda^2\overline{a} & 1-\frac{\lambda^2}{2}a^2 \end{pmatrix} .$$

The charged lepton and neutrino mass-terms are

$$M_{ij}^{(\ell)} \ell_i \, \ell_j^c \; + \; M_{ij}^{(\nu)} \nu_i \nu_j \, .$$

These matrices can be diagonalized as

$$M_{\Delta}^{(\ell)} \; = \; \mathbf{U}^{(\ell)\perp} M^{(\ell)} \mathbf{V}^{(\ell^c)}, \quad M_{\Delta}^{(\nu)} \; = \; \mathbf{U}^{(\nu)\perp} M^{(\nu)} \mathbf{U}^{(\nu)} \, ,$$

in terms of the unitary matrices $\mathbf{U}^{(\ell)}$, $\mathbf{V}^{(\ell^c)}$, $\mathbf{U}^{(\nu)}$ that connect the *current* and the *mass-eigenstates* (primed fields)

$$\ell \; = \; \mathbf{U}^{(\ell)} \ell', \; \ell^c \; = \; \mathbf{V}^{(\ell^c)} \ell^{c\prime}, \quad \nu \; = \; \mathbf{U}^{(\nu)} \nu' \, .$$

The neutrino charged current $J_\mu^{(+)} \; \propto \; \ell_i^\dagger \sigma_\mu \nu_i$ can be expressed in terms of the

Pontecorvo-Maki-Nakagawa-Sakata or simply *PMNS* ***-mixing matrix***

$$\mathcal{U}_{PMNS} \; \equiv \; \mathbf{U}^{(\ell)\dagger} \mathbf{U}^{(\nu)} \, .$$

For simplicity, we do not consider *CP* violation. In that case, the *PMNS*−matrix can be parametrized in terms of three mixing angles, namely the *``solar angle''* θ_{12}
the *``atmospheric angle''* θ_{23}
and the *``small''* angle θ_{13}
as

$$\mathcal{U}_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

where $\cos\theta_{ij} = c_{ij}$ and $\sin\theta_{ij} = s_{ij}$.

Assuming a trivial $\mathbf{U}^{(\ell)}$, $\mathcal{U}_{PMNS}$ is given by $\mathbf{U}^{(\nu)}$ and can be put in the form

$$\mathcal{U}_{PMNS} \;=\; \mathbf{U}(\theta_{23})\,\mathbf{U}(\theta_{13})\,\mathbf{U}(\theta_{12})\,,$$

where the $\mathbf{U}(\theta_{ij})$ unitary matrices describe rotations in the $(i,\,j)$-plane of flavor space.
For *Ansatz-I* we have

$$\sin\theta_{23} \;\approx\; \lambda\, a \,+\, \lambda^2\,\overline{a}, \quad \sin\theta_{12} \;\approx\; \lambda^3\, b\,, \quad \sin\theta_{13} \;\approx\; \lambda^4\,(c - ab)\;.$$

The coefficients $a,\ \overline{a},\ b,\ c$ are expressible in terms of the parametrers $e_i,\ c_i$ and ratios of the matrix elements $\hat{M}_{ij}$. Note the predicted hierarchy

$$\theta_{23} \;>\; \theta_{12} \;>>\; \theta_{13}\,.$$

We may also have the mass hierarchy

$$1 : \lambda : \lambda^5$$

corresponding to

Ansatz-II

$$Y^{(u)} = \begin{pmatrix} 0 & e_1\lambda^6 & 0 \\ e_1\lambda^6 & e_2\lambda^4 & 0 \\ 0 & 0 & e_3 \end{pmatrix}, \quad Y^{(s)} = \begin{pmatrix} c_1\lambda^6 & 0 & \\ 0 & c_2\lambda^3 & \\ 0 & 0 & c_3 \end{pmatrix}$$

and

$$\hat{M} = \begin{pmatrix} 0 & \hat{M}_{12} & 0 \\ \hat{M}_{12} & \hat{M}_{22} & \hat{M}_{23} \\ 0 & \hat{M}_{23} & \hat{M}_{33} \end{pmatrix}.$$

Note the two texture zeros in $\hat{M}$.

This Ansatz leads to neutrino mass eigenvalues

$$M_3 \approx M_3^{(0)} + \lambda^2 M_3^{(1)} + \ldots .$$

$$M_2 \approx \lambda M_2^{(0)} + \lambda^2 M_2^{(1)} + \ldots .$$

$$M_1 \approx \lambda^5 M_1^{(0)} + \lambda^6 M_1^{(1)} + \ldots .$$

The diagonalizing unitary matrix is

$$\mathbf{U} = \begin{pmatrix} 1 - \frac{a^2}{2}\lambda^4 & a\lambda^2 & -a\,b\,\lambda^3 \\ -a\,\lambda^2 & 1 - \frac{b^2}{2}\lambda^2 & b\,\lambda + c\,\lambda^2 \\ 2\,a\,b\,\lambda^3 & -b\,\lambda - c\,\lambda^2 & 1 - \frac{b^2}{2}\lambda^2 \end{pmatrix} .$$

This corresponds to a mixing matrix with mixing angles

$$\sin\theta_{23} \approx b\,\lambda + c\,\lambda^2, \ \sin\theta_{12} \approx a\,\lambda^2, \ \sin\theta_{13} \approx ab\,\lambda^3 .$$

Again the hierarchy

$$\theta_{23} >> \theta_{12} >> \theta_{13}$$

is true.

Although the mass patterns match the experimental values, as we anticipated earlier, neither Ansatz gives an entirely satisfactory mixing pattern. For instance, $\sin\theta_{12}$ is predicted to leading order to depend only on $Y^{(u)}$ entries $\lambda\,\frac{e_1}{2e_2}$, something the excludes maximal mixing as e_1, e_2 are already fixed by the quark Yukawa couplings.

For infinitesimal values of the mixing angles the mixing matrix obtained can be written to leading order as

$$\mathbf{U} = \begin{pmatrix} 1 & \theta_{12} & \theta_{13} \\ -\theta_{12} & 1 & \theta_{23} \\ -\theta_{13} & -\theta_{23} & 1 \end{pmatrix}$$

Returning to the neutrino mass formula, we may write

$$M^{(\nu)} = Y_u\, M_R^{-1}\, Y_u = \mathbf{V}\, Y_u^{(0)} \left(M_R^{(0)}\right)^{-1} Y_u^{(0)}\, \mathbf{V}^{\dagger}$$

where $Y_u^{(0)}$ and $M_R^{(0)}$ are the matrices employed in the Ansatze I, II, etc. that have led the given hierarchical eigenvalues, while **V** carries the hard component of the mixing. Then, the mixing matrice will be

$$\mathcal{U}_{PMNS} = \mathbf{V}\,\mathbf{U}$$

Assuming that

$$\mathbf{V} = \mathbf{U}(\eta_{23})\,\mathbf{U}(\eta_{12})$$

$$\mathcal{U}_{PMNS} = \mathbf{U}(\overline{\theta}_{23})\,\mathbf{U}(\overline{\theta}_{13})\,\mathbf{U}(\overline{\theta}_{12})$$

with

$$\overline{\theta}_{12} = \eta_{12} + \theta_{12}$$

$$\overline{\theta}_{13} = \sin\eta_{12}\,\theta_{23} + \cos\eta_{12}\,\theta_{13}$$

$$\overline{\theta}_{23} = \eta_{23} + \cos\eta_{12}\,\theta_{23} - \sin\eta_{12}\,\theta_{13}$$

For the particular case of *Ansatz-I*, we have

$$\sin\theta_{23} \approx \lambda\,a + \lambda^2\,\overline{a}, \quad \sin\theta_{12} \approx \lambda^3\,b\,, \quad \sin\theta_{13} \approx \lambda^4\,(c - a\,b)$$

and analogously for Ansatz-II.
No assumption has been made for the values of η_{23}, η_{12}, apart from the fact that a corresponding angle η_{13} was assumed vanishing.
Summarizing, we have shown that for particular Ansätze of the singlet couplings that determine the effective right-handed neutrino mass both the dominant part of neutrino mixing, aswell as the Cabbibo mixing, can be introduced, while the neutrino mass eigenvalues follow a hierarchical pattern parametrized by the Cabbibo parameter.

SUMMARY

REALIZATION OF SEESAW MECHANISM FLIPPED SU(5) + SINGLET SECTOR

$$\Downarrow$$

DERIVATION OF NEUTRINO MASS SCALE

ANSATZE FOR SINGLET SECTOR

$$\Downarrow$$

HIERARCHICAL NEUTRINO MASSES

$$1 : \lambda^{1,2} : \lambda^{n}$$

HIERARCHICAL NEUTRINO MIXING

$$\theta_{23} > \theta_{12} >> \theta_{13}$$