SUPERSYMMETRY
&
INFLATION

Outline of the lectures

Introduction to Cosmology

Some features of Supersymmetry

Inflation in general and in Susy

Apologies

Never underestimate the pleasure people have when they listen to something they already know

E. Fermi

Generalities I

In the last few decades Cosmology has become a Science, even at times, a precision Science. We can no longer laugh at Landau's joke:

Cosmology is often in error but seldom in doubt!

The more remarkable thing is how many aspects of the subjects are close to HEP. Findings in one field are likely to influence deeply the other.

The typical examples are Inflation and Dark Matter.

Dark energy although observed, and with a (anthropic) value, is even more perplexing in HEP.

Generalities II

Wigner's statement is very much to the point:

The unreasonable effectiveness of Mathematics in the Physical Sciences

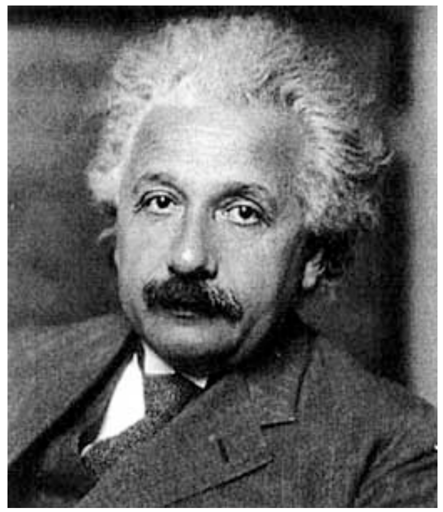
where we can apply it in particular to Cosmology

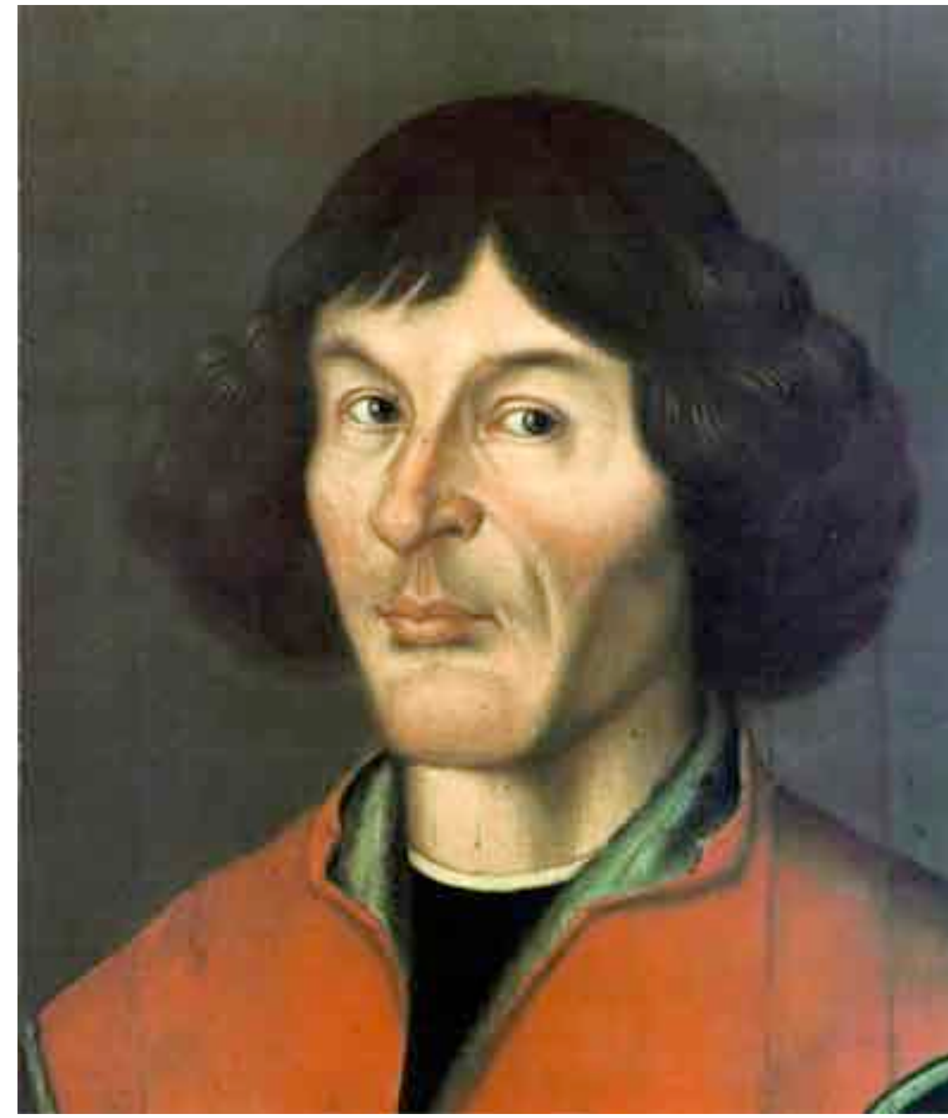
A remarkably consistent hypothesis, is that the laws of Physics are the same everywhere in the observable Universe.

As in the Athenian academy in the time of Plato: "Let no one ignorant of Geometry enter here"

GR has played a crucial role in our Understanding of the Cosmos

Heroes of Cosmology




The Cosmological Principle has turned out to work far better than expected

Human ingenuity

Human ingenuity

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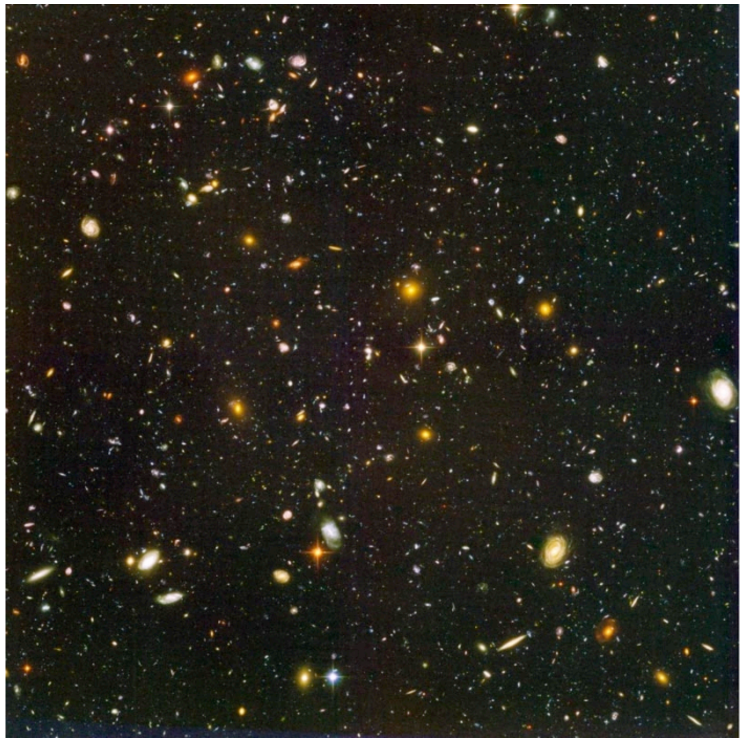
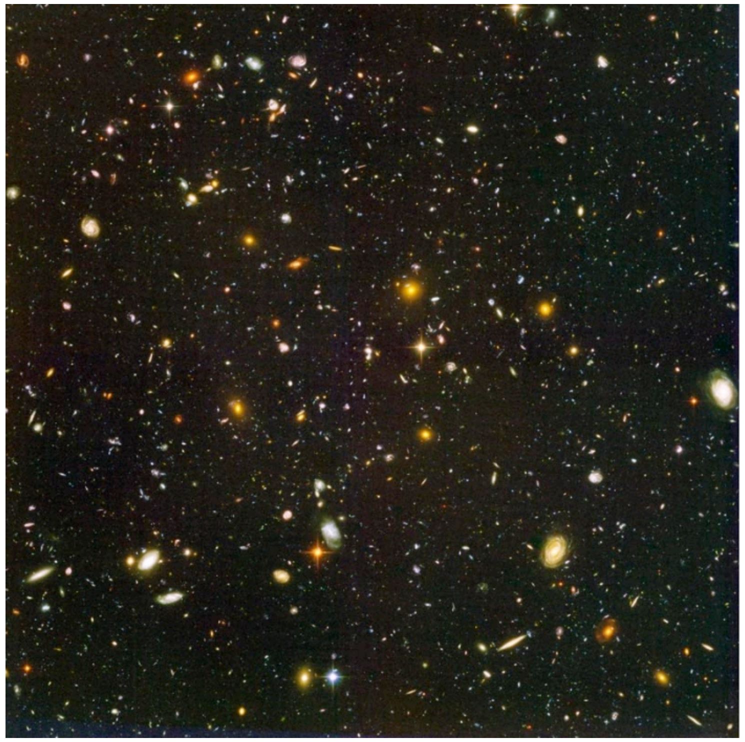
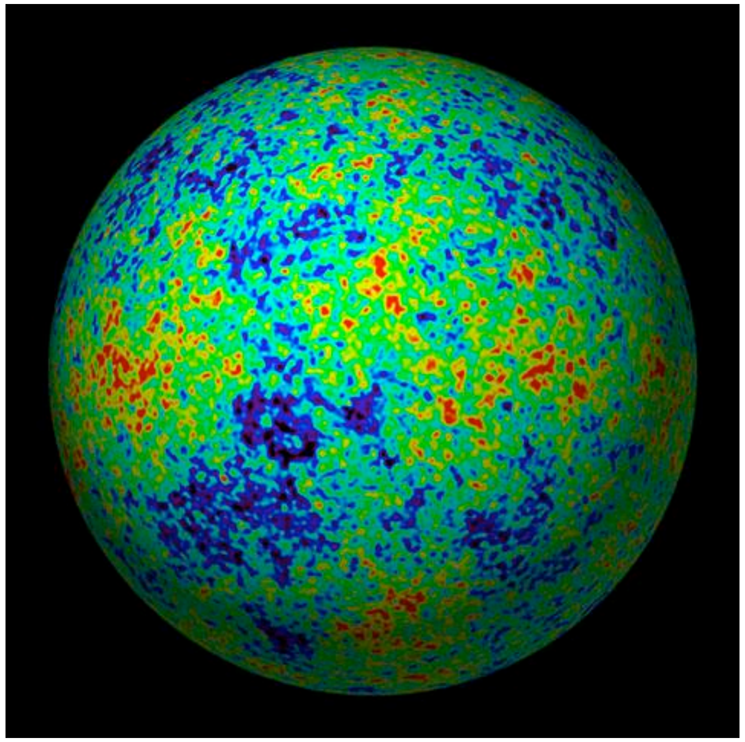
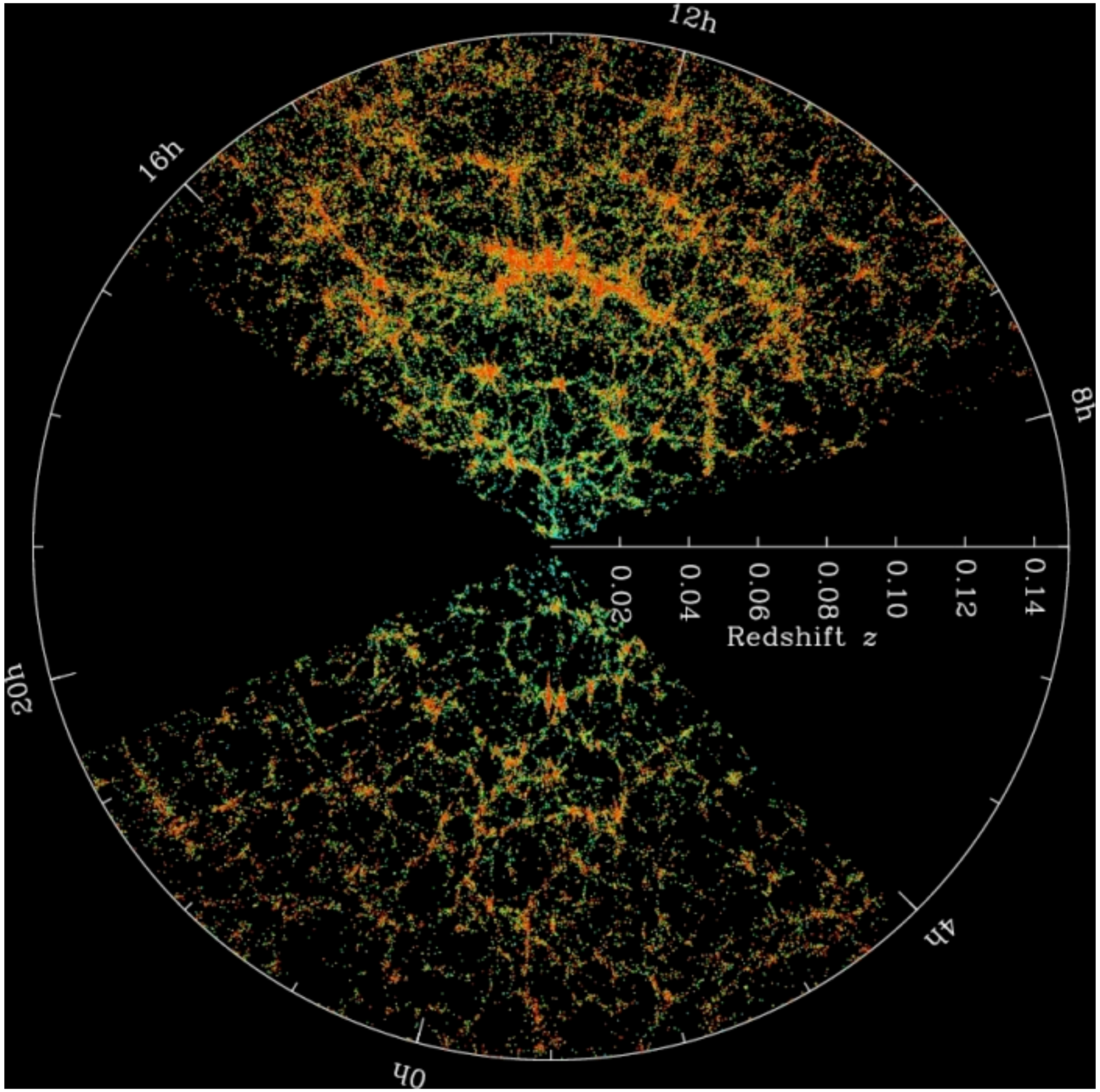


Human ingenuity





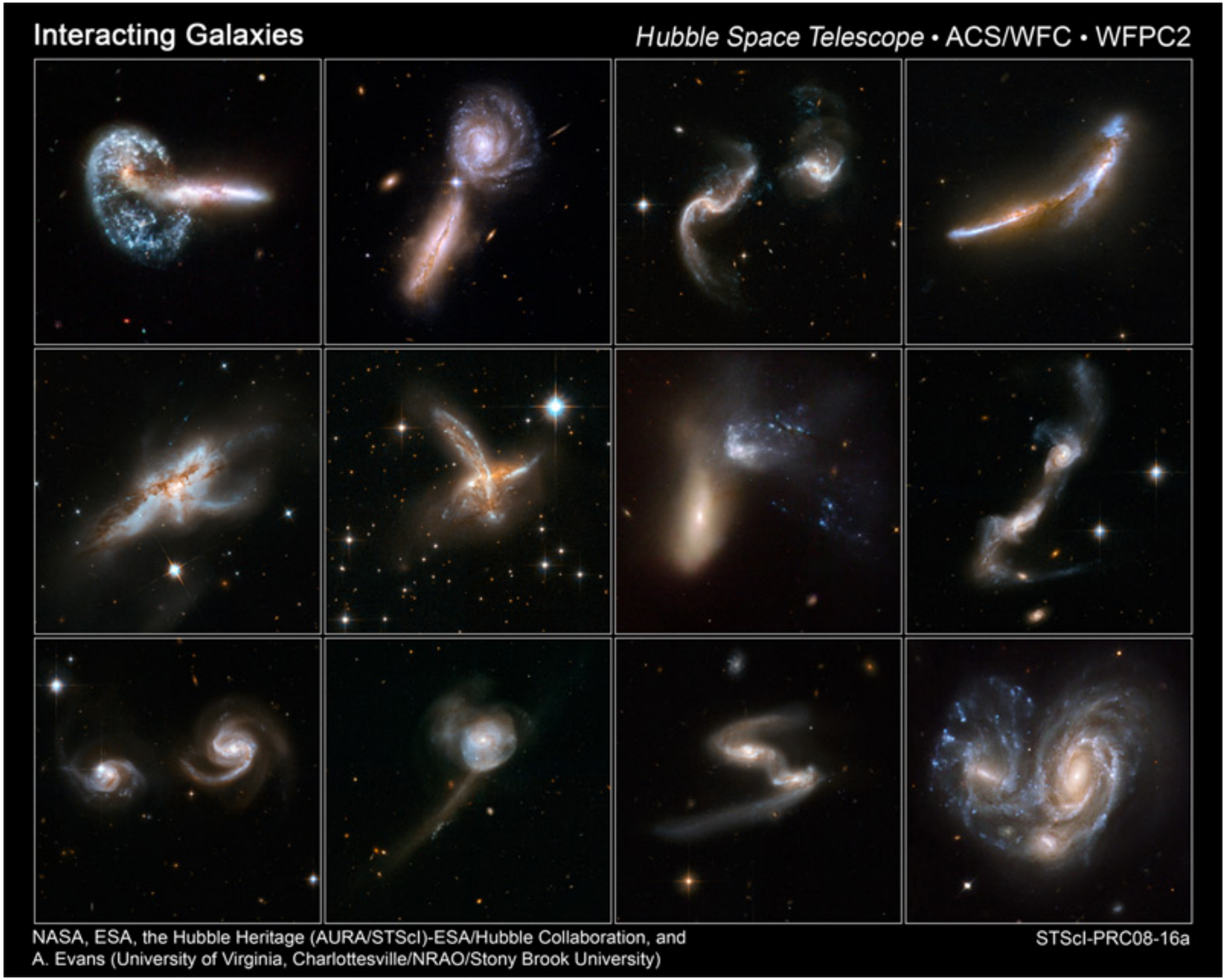
Galaxy surveys



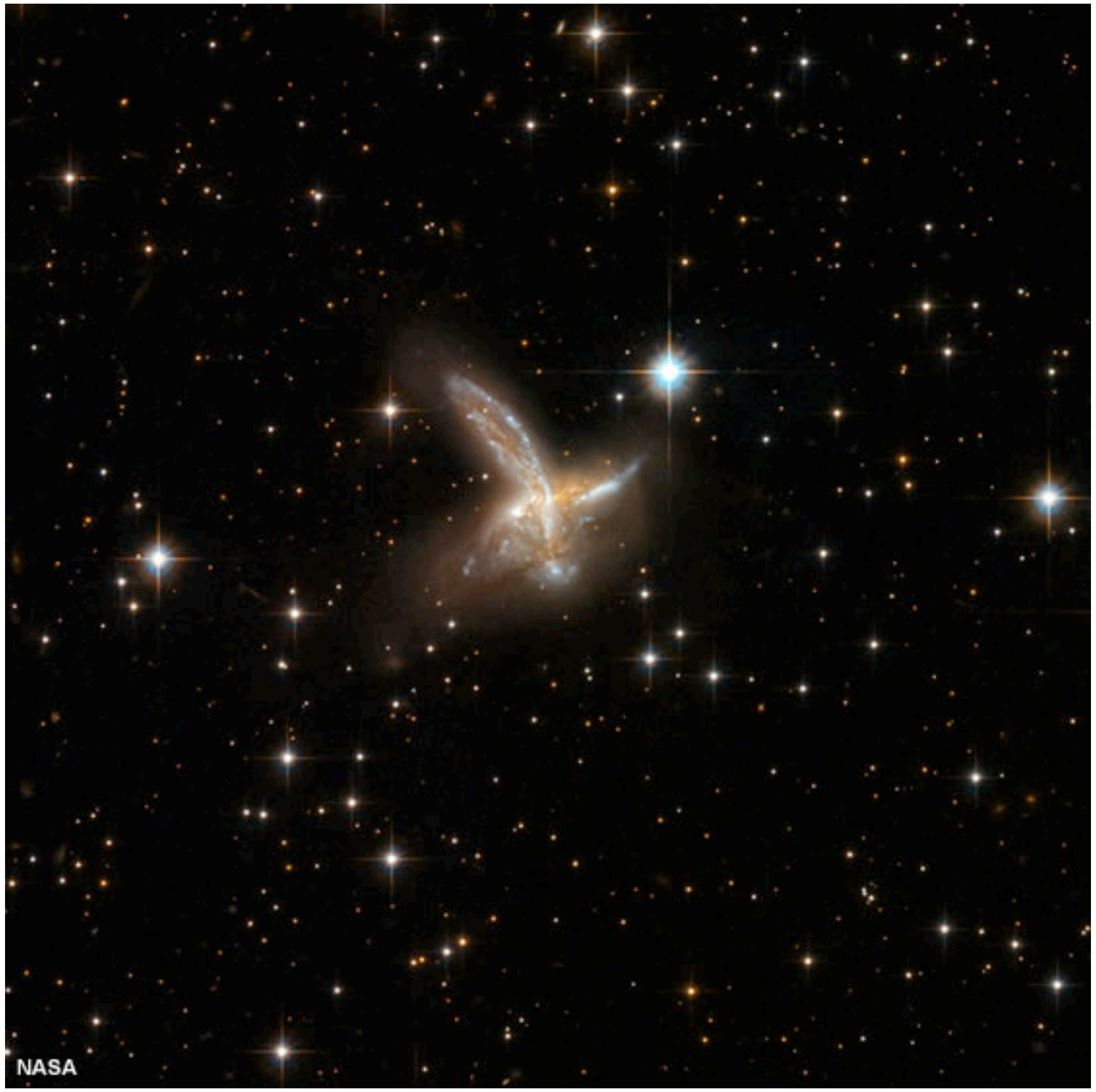
Plenty of time has elapsed, so that at low scales (intergalactic distance) nonlinearities begin to be important.

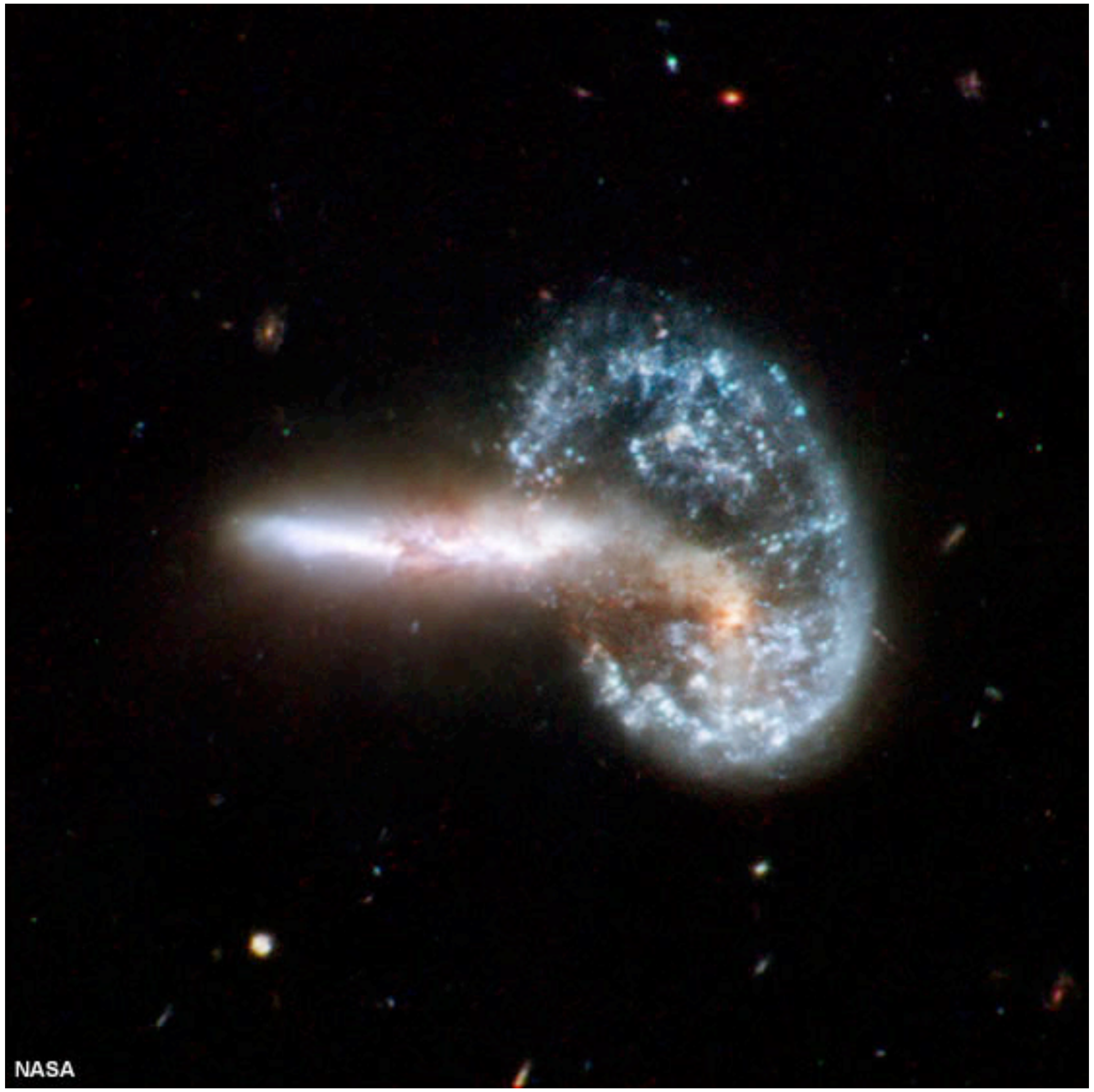
We will show later a better reason for the Cosmological Principle

Nonlinearity can be very beautiful!



Two more...



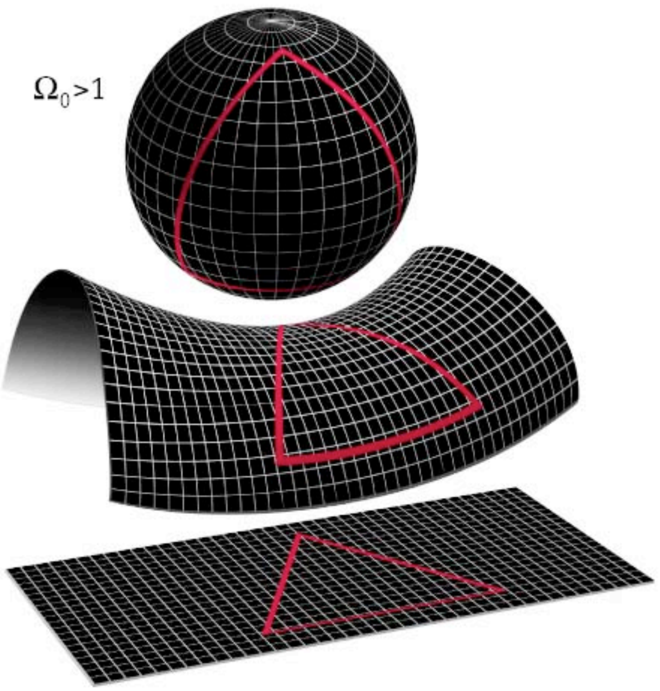


This is more the realm of Astrophysics. A nice feature of early cosmology is that we are frequently in the linear regime

FRW-I

$$ds^2 = -dt^2 + a(t)^2 \, ds_3^2 \qquad l(t) = a(t)L \qquad \frac{dl(t)}{dt} = \frac{\dot{a}}{a} l(t)$$

$$H \equiv \frac{\dot{a}(t)}{a(t)}$$ The Hubble parameter



$$H_0 = 100h \, Km \; s^{-1} Mpc^{-1} \qquad h \approx 2/3$$

Three dim. metric maximally symmetric. This is related to homogeneity and isotropy. The coordinates in the 3-space are called comoving. Imagine emitting at E, and receiving at O light of a given wavelength

$$\int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{r_e}^{r_o} ds_3, \qquad \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{dt}{a(t)} = \int_{r_e}^{r_o} ds_3$$

subtracting, and taking into account that the intervals are small:

$$\lambda_o = c\Delta t_o \qquad \lambda_e = c\Delta t_e \qquad z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \qquad \frac{\lambda_o}{\lambda_e} = 1 + z = \frac{a(t_o)}{a(t_e)}$$

Matter is well represented by a perfect fluid

$$T_{ab} = (p+\rho)u_a u_b + p g_{ab}$$

The Einstein equations are

$$G_{ab} = 8\pi G\, T_{ab}$$

$k = \pm 1, 0$ These are the possible curvatures distinguishing the space sections

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\dot\rho + 3H(\rho+p) = 0$$

$$p = w\,\rho$$

$$\dot\rho + 3\frac{\dot a(t)}{a(t)}(1+w)\rho = 0$$

$$\rho = \rho_0\left(\frac{a_0}{a}\right)^{3(1+w)}$$

$$S = \frac{1}{16\pi G}\int\sqrt{-g}(R-2\Lambda) + \int\sqrt{-g}\left(-\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right)$$

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi) \qquad\qquad p = \frac{1}{2}\dot\phi^2 - V(\phi)$$

Some consequences

$$H^2 = 8\pi G\,\rho - \frac{k}{a^2} \qquad \Omega \equiv \frac{\rho}{\rho_c} \qquad 1-\Omega = \frac{k}{a^2 H^2} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

$$\begin{aligned}
\rho_{c0} &= 1.88\,h^2 10^{-29} g\,cm^{-3} \\
&= 1.1\,h^2\,GeV\,m^{-3} \\
&= (3\,10^{-3} eV)^4\,h^2 \\
&= 2.775 h^2 10^{11} M_{Sun}\,Mpc^{-3}
\end{aligned}$$

First problem in standard cosmology. The flatness problem

$$|1-\Omega_0| \sim O(10^{-2})$$

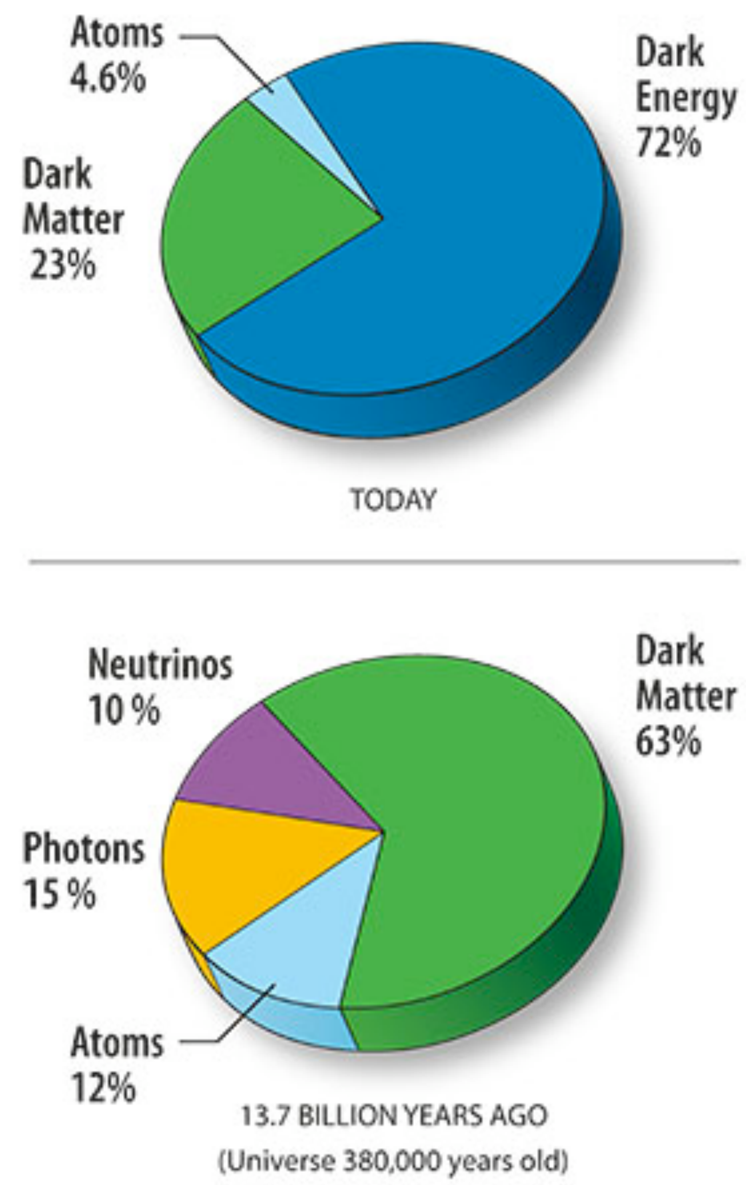
For this to be true at present, requires a remarkable fine tuning at previous times. The value 1 is unstable, and hence it requires a severe fine-tuning at early times to have the value today, unless the Universe has naturally k=0, i.e. it is flat.

Composition of the Universe

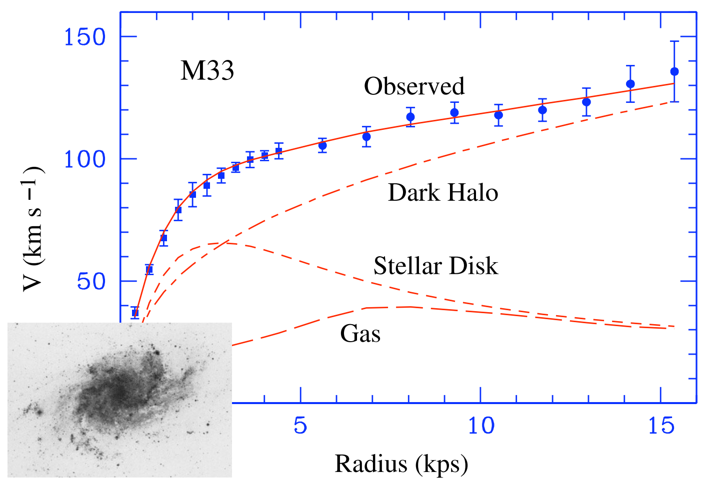
The energy density term in the FRW equations requires experimental input, that determines how energy is distributed in different epochs. The information on the composition of the Universe has required many years of observation. The last shocking news came from the supernova projects and the analysis of WMAP data.

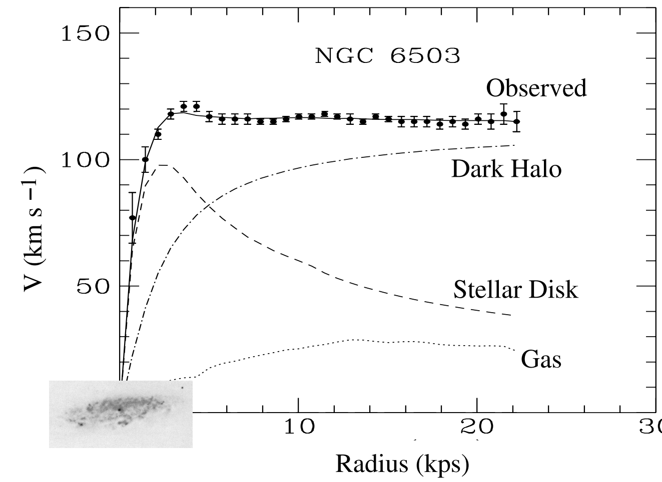
The pie charts show our current understanding of the distribution of different types of matter.

Whenever possible it is very important to have independent determinations of the different component abundances.



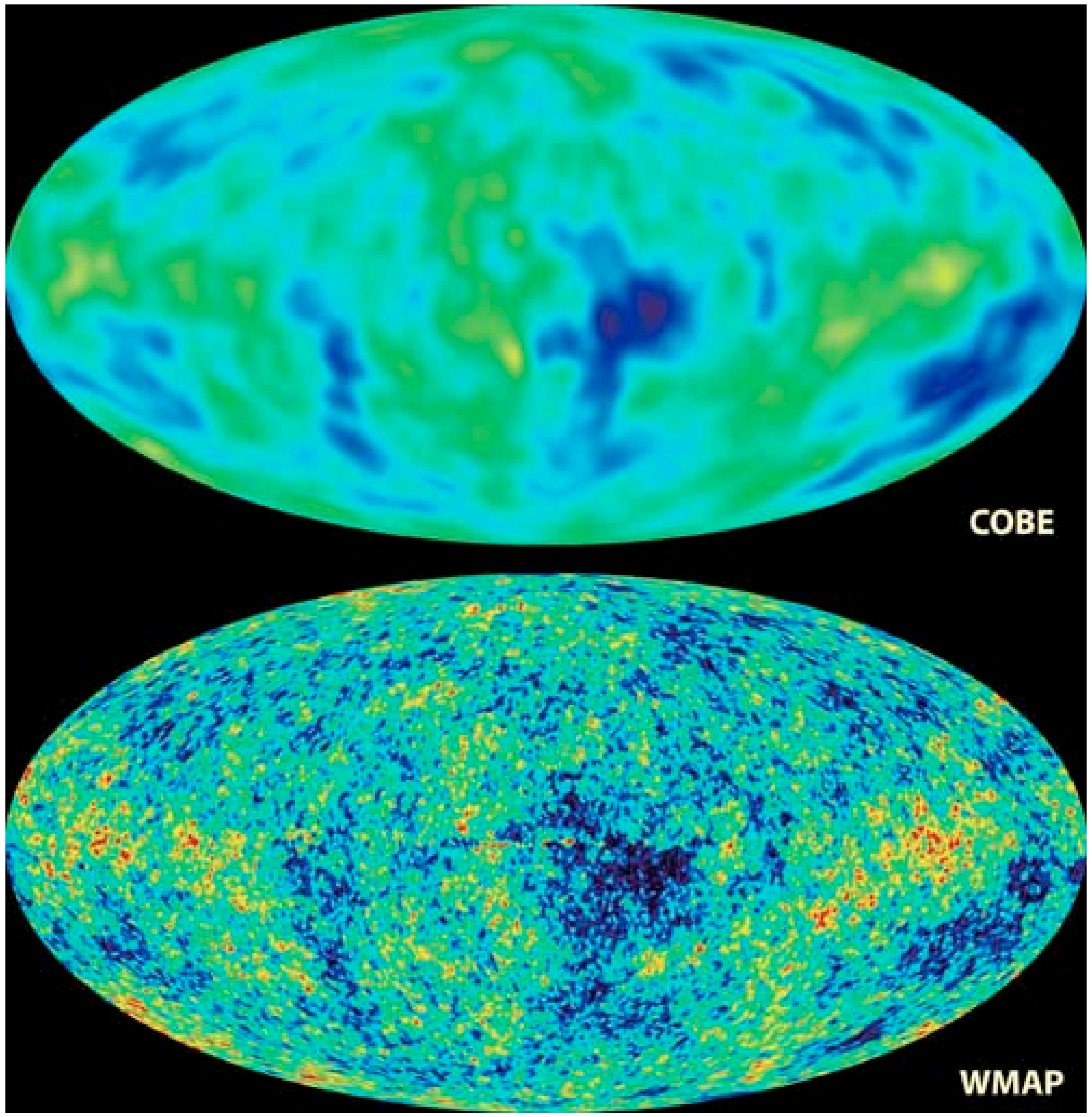
DM evidence

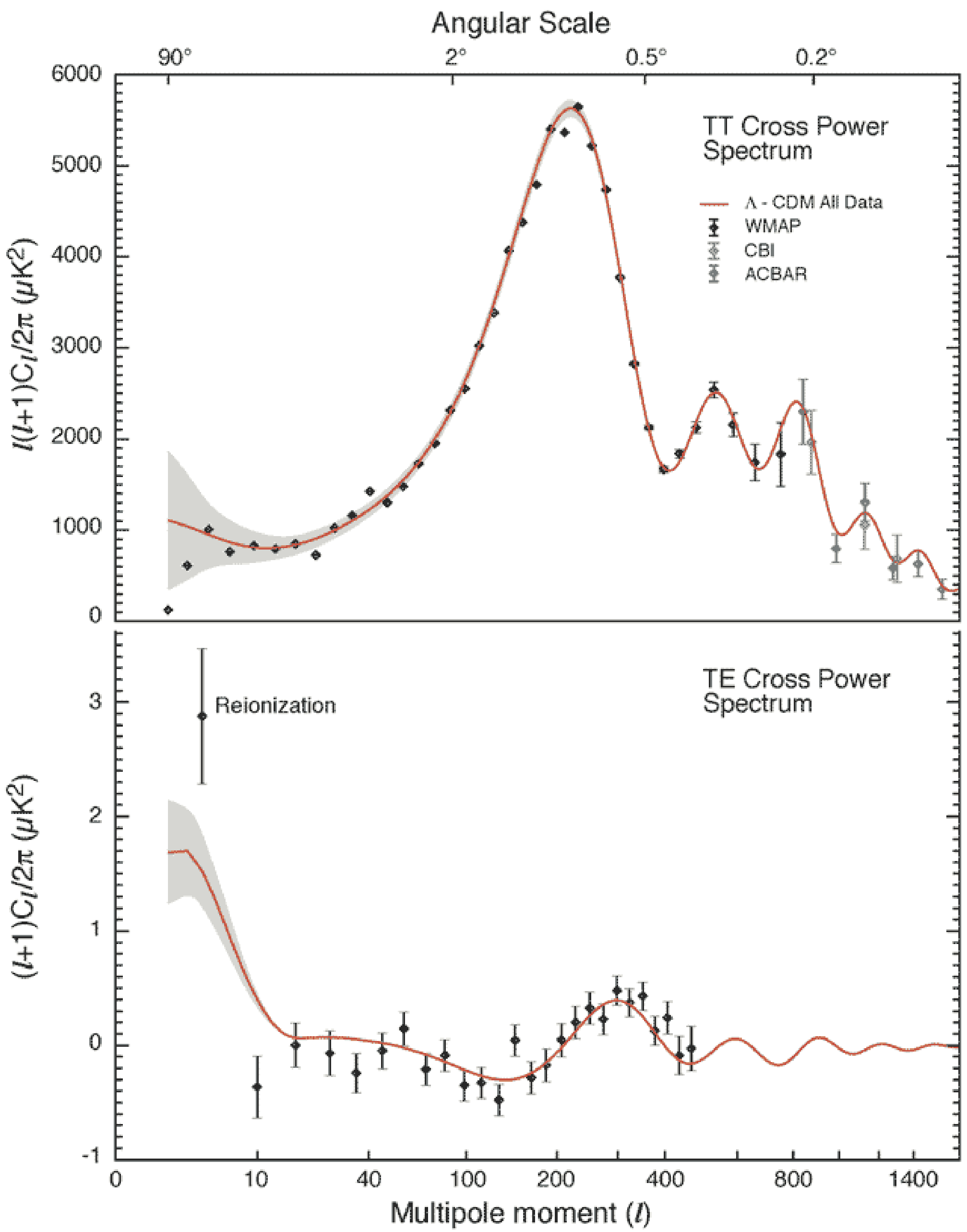




There is independent evidence for CDM (see Leszek's lecture, Bullet cluster), virial theorem in globular clusters, structure formation...

COBE + WMAP





Spectacular measurement and test of the hot BB theory. What is the origin of these fluctuation? Are they the seed for structure formation?

Temperature fluctuations

$$\langle T \rangle = \frac{1}{4\pi} \int T(\theta, \phi) \sin\theta d\theta d\phi = 2.726K$$

Challenge!

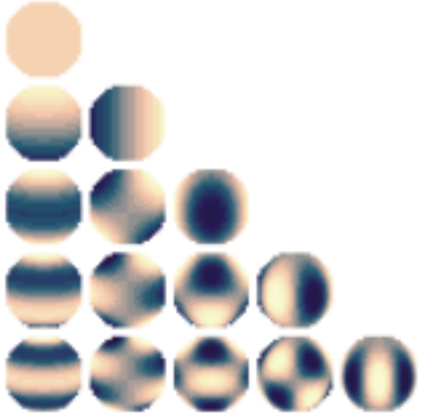
$$\frac{\Delta T}{\langle T \rangle} = \frac{\Delta T}{T}(\theta, \phi) = \frac{T - \langle T \rangle}{\langle T \rangle}$$

rms 1 in 10^5

Even bigger challenge!

Expand in spherical harmonics

$$\frac{\Delta T}{\langle T \rangle} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi)$$



$$C(\theta) = \langle \frac{\Delta T}{T}(n) \frac{\Delta T}{T}(n') \rangle = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos\theta)$$

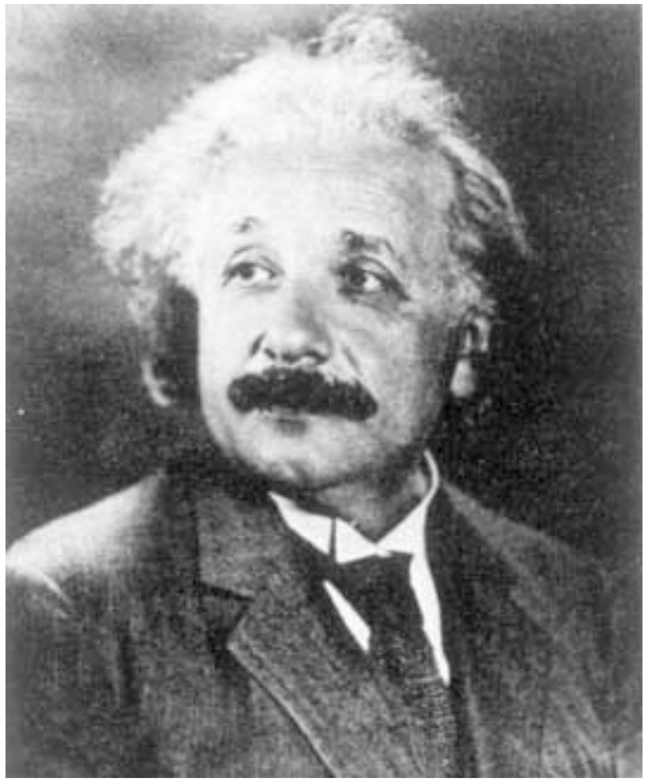
Legendre polynomials

$$C_l = \frac{1}{2l+1} \sum_l |a_{lm}|^2$$

Angular power spectrum

(Courtesy of Licia Verde)

Enter Dr. Einstein



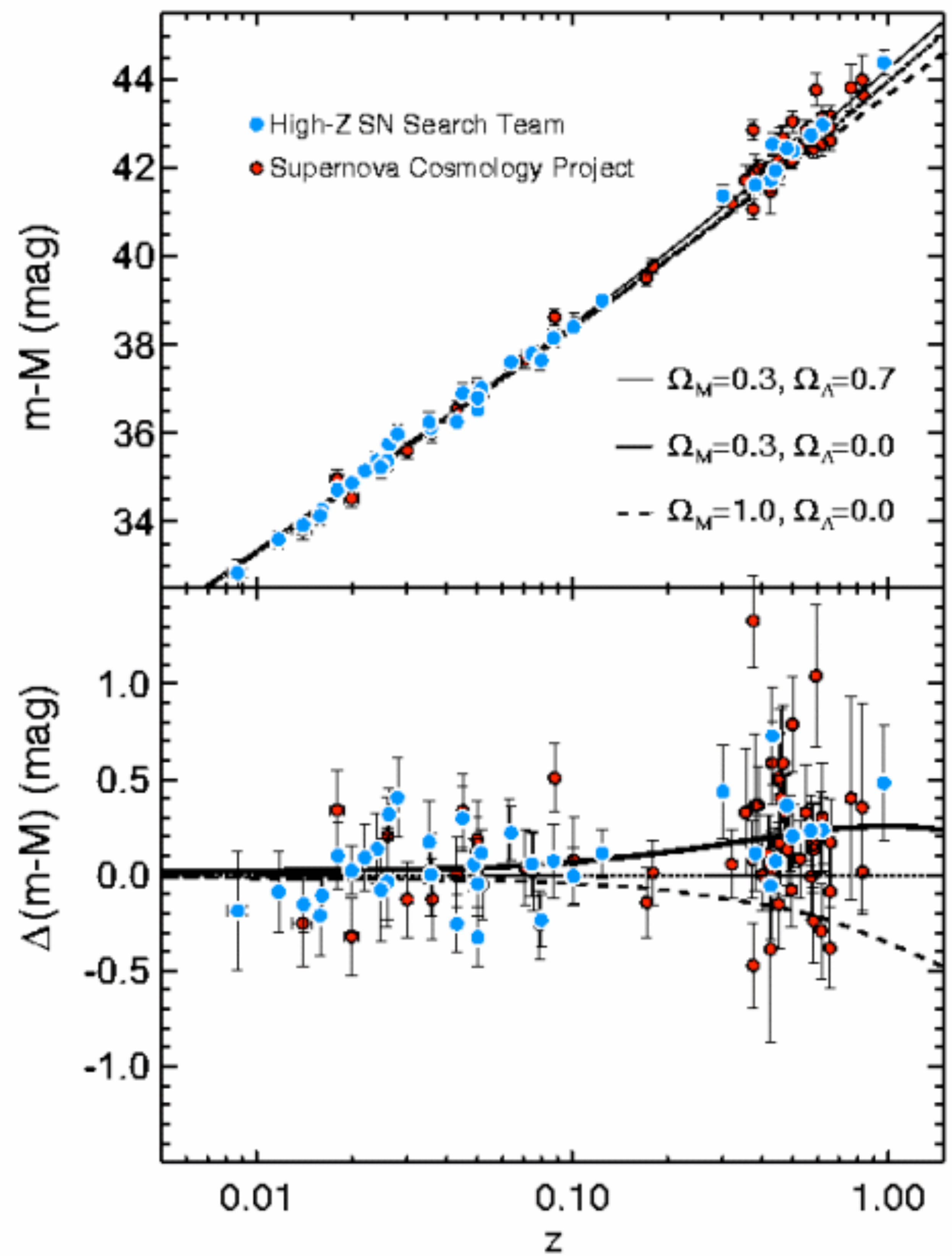
$$\Omega_\Lambda + \Omega_{DM} + \Omega_B = 1$$
$$73\% + 22.6\% + 4.4\% = 100\%$$

Could the Universe have been created out of nothing?

Independent evidence for the CC

Apart from anthropic arguments, we have no clue of why such a cosmological constant should be present

There are many contributions to it from QFT, all many orders of magnitude larger than the observed value. Why is it so small?



Hot BB scenario

The successes of the Big Bang scenario are generally accepted. The Universe evolved from a hot plasma with radiation and matter in equilibrium at early times, to a cold Universe nowadays. Along the way the model is able to explain a number of crucial things:

Primordial nucleosynthesis of light elements: H, D, He, Li,... and few more, the heavy elements like C, N, O, P, S... are cooked in stars. The primordial abundances are in good agreement with observation.

As the Universe cools, there is a value of z of the order of 1100, where we reach the surface of last scattering LSS (in reality a region) where matter decouples from radiation. The Universe becomes transparent and the radiation is decoupled and expands with a temperature decreasing at the same rate as the frequences get red-shifted.

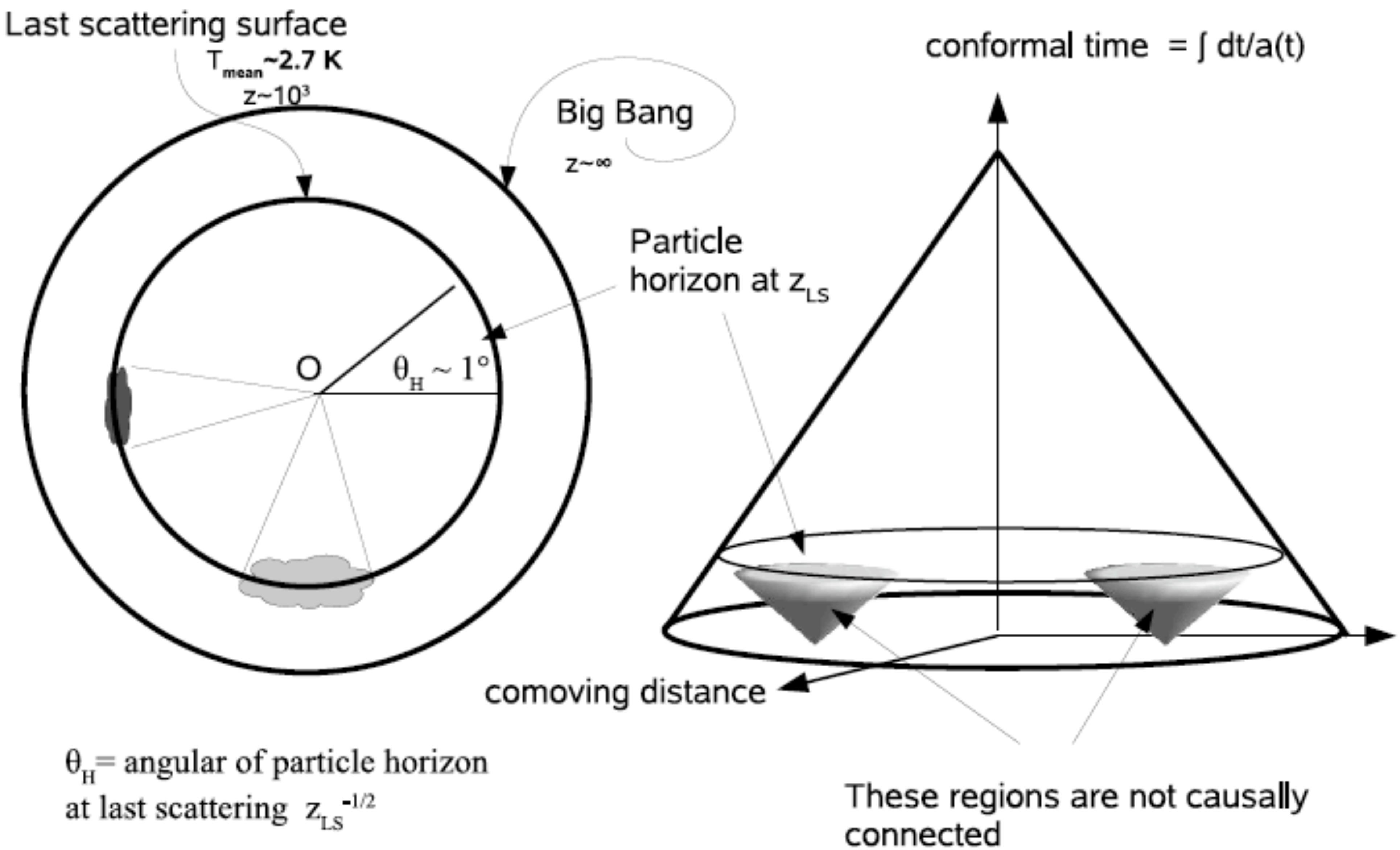
The free radiation is what we see in the CMBR, and its T-fluctuations gives as the seeds of structure formation.

It is crucial that the CDM freezes out before, and begins to collapse, so that the gravitational valleys it generated are crucial to generate sufficient fluctuations in the baryonic fluids after decoupling. Without CDM there are not enough fluctuations to generate in principle the structure we see (non-trivial simulations).

A major problem however, is the "horizon problem". In the standard scenario where the universe evolve from a hot phase, there was not way of explaining the extreme homogeneity of the CMBR.

Horizon problem

$$\frac{dt^2}{a(t)^2} - ds_3^2 = 0$$



θ_H= angular of particle horizon at last scattering $z_{LS}^{-1/2}$

(Courtesy of Michel Tytgat)

Structure formation

$$\Delta = \frac{\delta\rho}{\rho}$$

This is the density contrast. For "short" distances, it is big, as we observe. For larger scales we can still use perturbation theory and compute its dependence on z (the redshift) and cosmological parameters.

In the newtonian theory, one is led to the Jeans instability and length, which grow exponentially.

In the context of an expanding universe, the growth of instabilities is remarkably tamed to power-like behavior.

We use the fluid approximation when the contrast is <<1, hence we need four equations to proceed:

1. The continuity equations, i.e. energy conservation

2. Euler's equation.

3. A way of getting the gravitational potential from the mass distribution.

4. An equation of state relating pressure and density

Elaboration

Ignore gravity, and expansion, but not pressure, then we get the propagation of sound

$$\Delta(t) = \Delta_k(t)e^{ikt} \qquad\qquad \ddot{\Delta} + c_s^2 k^2 \Delta = 0$$

Including static gravity, and pressure:

$$\ddot{\Delta} + (c_s^2 k^2 - 4\pi G\rho)\Delta = 0 \qquad\qquad k_J = \sqrt{4\pi G\rho / c_s^2}$$

For small enough k, i.e. large distances, we get the Jeans instability with exponentially growing solutions

Next include the effect of expansion. The wave equation is:

$$\ddot{\Delta} + 2\, H(t)\dot{\Delta} + (c_s^2 k^2 - 4\pi G\rho)\Delta = 0$$

This will damp modes with k < H. The most relevant effect is that due to expansion, the gravitational attraction is reduced by expansion. Consider for instance matter domination, with the density like a^{-3}, then:

$$\Delta \sim t^{2/3} \propto a(t)$$

(Courtesy of Michel Tytgat)

Inhomogeneities of CMB

We know that the density contrast is:

$$\Delta \sim 1 \qquad \text{on scales } \mathcal{O}(10Mpc)$$

$$a(t) \sim \frac{1}{1+z}$$

From the analysis in the previous page, we expect:

$$\Delta \sim 10^{-3} \qquad \text{at } z \sim 10^{3}$$

This corresponds to red-shifts near the LSS, or recombination. For larger values of z, we have the baryon-photon fluid, with fully ionized hydrogen, Compton scattering, Coulomb scattering (e p)

Photon pressure fights gravitational attraction, and this generates acoustic oscillation of the photon fluid. The beautiful picture of the CMBR is like "seing sound". With this and much more analysis one can read the acoustic oscillations in the WMAP data.

Acoustic Oscillations ⟷ Heating and Cooling of photon fluid

Until the LSS

Inflation (at last)

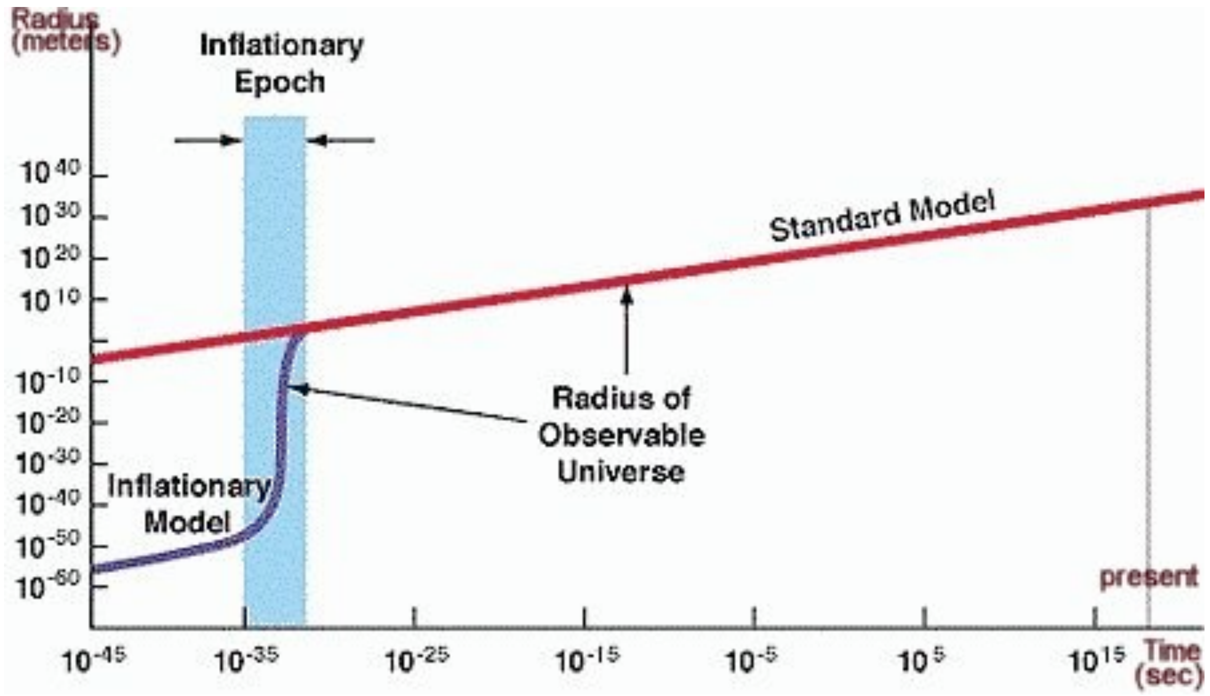
The main idea is to introduce a phase of accelerated expansion at the very early Universe.

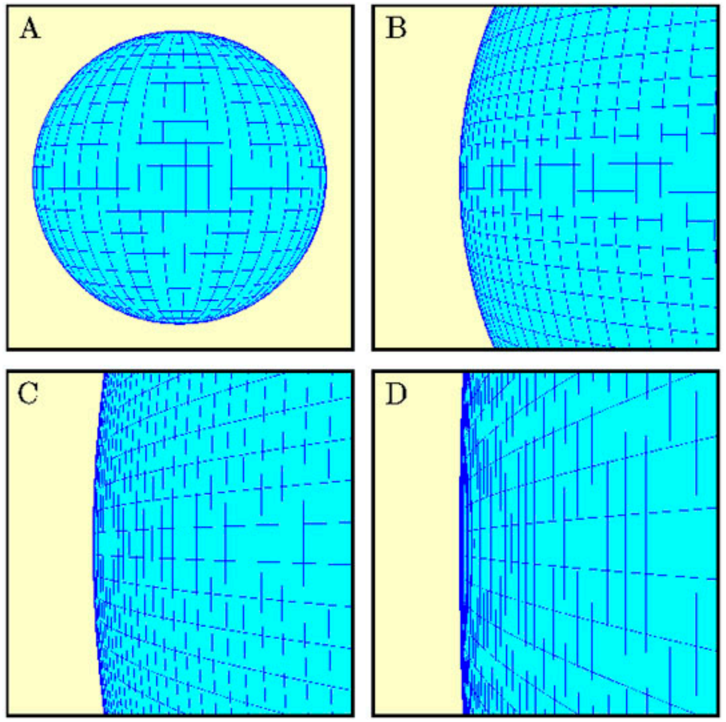
Solves the flatness problem

Solves the horizon problem

Predicts adiabatic fluctuations

Generates a near scale invariant spectrum of density fluctuations, and provides the primordial fluctuations as stretched quantum fluctuations





If you believe in GUTs...

A large number of relics could be generated that would overcome the energy density of the Universe and close it, or in any case produce something far different from what we see

Monopoles and other exotic objects... This is perhaps less severe that the horizon and flatness problems from the cosmological/HEP point of view. We like unification, but there is no reason why Nature should fulfill our wishes...

Accelerating the Universe

We need to violate the dominanat energy condition for a sufficiently long time.

If we take during inflation H approx. constant, the number of e-foldings:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$p < -\frac{1}{3}\rho \quad \Rightarrow \quad \boxed{\ddot{a}(t) > 0}$$

$$|1-\Omega| \propto \dot{a}^{-2}$$

$$N = \log\left(\frac{a(t_f)}{a(t_i)}\right) = H(t_f - t_i)$$

$$|1-\Omega(t_f)| = e^{-2N}|1-\Omega(t_i)|$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

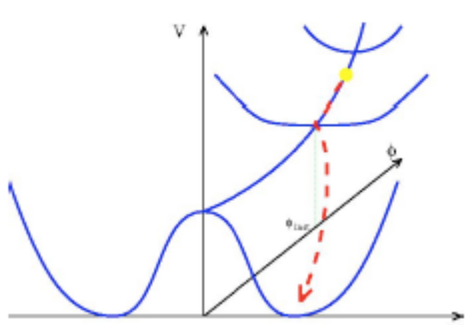
Slow roll paradigm $\quad \dot{\phi}^2 << V(\phi)$

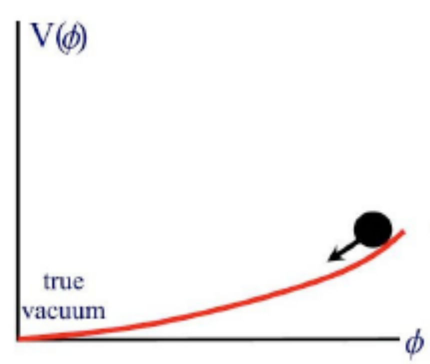
Number of e-foldings could be 50-100, making a huge Universe.

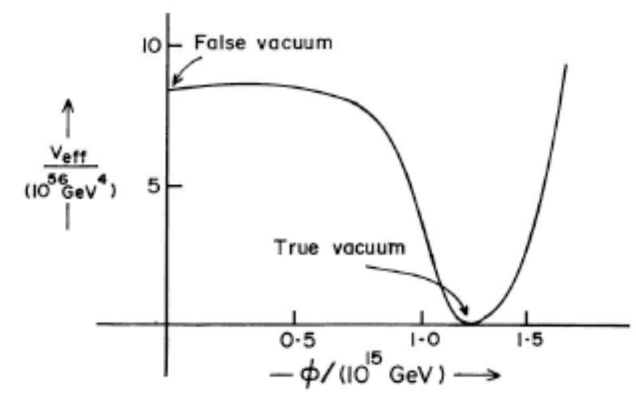
We can use QFT to construct some models

$$S = \frac{1}{16\pi G}\int\sqrt{-g}(R-2\Lambda) + \int\sqrt{-g}\left(-\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right)$$

$$\frac{V''}{V} << 1, \qquad \frac{V'}{V} << 1$$







(Courtesy of Licia Verde)

Slow roll, more details

$$H^2 = \frac{8\pi G}{3}\rho \qquad H = \frac{\dot{a}}{a} \qquad \rho \approx V(\phi)$$

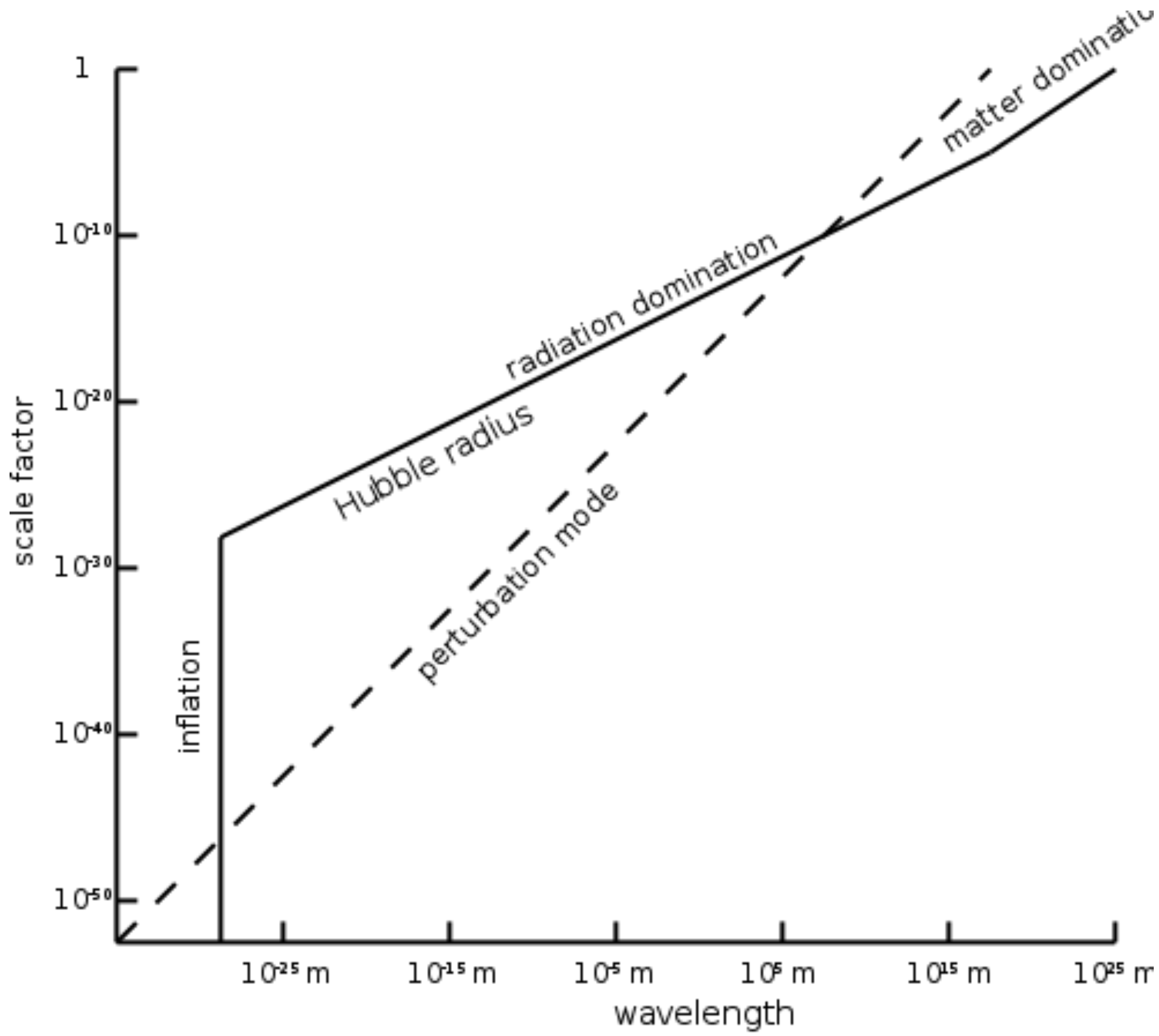
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad \rightarrow \quad 3H\dot{\phi} \approx -V'(\phi) \qquad \epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = M_{Pl}^2 \frac{V''}{V}$$

During inflation, it is a good approximation to treat the inflaton field as moving in backgroun De-Sitter space, with H constant. The quantum fluctuations of the inflaton are the source of primordial inhomogeneities. The existence of this quantum fluctuations are similar to the Hawking radiation in dS-space. Schematically the picture shows what happens to the fluctuations. Remember that in dS there is a particle horizon 1/H

$$\langle \phi(x)^2 \rangle = \frac{H^2}{2\pi}$$

$$L_{phy} = a(t)\, L_{com}$$

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$



Power spectrum

In flat De Sitter coordinates we can Fourier transform F.T. To compute the power spectrum, we quantize the field in dS space. Then, expanding the field in oscillators:

$$\mathcal{P}_\phi(k) = V \frac{k^3}{2\pi^2} \langle \phi(k)\, \phi(k)^\dagger \rangle \qquad \langle\, \phi(x)^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\phi(k)$$

Precisely: $$\mathcal{P}_\phi(k) = \left(\frac{H}{2\pi}\right)^2_{aH=a_0 k}$$

Finally, the curvature perturbations can be computed using the perturbed Einstein equations, and a number of laborious and subtle steps, we get the power spectrum for the induced density perturbations (at horizon exit)

$$\mathcal{P}_\mathcal{R}(k) = \left[\left(\frac{H}{\dot\phi}\right)\left(\frac{H}{2\pi}\right)\right]^2_{aH=a_0 k} = \frac{1}{4\pi^2}\left(\frac{H^2}{\dot\phi}\right)^2_{t=t_k}$$

Using the slow-roll equations and defining the spectral index:

$$\frac{d\log \mathcal{P}_\mathcal{R}(k)}{d\log k} \equiv n(k) - 1$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{1}{12\pi^2}\frac{1}{M_{Pl}^6}\frac{V^3}{V'^2} = \frac{1}{24\pi^2}\frac{1}{M_{Pl}^4}\frac{V}{\epsilon}$$

$$\mathcal{P}_\mathcal{R}(k) \sim (\frac{k}{k_p})^{n-1}$$

n=1 is Harrison-Zeldovich

BREATHE!!

Supersymmetry and its breaking

In the standard treatment of global supersymmetry the order parameter of supersymmetry breaking is associated with the vacuum energy density. More precisely, in local Susy, the gravitino mass is the true order parameter.

Having a vacuum energy density will also break scale and conformal invariance.

Witten's index is a way of computing if SUSY can be broken. Obviously in models with a nonzero index, the rigid vacuum is supersymmetric. The index has been useful in analyzing field theories, and also in showing that in some cases the claims of non-perturbative breaking of the symmetry were not possible.

In the models where we have breaking built in, the index is always zero.

When supergravity is included the breaking mechanism is more subtle, and the scalar potential far more complicated.

$$\{Q, \bar{Q}\} = 2\sigma^\mu P_\mu$$

$$\frac{1}{4} \langle 0| \,\mathrm{tr}\{Q, \bar{Q}\}\, |0\rangle = \langle 0|H|0\rangle$$

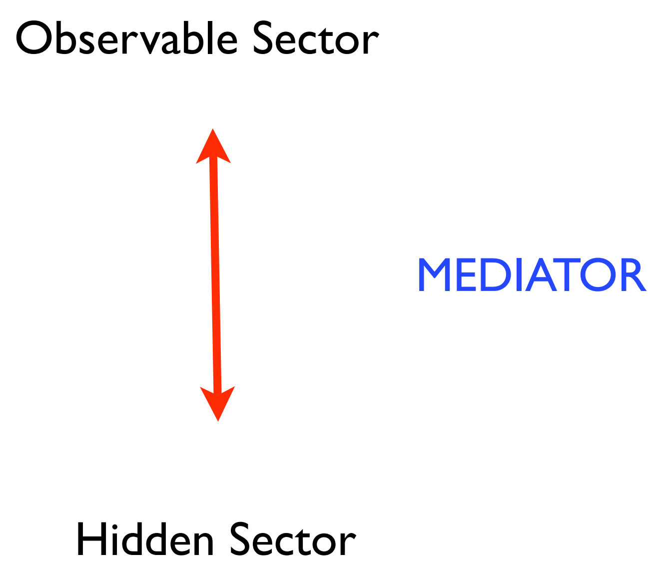
$$\Delta \;=\; \mathrm{tr}\,(-1)^F$$

SSB Scenarios

Gauge mediation

Gravity mediation

Anomaly mediation



It is normally assumed that SSB takes places at scales well below the Planck scale. The universal prediction is then the existence of a massless goldstino that is eaten by the gravitino. However in the scenario considered, the low-energy gravitino couplings are dominated by its goldstino component and can be analyzed also in the global limit.

This often goes under the name of the Akulov-Volkov lagrangian, or the non-linear realization of SUSY

Flat directions

One reason to use SUSY in inflationary theories is the abundance of flat directions. Once SUSY breaks most flat directions are lifted, sometime by non-perturbative effects. However, the slopes in the potential can be maintained reasonably gentle without excessive fine-tuning.

Apart from flat directions in the original UV fields, in String-like theories one often encounters moduli fields with flat or nearly flat directions, where inflation can take place, and in some cases it is only instanton effects who do that. Recall the no-scale models...

$$G \to G^c$$

There is a theorem (ma non troppo) which holds in many general circumstances which implies the existence of flat directions. If a SUSY theory is invariant under a given group, its potential is invariant under its complexification. This automatically implies the existence of flat directions, pseudo-goldstone bosons etc. Nice directions to inflate. The first paper on SUSY and inflation carried out the title: Inflations cries out for Supersymmetry.

Most models of supersymmetric inflation are hybrid models (multi-field models, chaotic, waterfall...)

In a remarkable recent paper, Komargodski and Seiberg have provided simple and elegant way to understand supersymmetry breaking, and how to construct systematically the low-energy goldstino couplings. (0907.2441).

The literature on these AV-type actions is rather large and very technical. The effective lagrangian formulation in the above paper provides also simple and practical ways of obtaining the consequences of supersymmetry breaking.

The starting point of their analysis is the Ferrara-Zumino (FZ) multiplet of currents that contains the energy-momentum tensor, the supercurrent and the R-symmetry current

$$J_\mu = j_\mu + \theta^\alpha S_{\mu\alpha} + \overline{\theta}_{\dot\alpha} \overline{S}^{\dot\alpha}_\mu + (\theta \sigma^\nu \overline{\theta})\, 2T_{\nu\mu} + \dots$$

$$\overline{D}^{\dot\alpha} J_{\alpha\dot\alpha} = D_\alpha X$$

$$X = x(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\psi_\alpha = \frac{\sqrt{2}}{3} \sigma^\mu_{\alpha\dot\alpha} \overline{[illegible]}^{\dot\alpha}_\mu \qquad [illegible] = \frac{2}{3}[illegible] + [illegible]_\mu [illegible]^\mu$$

General Lagrangian

$$S=\int d^4\theta K(\Phi^i,\bar{\Phi}^{\bar{i}}) + \int d^2\theta W(\Phi^i) + \int d^2\bar{\theta}\bar{W}(\bar{\Phi}^{\bar{i}})$$

$$J_{\alpha\dot{\alpha}} = 2g_i(D_\alpha\Phi^i)(\bar{D}_{\dot{\alpha}}\bar{\Phi}) - \tfrac{2}{3}[D_\alpha,\bar{D}_{\dot{\alpha}}]K + i\partial_\alpha(Y(\Phi) - \bar{Y}(\bar{\Phi}))$$

$$X = 4W - \frac{1}{3}\overline{D}^2 K - \frac{1}{2}\overline{D}^2 Y(\Phi)$$

X is a chiral superfield, microcopically it contains the conformal anomaly (the anomaly multiplet), hence it contains the order parameter for SUSY breaking as well as the goldstino field.

The key observation is:

$$X \to X_{NL}$$

$$UV \to IR$$

$$X_{NL}^2 = 0$$

$$SPoincare/Poincare$$

Some consequences

$$L = \int d^4\theta \, X_{NL}\overline{X}_{NL} + \int d^2\theta \, f \, X_{NL} + c.c.$$

$$X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F$$

This is precisely the Akulov-Volkov Lagrangian

Coupling goldstinos to other fields

We can have two regimes of interest. Recall that a useful way to express SUSY breaking effects in Lagrangians is the use of spurion fields. The gluino mass can also be included...

$$m_{soft} << E << \Lambda$$

The goldstino superfield is the spurion

$$\int d^4\theta \left| \frac{X_{NL}}{f} \right| m^2 \, Q e^V \overline{Q} + \int d^2\theta \frac{X_{NL}}{f} (B\, Q\, Q + A Q\, Q\, Q\,) + c.c.$$

$$E << m_{soft}$$

Integrate out the massive superpartners adding extra non-linear constraints

$$X_{NL}^2 = 0, \qquad X_{NL}\, Q_{NL} = 0$$

For light fermions, and similar conditions for scalars, gauge fields,...

Our proposal

Our suggestion is to try and build up a realistic (in the sense of compatibility with the observational constraints) inflationary scenario based on the previous analysis of supersymmetry breaking. We make the following assumptions:

We will identify in the UV the inflaton field with the scalar component of the X superfield. We do not need to think of the inflaton as any extra fundamental field. The field X can be identified independently of how SUSY is broken. This is different from many scenarios in the literature. We identify the vacuum energy driven inflation with the SUSY breaking order parameter. We will assume f << M^2, inflation takes place at energies well below the Planck scale. This condition forces initial conditions for slow roll for x much smaller than Planck.

$$\sqrt{f} << E << M_{Pl}$$

We identify the slow roll with the IR flow of the X-superfield into the non-linear goldstino superfield X_{NL}. A way to model the flow is to start with the simplest dynamics for X in the UV:

$$W = c + fX$$

The canonical Kahler potential gets quantum corrections

$$X\overline{X} + \frac{c_1(X\bar{X})^2}{\Lambda^2} + \frac{c_2(X^3\bar{X}^3 + cc)^2}{\Lambda^2} + O(\frac{1}{\Lambda^3})$$

where we include the scale defining the UV completion of the theory. The low energy IR superfield is defined by integrating out the scalars.

Since we are trying to identify the slow roll with the flow of X to the IR, the corrected scalar potential has to satisfy the conditions relative to the slow roll. If inflation starts well below the Planck scale, it may not require much fine tuning in the parameters c1, c2 . Inflation will end when the X field approaches its minimum. This lead to the cosmological constant problem. This could be solved at the end of inflation, tuning the SUGRA potential

$$\frac{V''}{V} << 1, \qquad \frac{V'}{V} << 1$$

$$V_{SUGRA} = e^{K/M^2}\left(G^{-1}\, DW\overline{D}W - \frac{3}{M^2}|W|^2\right)$$

At the end of inflation the X-field reaches the stable vacuum, the inflaton becomes a two goldstino state. In the IR, reheating is driven by the interactions of the goldstino superfield to matter, written before. In SUGRA models one has to be careful with the "eta" problem.

We have looked at the numerics of simple examples to assess the level of fine tuning to fit current cosmological data, as well as what are the generic predictions of the simplest models based on the assumptions above.

Features of Inflation

Theory of initial conditions: Horizon, flatness, monopoles...

Theory of density perturbations and irregularities, the most promising to vindicate inflation.

It seems our current paradigm with some clear predictions.

We are still missing a derivation from 1st principles, or rather some "natural" model of inflation without fine tuning and other unnatural patterns.

The number of possible scenarios, including also strings, branes, ekpyrotic etc is enormous. Each of them has to provide an inflaton with its potential together with a graceful exit and enough reheating.

If one is more ambitious, then one also wants to solve the baryogenesis-leptogenesis problem, make sure that BBN is not affected, get some additional prediction on the polarization of the CMB etc.

This requires an UV theory, an also some detail Particle Theory. Most models of supersymmetric inflation are hybrid models (multi-field models, chaotic, waterfall...)

Single field inflation

Hybrid inflation

Multifield inflation

Chaotic inflation

Radion ...

Moduli fields ...

Tachyon fluids...

Pseudogoldstone bosons

Guth et al
Copeland et al.
Dvali et al.
Ross Sarkar et al.
and many more!

Thank you