Towards particle physics models from fuzzy extra dimensions

Athanasios Chatzistavrakidis
National Technical University and NCSR Demokritos, Athens

Joint work with H.Steinacker and G.Zoupanos
"Particles and Cosmology", Warsaw, February 2010

Overview

Motivation

Orbifolds of $\mathcal{N} = 4$ SYM

Fuzzy sphere and fuzzy gauge theory

Dynamical generation of (twisted) fuzzy sphere

Particle physics models

Conclusions

Motivation

Unification of fundamental interactions

Standard Model

- Remarkably successful...
- ...but it contains large number of free parameters
  - mainly related to the Higgs and Yukawa sectors

Exciting proposal: Extra dimensions may exist in nature and unification might be achieved in higher dimensions

Dimensional Reduction to four dimensions $\rightsquigarrow$ try to make contact with low energy phenomenology

Motivation

Starting from an $\mathcal{N}=1$ susy theory in higher dimensions and performing a toroidal reduction $\rightsquigarrow$ $\mathcal{N}=4$ susy in 4D

In order to achieve $\mathcal{N}=1$ susy in four dimensions...

- ...use appropriate manifolds to describe the extra dims (e.g. Calabi-Yau, $SU(3)$-structure manifolds)
- ...use orbifold techniques to project $\mathcal{N}=4$ SYM to a theory with less susy (Kachru-Silverstein '98)

Here: starting from $\mathcal{N}=4$ SYM study the possibility to reveal vacua which...

- ...dynamically develop fuzzy extra dimensions (i.e. matrix approximations to smooth manifolds)
- ...could be realistic

Motivation

Important results from the dimensional reduction of higher-dimensional gauge theories on fuzzy manifolds (Aschieri,Madore,Manousselis,Zoupanos '04,'05):

- Appearance of non-abelian gauge theories in four dimensions starting from an abelian gauge theory in higher dimensions.
- Renormalizability of the theory both in four as well as in higher dimensions.

Difficulty: Chiral Fermions.

In order to further justify the renormalizability and try to obtain chiral theories...

...Reverse approach:

- Start with 4D renormalizable gauge theory with appropriate content of scalars.
- find minima of the potential and determine vacua where fuzzy extra dimensions are dynamically generated.

Inclusion of fermions $\rightsquigarrow$ mirror models.

Further step in order to achieve chirality: Perform orbifold projection...

$\mathcal{N} = 4$ SYM theory

- Gauge group: $SU(3N)$
- Spectrum:
  - Gauge fields $A_\mu, \mu = 1, \ldots, 4$
  - 6 real scalars ϕ_a (or 3 complex $\phi_i, i = 1, 2, 3$)
  - 4 Majorana fermions $\psi_p, p = 1, \ldots, 4$
- Also, global $SU(4)_R$ R-symmetry:
  - gauge fields $\to$ singlets
  - real scalars $\to$ in **6**
  - fermions $\to$ in **4**
- Interactions encoded in the superpotential:

$$W_{\mathcal{N}=4} = Tr(\epsilon_{ijk}\Phi^i\Phi^j\Phi^k),$$

where Φ^i the three adjoint chiral supermultiplets.

Orbifolds

Consider the discrete group $\mathbb{Z}_3$ as subgroup of $SU(4)_R$

- maximal embedding in $SU(4)_R \rightsquigarrow$ no susy
- embedding in $SU(3)$ subgroup $\rightsquigarrow \mathcal{N} = 1$ susy
- embedding in $SU(2)$ subgroup $\rightsquigarrow \mathcal{N} = 2$ susy

Here we discuss the case of $\mathcal{N} = 1$ models.

$\mathbb{Z}_3$ acts non-trivially on the various fields, depending on their transformation properties under the R-symmetry and the gauge group.

Orbifold projection: keep the fields which are invariant under the (combined) $\mathbb{Z}_3$ action.

The projected theory has the following spectrum:

- Gauge group: $SU(N) \times SU(N) \times SU(N)$.
- Complex scalars and fermions ($\mathcal{N} = 1$ susy):
$$(N, \overline{N}, 1) + (\overline{N}, 1, N) + (1, N, \overline{N}).$$
  Chiral representations...
- Three families.

The projected superpotential has the same form as before, encoding the interactions among the surviving fields in the projected theory.

Search for vacua of the projected theory which can be interpreted as dynamically generated fuzzy extra dimensions.

The fuzzy sphere

A matrix approximation of the usual sphere (Madore '91).

- Ordinary sphere: $S^2 \subset \mathbb{R}^3, \quad x_1^2 + x_2^2 + x_3^2 = R^2$.
- fuzzy sphere S_N^2 = the noncommutative manifold with coordinate functions X_i being $N \times N$ matrices proportional to the generators of $SU(2)$ in the N-dim representation.
  - $X_i X_i = R^2$
  - Commutation relations: $[X_i, X_j] = i\epsilon_{ijk} X_k$
  - In the limit $N \to \infty$ the usual sphere is retrieved.

Differential calculus

In order to be able to define derivatives we need the appropriate differential calculus.

Important property: the action of $SU(2)$ remains intact $\rightarrow$ natural differential calculus.

Derivations of a function f along X_i: $e_i(f) = [X_i, f]$.

In the $N \rightarrow \infty$ limit the derivations become $e_i = \epsilon_{ijk} x_j \partial_k$
($\equiv$ angular momentum operators).
x : the commutative coordinates.
∂ : the ordinary partial derivative.

Gauge theory

Gauge fields can be naturally defined on the fuzzy sphere (Madore,Schraml,Schupp,Wess '00).

- Consider field $\phi(X_i)$, $X_i =$ the noncommuting coordinates...
- ...and infinitesimal gauge trans'n: $\delta\phi(X_i) = \lambda(X_i)\phi(X_i)$.
- Notice that: $\delta(X_i\phi) = \delta X_i\phi + X_i\delta\phi = X_i\lambda\phi \neq \lambda X_i\phi$
  Not a covariant operation.
- Covariance suggests the introduction of "covariant coordinates"

$$\phi_i = X_i + A_i$$

(as in ordinary gauge theory $\rightarrow$ covariant derivative).

- A_i is naturally interpreted as the gauge potential of noncommutative gauge theory, since it transforms as $\delta A_i = -[X_i, \lambda] + [\lambda, A_i]$
- The covariant field strength has the form

$$F_{ij} = [\phi_i, \phi_j] - \epsilon_{ijk}\phi_k$$

and it can be used to define the gauge theory on the fuzzy sphere

$$\mathcal{L} = -\frac{1}{4} F_{ij}^{\dagger} F_{ij}.$$

Twisted fuzzy sphere

Variant of the fuzzy sphere, compatible with the orbifolding.

Defined by the relations:

$$\begin{aligned} [\phi_i, \phi_j] &= i\epsilon_{ijk}(\phi_k)^\dagger, \\ \phi_i \phi_i &= R^2. \end{aligned}$$

Relation to the ordinary S^2_N via

$$\phi_i = \Omega\, \tilde{\phi}_i,$$

for some $\Omega \neq 1$ which satisfies $\Omega^3 = 1, \quad [\Omega, \phi_i] = 0, \quad \Omega^\dagger = \Omega^{-1}$ and $(\tilde{\phi}_i)^\dagger = \tilde{\phi}_i$.

Then $[\tilde{\phi}_i, \tilde{\phi}_j] = i\epsilon_{ijk}\tilde{\phi}_k \qquad \rightsquigarrow S^2_N$

The mechanism...

The (scalar) potential of the projected theory is

$$V_{\mathcal{N}=1}^{(proj)}(\phi) = -\frac{1}{4} Tr([\phi_i, \phi_j]^2).$$

Clearly there is no vacuum with non-vanishing commutators.

Required modification: add $\mathcal{N} = 1$ Soft Susy Breaking (SSB) terms

$$V_{SSB} = -\frac{1}{2}\delta_i^j (\phi^i)^\dagger \phi_j - \frac{1}{2}\epsilon^{ijk}\phi_i\phi_j\phi_k + h.c.$$

SSB potential is necessary for the theory to have a chance to become realistic.

Vacuum

The full potential is:

$$V = V_{\mathcal{N}=1}^{(proj)} + V_{SSB} + V_D,$$

where V_D includes the D-terms.

The vacuum of the model is given by the minimum of the potential.

The potential can be brought in the form:

$$V = -\frac{1}{4} F_{ij}^{\dagger} F_{ij},$$

where we have defined

$$F_{ij} = [\phi_i, \phi_j] - i\epsilon_{ijk}(\phi_k)^{\dagger}.$$

Vacuum

The minimum is obtained when the twisted fuzzy sphere relation is satisfied.

In view of the decomposition:

$$SU(N) \supset SU(N-n) \times SU(n) \times U(1),$$

such a vacuum may be expressed as

$$\phi_i = \Omega \left(\mathbb{1}_3 \otimes (\lambda_i^{(N-n)} \oplus 0_n) \right),$$

with $\lambda_i^{(N-n)}$ the generators of the corresponding representation of $SU(2)$.

- The gauge symmetry $SU(N)^3$ is broken down to $SU(n)^3$.
- Moreover, there exists a finite Kaluza-Klein tower of massive states.
- Therefore the vacuum can be interpreted as spontaneously generated fuzzy extra dimensions.
- At this intermediate scale the theory behaves as a higher dimensional theory.
- The fluctuations of the vacuum correspond to the internal components of the higher-dimensional gauge field.

$SU(3)^3$

- Let us consider the case: $N = n + 3$
- The relevant decomposition of each $SU(N)$ factor is: $SU(N) \supset SU(n) \times SU(3) \times U(1)$
- Then the decomposition of the $(N, \overline{N}, 1) + (\overline{N}, 1, N) + (1, N, \overline{N})$ of $SU(N)^3$ is:

$$
\begin{aligned}
&SU(n) \times SU(n) \times SU(n) \times SU(3) \times SU(3) \times SU(3) \\
&(n, \bar{n}, 1; 1, 1, 1) + (1, n, \bar{n}; 1, 1, 1) + (\bar{n}, 1, n; 1, 1, 1) + \\
&+(1, 1, 1; 3, \bar{3}, 1) + (1, 1, 1; 1, 3, \bar{3}) + (1, 1, 1; \bar{3}, 1, 3) + \\
&+(n, 1, 1; 1, \bar{3}, 1) + (1, n, 1; 1, 1, \bar{3}) + (1, 1, n; \bar{3}, 1, 1) + \\
&+(\bar{n}, 1, 1; 1, 1, 3) + (1, \bar{n}, 1; 3, 1, 1) + (1, 1, \bar{n}; 1, 3, 1).
\end{aligned}
$$

- Applying the above mechanism we can write down the vacuum solution:

$$\phi_i = \Omega\, [\mathbb{1}_3 \otimes (\lambda_i^{(N-3)} \oplus 0_3)],$$

interpreted in terms of twisted fuzzy spheres $\tilde{S}^2_{N-3}$.

- This vacuum solution amounts to vevs for the fields:

$$\langle(n,\overline{n},1;1,1,1)\rangle, \langle(1,n,\overline{n};1,1,1)\rangle, \langle(\overline{n},1,n;1,1,1)\rangle.$$

- The gauge group is broken down to

$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

- The matter fields transform as $(\overline{3},1,3)+(3,\overline{3},1)+(1,3,\overline{3})$ $\rightsquigarrow$ chiral fermions

The quarks of the first family transform under the gauge group as

$$q=\begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3,\overline{3},1), \quad q^c=\begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (\overline{3},1,3),$$

and the leptons transform as

$$\lambda=\begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1,3,\overline{3}).$$

- Concerning the rest of the fermions we can form invariants (Yukawas)...
  - $(1, n, \overline{n}; 1, 1, 1)\langle(n, \overline{n}, 1; 1, 1, 1)\rangle(\overline{n}, 1, n; 1, 1, 1)$ + cyclic
  - $(\overline{n}, 1, 1; 1, 1, 3)\langle(n, \overline{n}, 1; 1, 1, 1)\rangle(1, n, 1; 1, 1, \overline{3})$ etc.

$\rightsquigarrow$ finite Kaluza-Klein tower of massive fermionic modes

- Also: the 1-loop β-function of the model is zero
- Phenomenological studies on $SU(3)^3$ have shown that it can be rendered finite even to all orders and it leads to predictions (Ma-Mondragon-Zoupanos '04)

Similarly, other particle physics models can be constructed.

- $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam gauge group)
- matter fields in $(4,1,2)+(\bar{4},2,1)+(1,2,2)$
- quarks and leptons:

$$f \sim (4,2,1) = \begin{pmatrix} d & u \\ d & u \\ d & u \\ e & \nu \end{pmatrix},$$

$$f^c \sim (\bar{4},1,2) = \begin{pmatrix} d^c & d^c & d^c & e^c \\ u^c & u^c & u^c & \nu^c \end{pmatrix},$$

- $SU(4)^3$ with matter fields in $(4,1,\bar{4})+(\bar{4},4,1)+(1,\bar{4},4)$
- quarks and leptons:

$$f = \begin{pmatrix} d & u & y & x \\ d & u & y & x \\ d & u & y & x \\ e & \nu & a & v \end{pmatrix} \sim (4,\bar{4},1), \quad f^c = \begin{pmatrix} d^c & d^c & d^c & e^c \\ u^c & u^c & u^c & \nu^c \\ y^c & y^c & y^c & a^c \\ x^c & x^c & x^c & v^c \end{pmatrix} \sim (\bar{4},1,4).$$

Further breaking

- Study whether the breaking of $SU(3)^3$ to the MSSM and furthermore to the $SU(3) \times U(1)_Q$ can be performed in the above spirit.
- Require that the breaking takes place...:
  - ...without adding any additional supermultiplets
  - ...at one step, i.e. without breaking first to an intermediate gauge group

- The field content consists of 3 families in

$$(\overline{3}, 1, 3) + (3, \overline{3}, 1) + (1, 3, \overline{3})$$

- The superpotential is

$$W^{(proj)}_{\mathcal{N}=1} = YTr(\lambda q^c q) + Y' \epsilon_{ijk} \epsilon_{abc} (\lambda_{ia} \lambda_{jb} \lambda_{kc} + q^c_{ia} q^c_{jb} q^c_{kc} + q_{ia} q_{jb} q_{kc}).$$

- The scalar potential includes the corresponding SSB terms
- we can give vevs to neutral directions of

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix},$$

i.e. ν, ν^c, S (GUT breaking) and N, N^c (EW breaking).

- In order to apply our mechanism we need trilinear terms involving the fields which can acquire vev.
- These terms exist and they are: $\nu \nu^c N^c$ and $N N^c S$.
- Therefore we are able to determine a vacuum which spontaneously generates twisted fuzzy spheres and at the same time breaks the gauge symmetry $SU(3)^3$ down to the MSSM and further to the $SU(3)_c \times U(1)_Q$.

Conclusions

- Extra dimensions serve as an arena for unification of fundamental interactions.
- The attempt to make contact between physics studied in accelerators and fundamental theories in higher dimensions is still under way.
- We show that within a four-dimensional and renormalizable theory, fuzzy extra dimensions can be generated dynamically...
- ...leading to low-energy models which have phenomenological relevance.
- Using orbifold techniques we build chiral unification models with fuzzy extra dimensions.
- The spontaneous symmetry breaking down to the MSSM and $SU(3)_c \times U(1)_Q$ is performed by higgsing of fuzzy spheres.