STABILIZING EXTRA DIMENSIONS

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Warsaw, October 19th 2009

Collaboration with J.A.Cabrer and M.Quirós

OUTLINE

- Features of Warped Extra Dimensions
- Stabilizing Models with 2 branes
- Soft Wall models (Models with 1 brane)
- Stabilizing the Soft Wall
- (Soft Walls and the Cosmological Constant)

OPEN QUESTIONS IN THE SM (AND BEYOND)

- What is the origin of Electroweak Symmetry Breaking?
- Why is the scale of the Z and W bosons 10^{17} times smaller than the Planck mass? (Hierarchy Problem)
- Why is there such a huge hierarchy in the masses of the Standard Model fermions?
- What is the origin of neutrino masses?
- If there is Supersymmetry, how is it broken?
- If there is a Grand Unified Theory, how is it broken to the SM, and why are there no colored Higgses?

All these issues can be addressed in models with Extra Dimensions

RS MODELS

Randall & Sundrum '99

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$$ds^2 = e^{-2\,k\,y} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$

Warp factor

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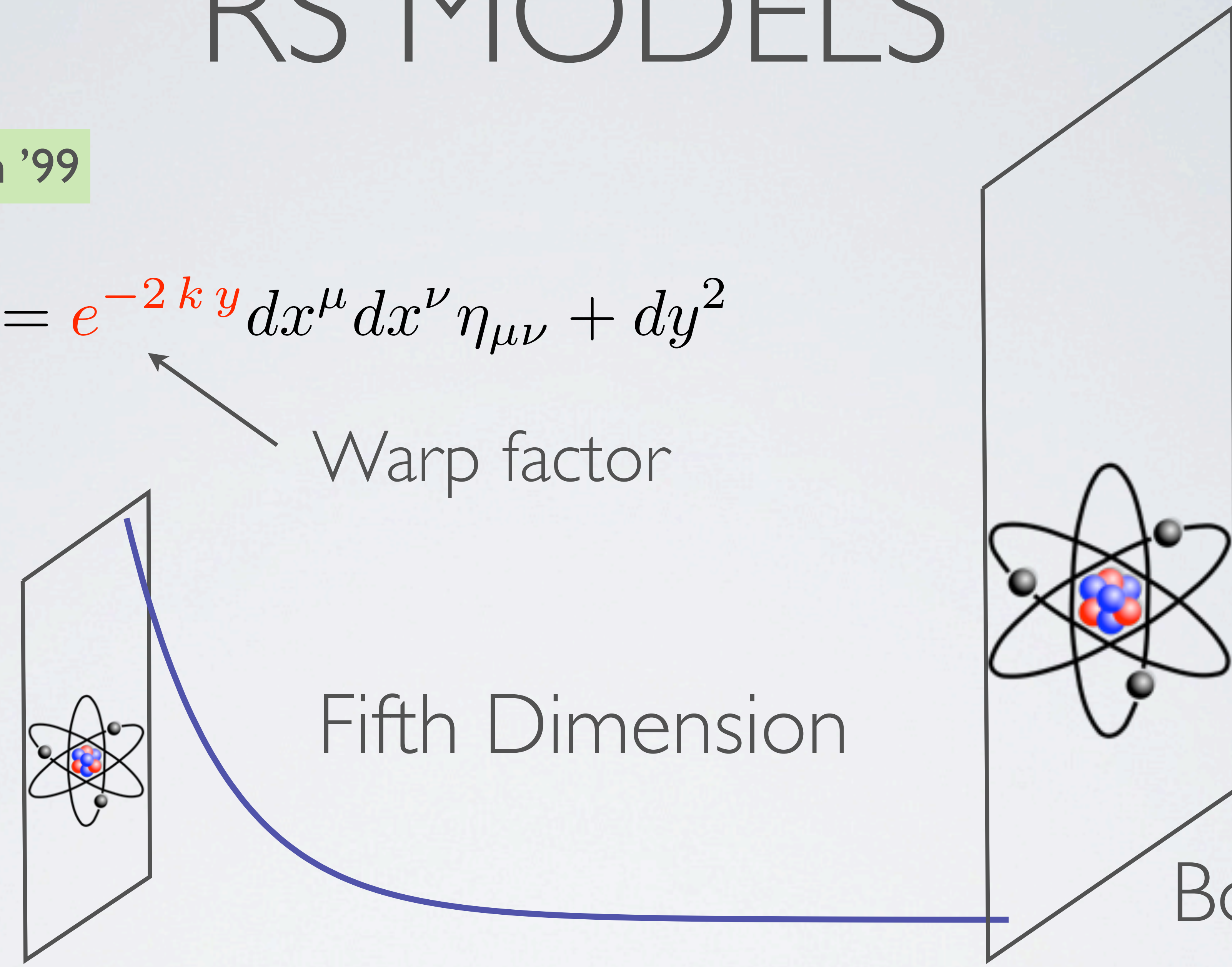

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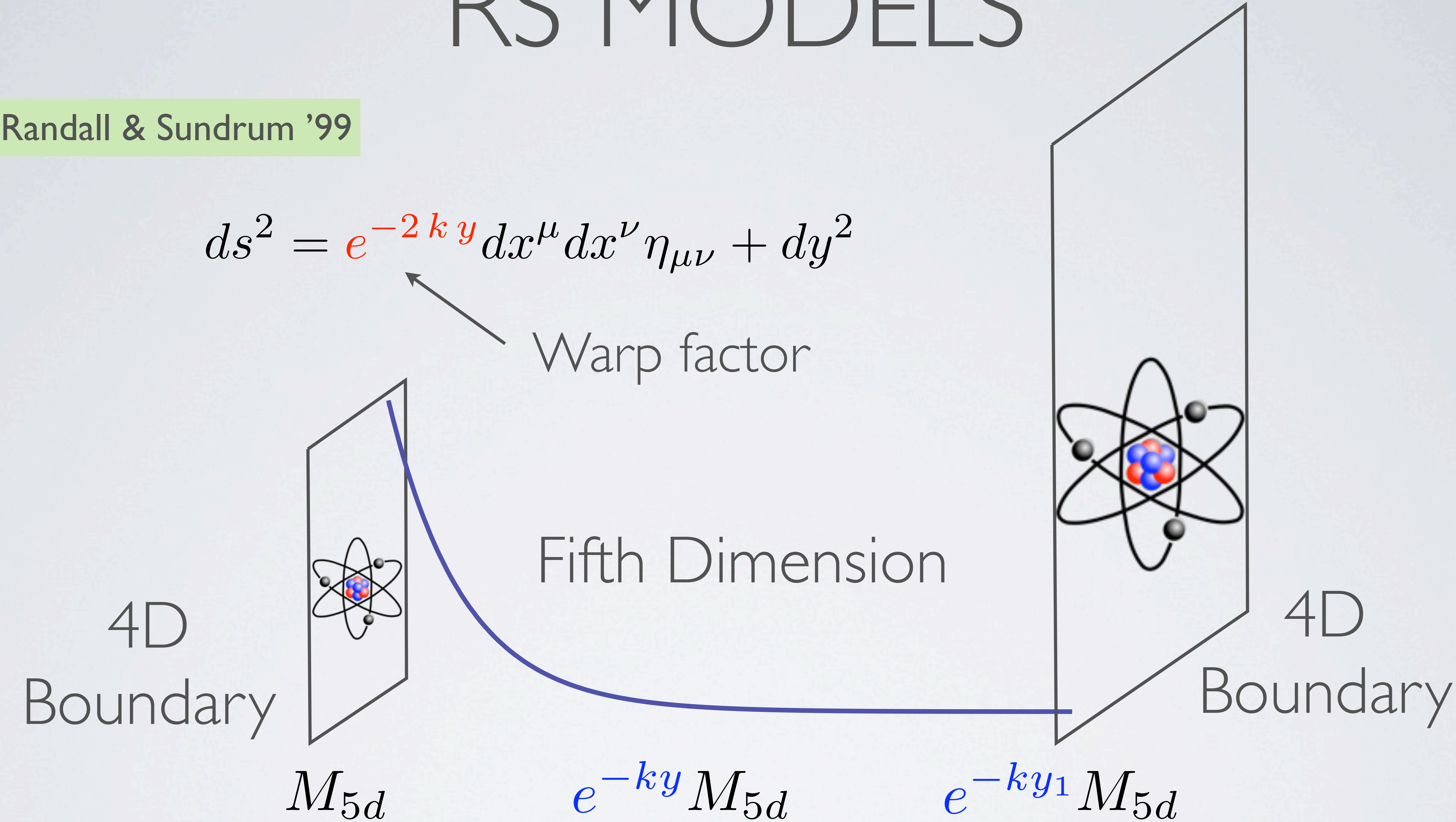

Fundamental cutoff scale is redshifted

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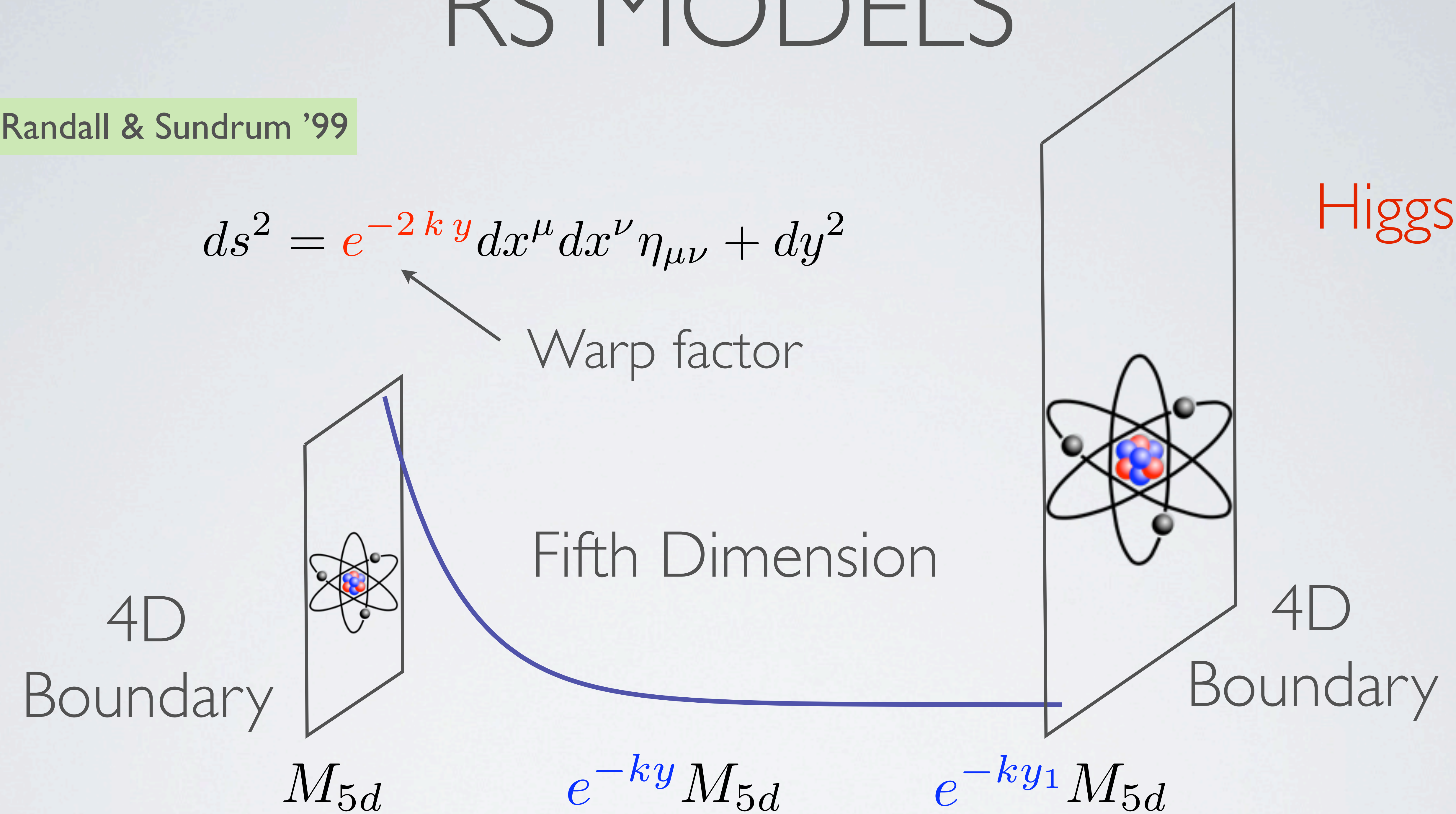

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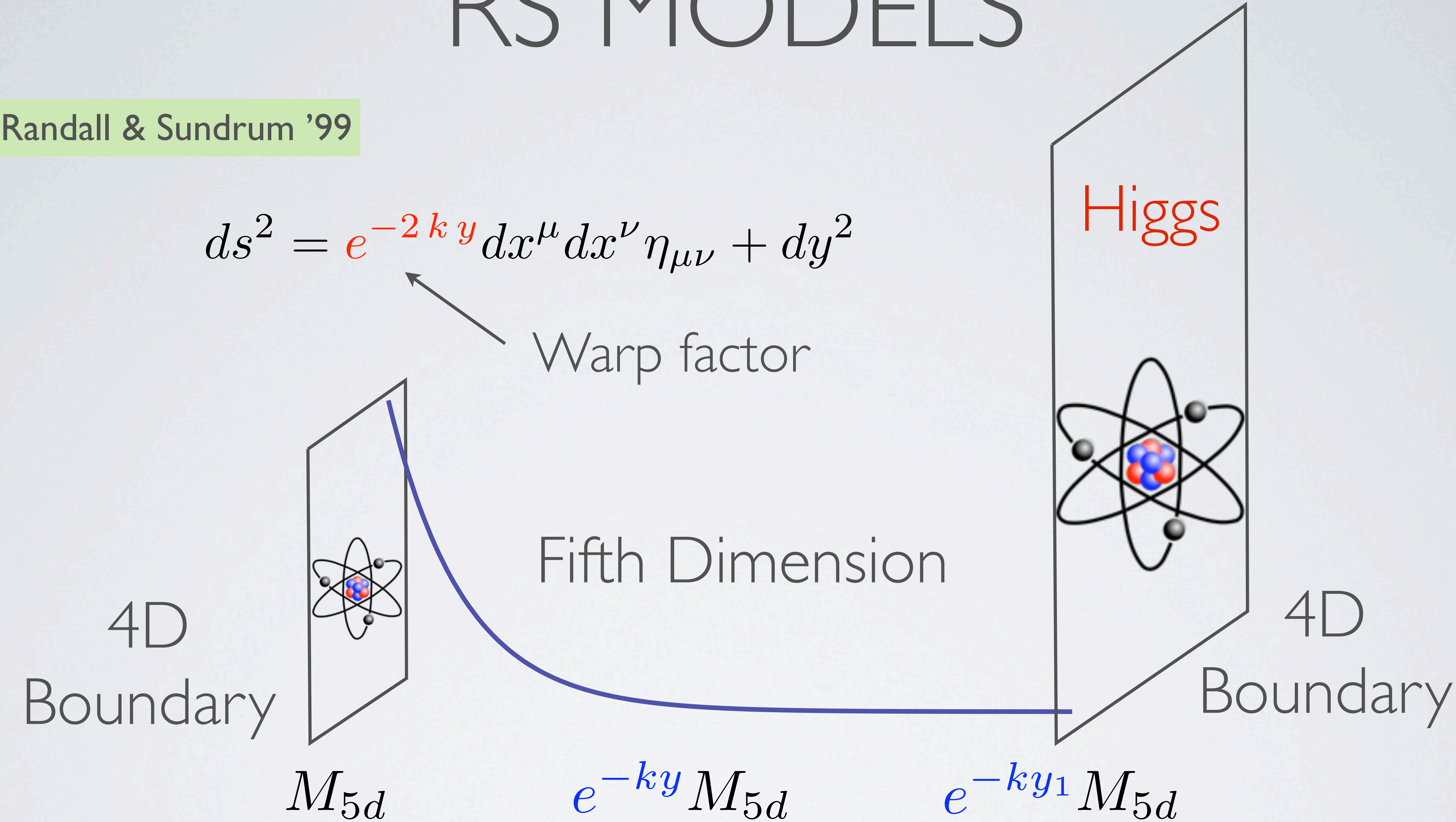

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FERMION MASSES

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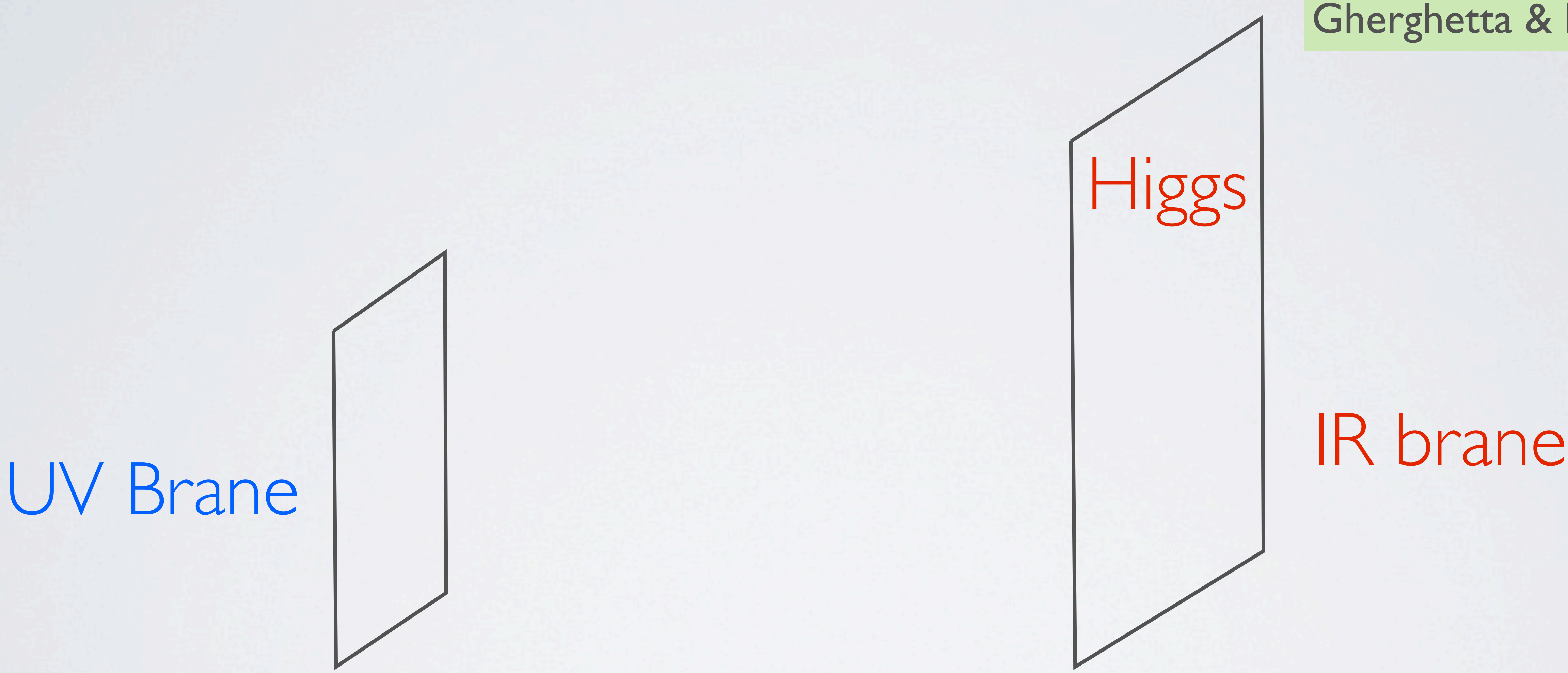

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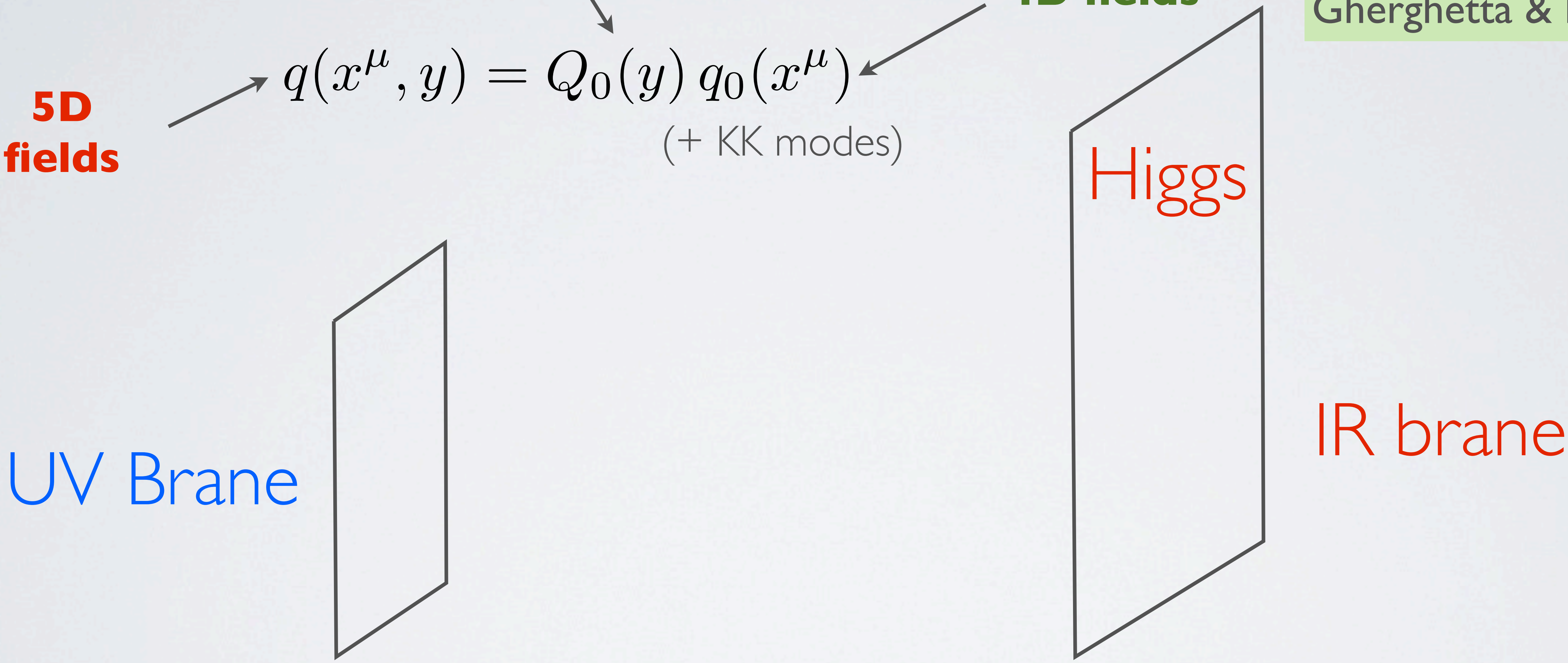
Wave functions

4D fields

5D fields

$$q(x^\mu, y) = Q_0(y)\, q_0(x^\mu)$$

(+ KK modes)



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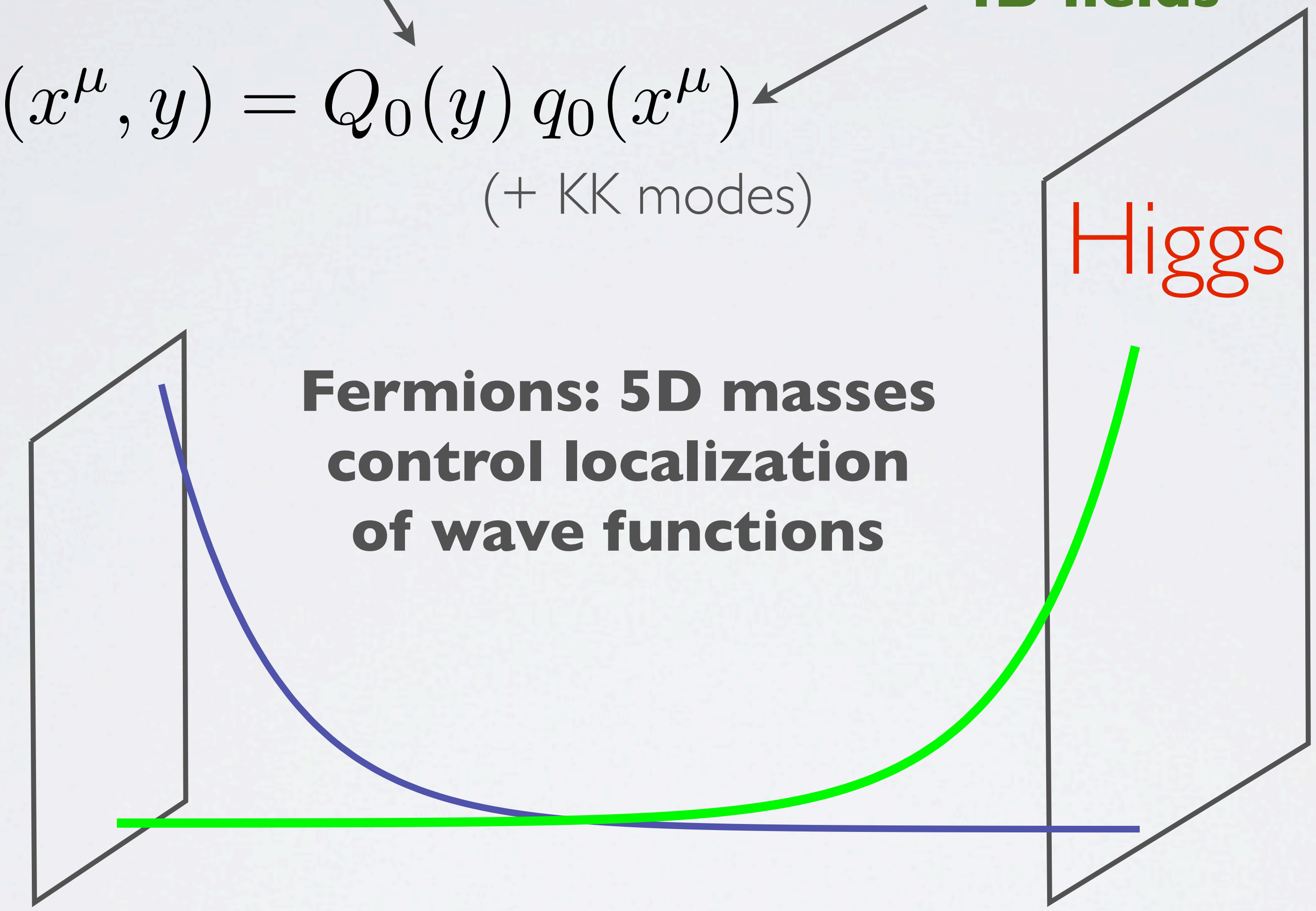
Wave functions

5D fields → $q(x^\mu, y) = Q_0(y)\, q_0(x^\mu)$ ← **4D fields**

(+ KK modes)

Fermions: 5D masses control localization of wave functions

UV Brane



IR brane

$$Y_{4D} = Y_{5d}\, Q_0(y_1)\, U_0(y_1)$$ →

- **Localize heavy fermions on IR brane**
- **Localize light fermions on UV brane**

SUMMARY RS MODELS

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- Extra Dimensions with WARPED background successful for
  - Explaining electroweak hierarchy
  - Explaining fermion mass hierarchy

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- Extra Dimensions with WARPED background successful for
  - Explaining electroweak hierarchy
  - Explaining fermion mass hierarchy
- Other features
  - Distinctive Collider Signature (KK gravitons)
  - Dual to strongly coupled gauge theories in 4D
  - “Modelling QCD”

PROBLEMS

- Pure 5D Gravity with negative Cosmological constant (and appropriate brane tensions) has RS as a solution.

- BUT: Interbrane distance is UNDETERMINED

- There is an extra massless mode (RADION)

$$g_{MN} = g_{MN}^{RS} + \begin{pmatrix} h_{\mu\nu} & \\ & h_{55} \end{pmatrix}$$

- Both brane tensions need to be fine tuned

STABILIZATION

- The question of Radius stabilization
  - What determines DISTANCE between UV and IR brane?
  - How can I generate a POTENTIAL and a MASS for the Radion?
  - What ensures that the 4D metric is FLAT?
  - How NATURAL is it?

SUPERPOTENTIAL METHOD

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- Gravity + scalar field
  with bulk and brane potential

- Solve Einstein equations
  coupled to scalar

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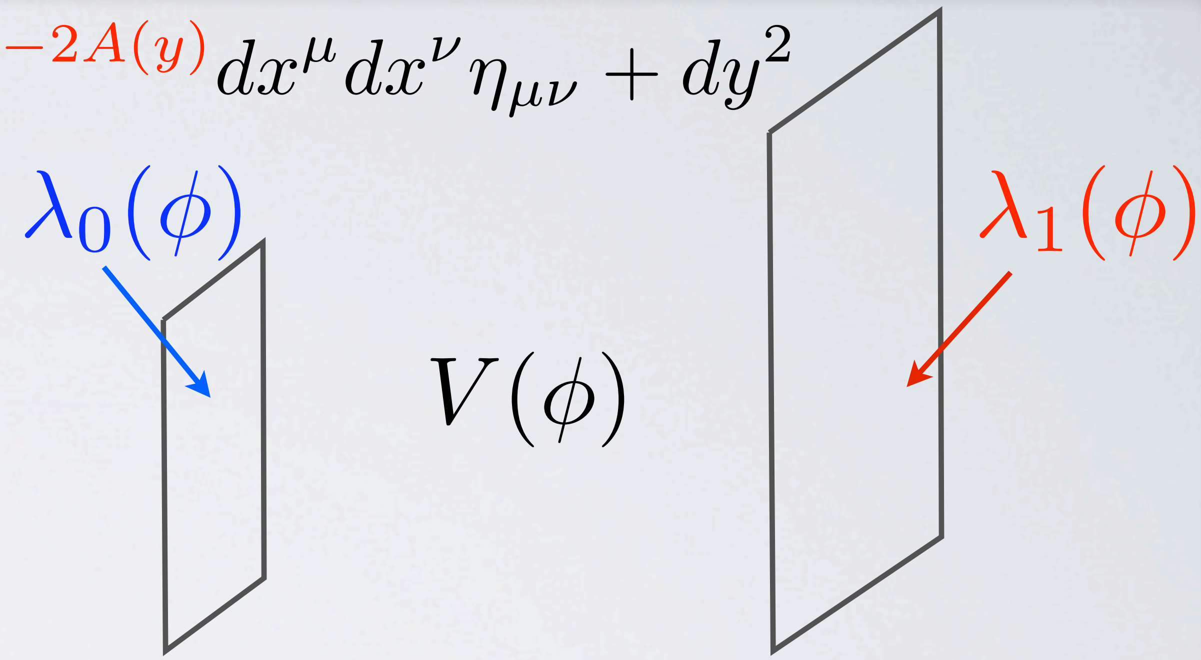

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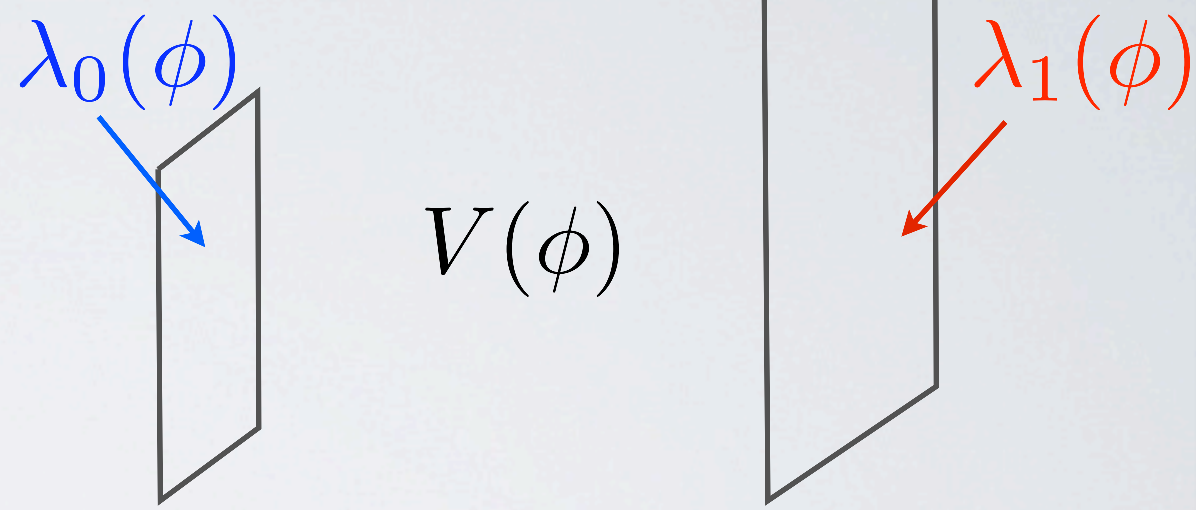

- Solve Einstein equations
  coupled to scalar

- Define a "Superpotential" $V(\phi) = 3W'(\phi)^2 - 12W^2(\phi)$ **NO SUSY INVOLVED**

- Einstein equations become $\phi'(y) = W'(\phi) \qquad A'(y) = W(\phi)$

- Boundary values from Minimizing the 4D potentials

$$V_i(\phi) = \lambda_i(\phi) - 6\,W(\phi)$$

DeWolfe et al '99, Brandhuber & Sfetsos '99

STABILIZATION

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- Solve to get bulk profiles $\phi'(y) = W'(\phi)$ $A'(y) = W(\phi)$
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- Notice that $e^{k\,y_1} = 10^{16} \Longrightarrow k\,y_1 \approx 37$
- Choose some suitable W such that

$$k\,y_1 = \int_{\phi_0}^{\phi_1} \frac{1}{W'} \approx 37$$

- Now shift superpotential $W \to W + k$

$$A(y) \to A(y) + k\,y$$

- Adds warping without changing the value of ky_1

DeWolfe et al '00,
Cabrer, GG & Quirós '09

GOLDBERGER WISE

Goldberger & Wise '99

- Take $W'(\phi) = k\, b\, \phi$

- Then $ky_1 = b^{-1} \log \phi_1/\phi_0 \approx 37$

- Moderate fine tuning necessary

- Metric $A(y) = ky + \frac{1}{2}\phi_0^2\,(e^{2\,b\,k\,y} - 1)$

Warping **Exact Backreaction**

- Backreaction small:

$$\frac{BR}{ky} < .01 \quad \text{for} \quad b^{-1} = 37,\ \phi_0 = 1/e,\ \phi_1 = 1$$

Do we need two branes?

GAUGE/GRAVITY DUALITY

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- Gravity/Gauge theory correspondence stipulates that the 5D theory is dual to a strongly coupled 4D gauge theory that
    - is approximately conformal in the UV
    - has large number of colors
    - describes the same physics as 5D theory

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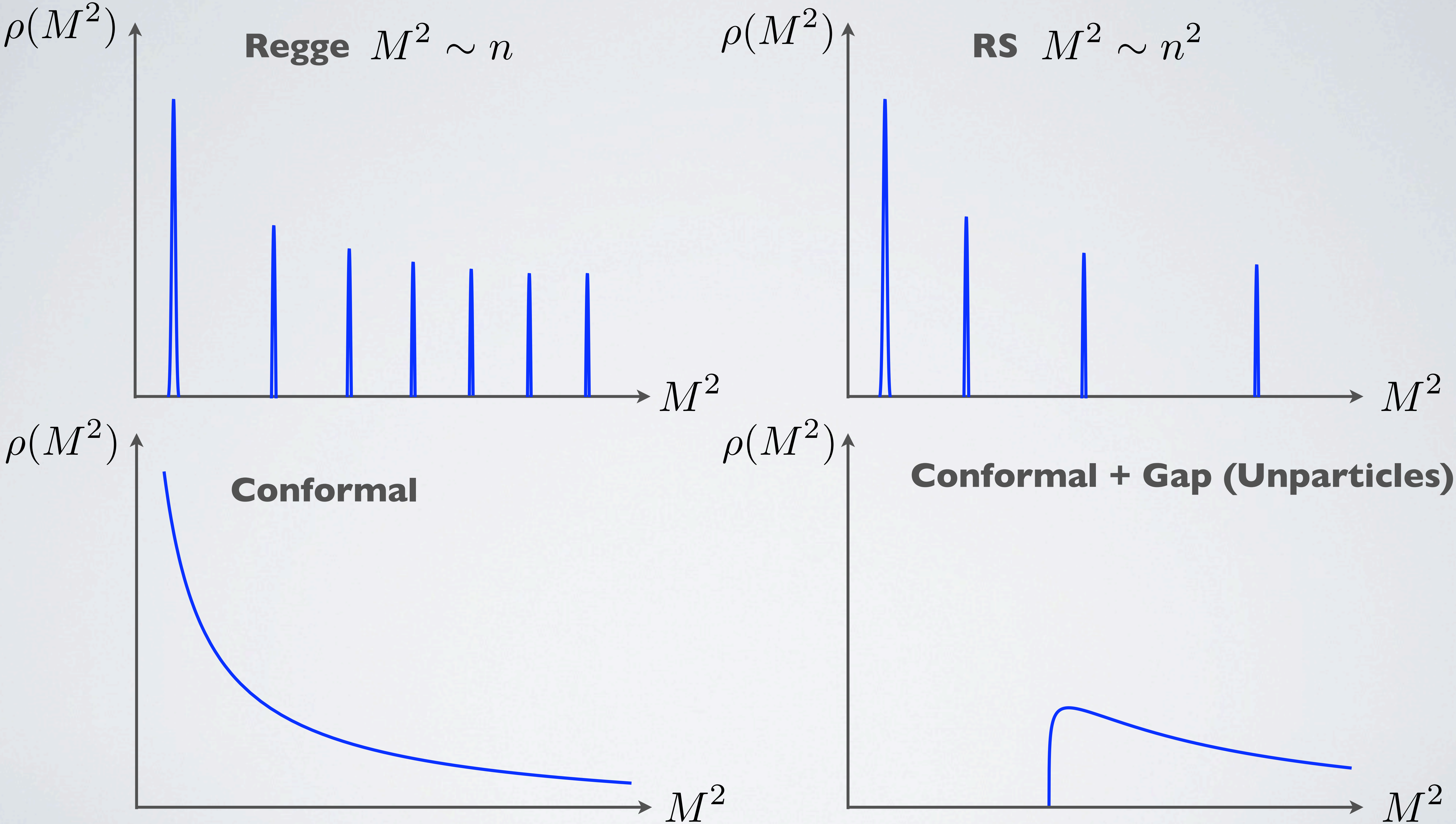
- Gravity/Gauge theory correspondence stipulates that the 5D theory is dual to a strongly coupled 4D gauge theory that
  - is approximately conformal in the UV
  - has large number of colors
  - describes the same physics as 5D theory
- KK modes correspond to resonances of gauge theory
  - RS with two branes: KK spectrum is roughly $m_n^2 \sim n^2$
  - 4D strongly coupled gauge theories have many more possibilities.

POSSIBLE SPECTRA



IR brane can be replaced by SOFT WALL

SOFT WALLS

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Soft Walls models only possess a single (UV) brane, but nevertheless exhibit a finite length in the 5th dimension. The IR brane is replaced by a curvature singularity at which the metric vanishes.

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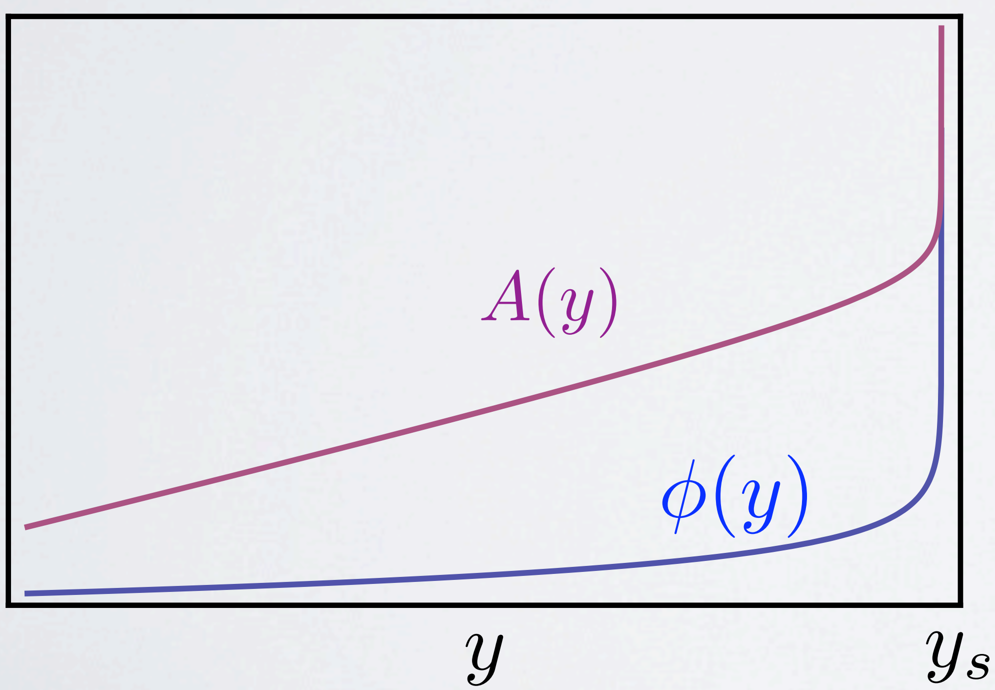

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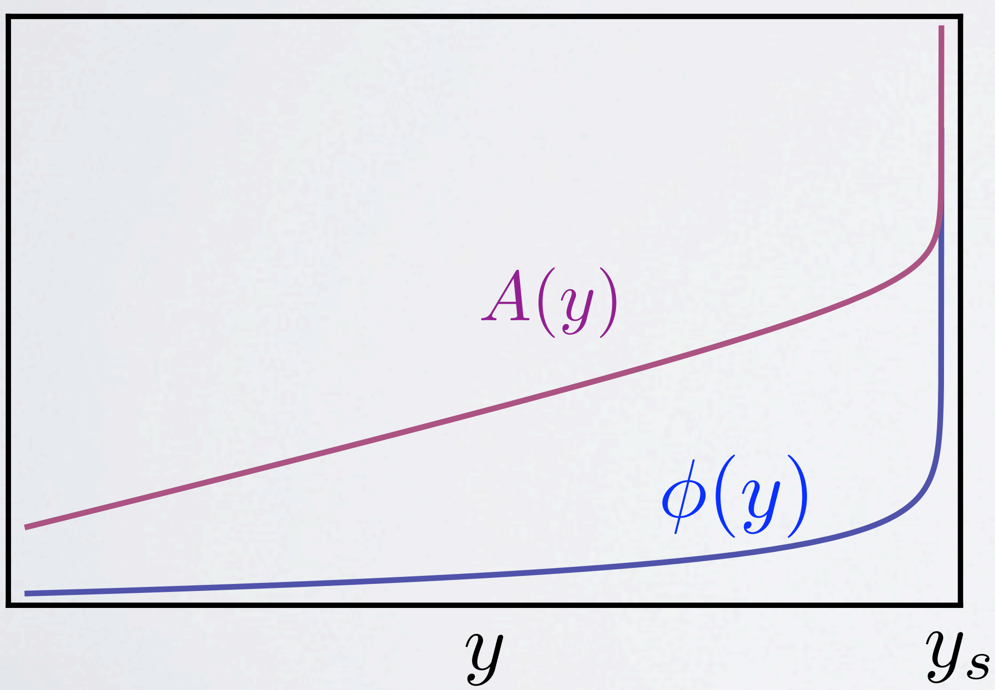

Profiles diverge at finite y if $W(\phi) \sim \phi^2$ or faster!

APPLICATIONS

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- Things that CAN be done with Soft Walls
  - Electroweak Breaking

    Batell, Gherghetta & Sword '08, Falkowski & Perez-V. '08, Cabrer, GG & Quiros (in progr.)

  - Strong interactions (AdS/QCD)

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    Karch et al '06, Gursoy et al '07, Batell & Gherghetta '08,
- Things that CANNOT be done with Soft Walls
  - Solve Cosmological Constant problem

    Arkani-Hamed et al '00, Kachru, Schulz & Silverstein '00 , Csaki et al '00

    Forste et al '00, Cabrer, GG & Quirós '09

SPECTRA WITH SOFT WALLS

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- Even though the physical length is finite, the conformal length might be either finite or infinite:

Proper Length coordinates	Conformally flat coordinates
$ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$	$ds^2 = e^{-2A(z)} (dx^\mu dx^\nu \eta_{\mu\nu} + dz^2)$

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- In the conformally flat frame, the KK spectrum of any bulk field follows a Schrödinger Equation

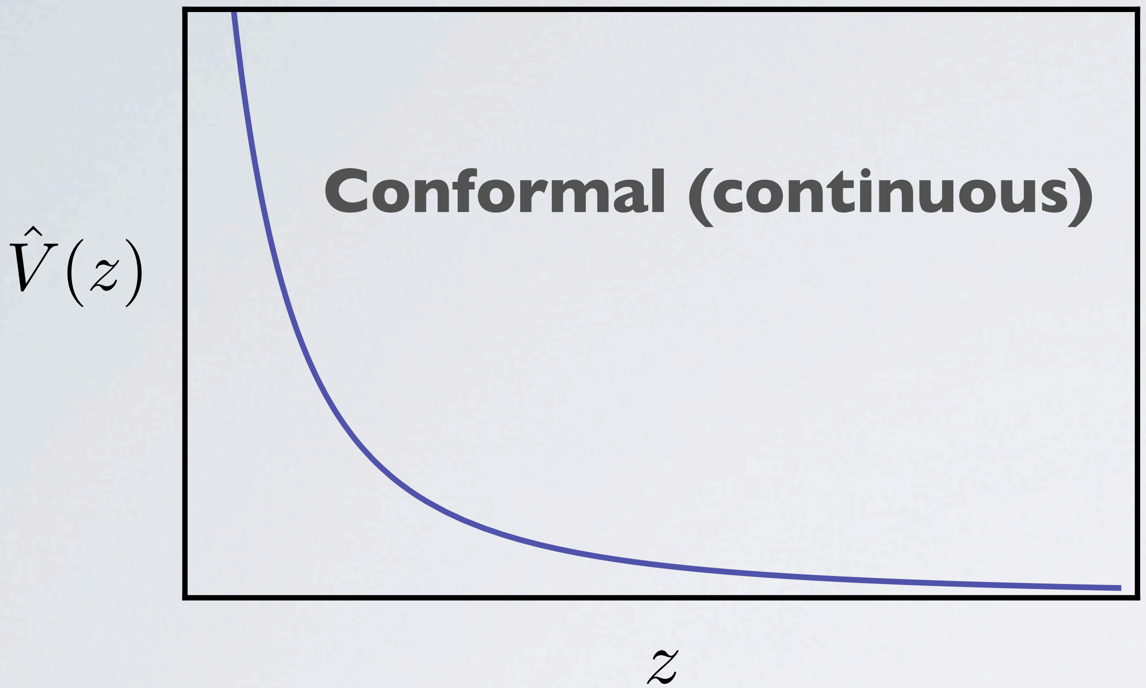
$$-\psi''(z) + \hat{V}(z)\psi(z) = m^2\psi(z)$$

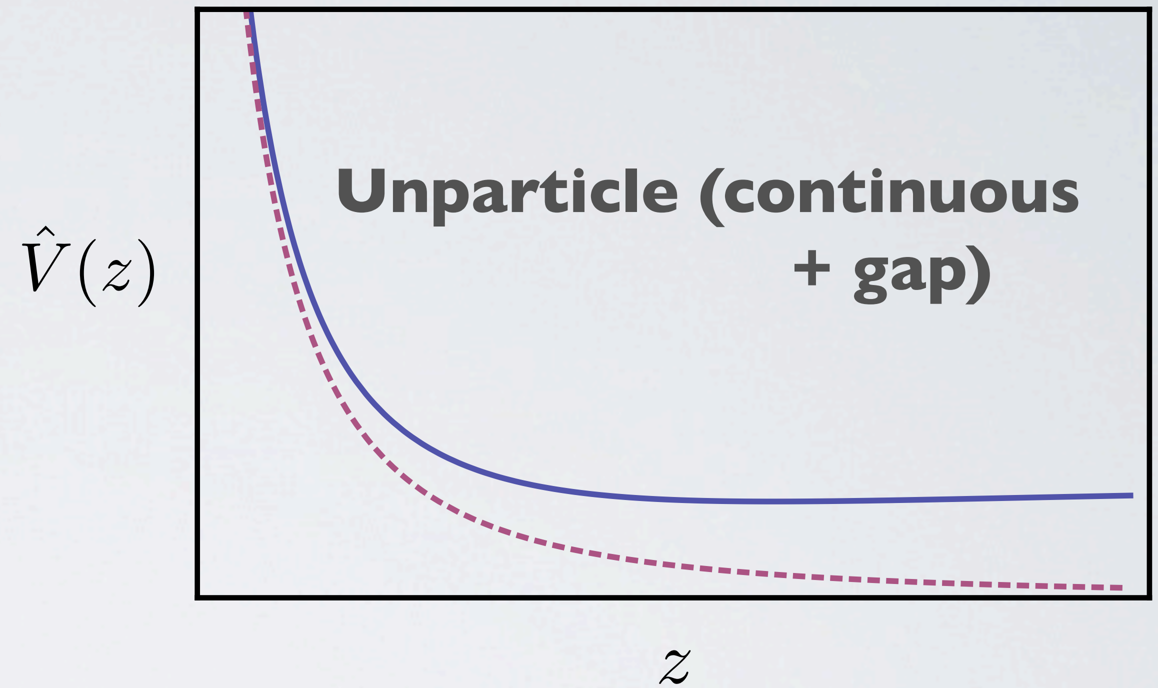
$\uparrow$

Depends on the background

NON CONFINING POTENTIALS

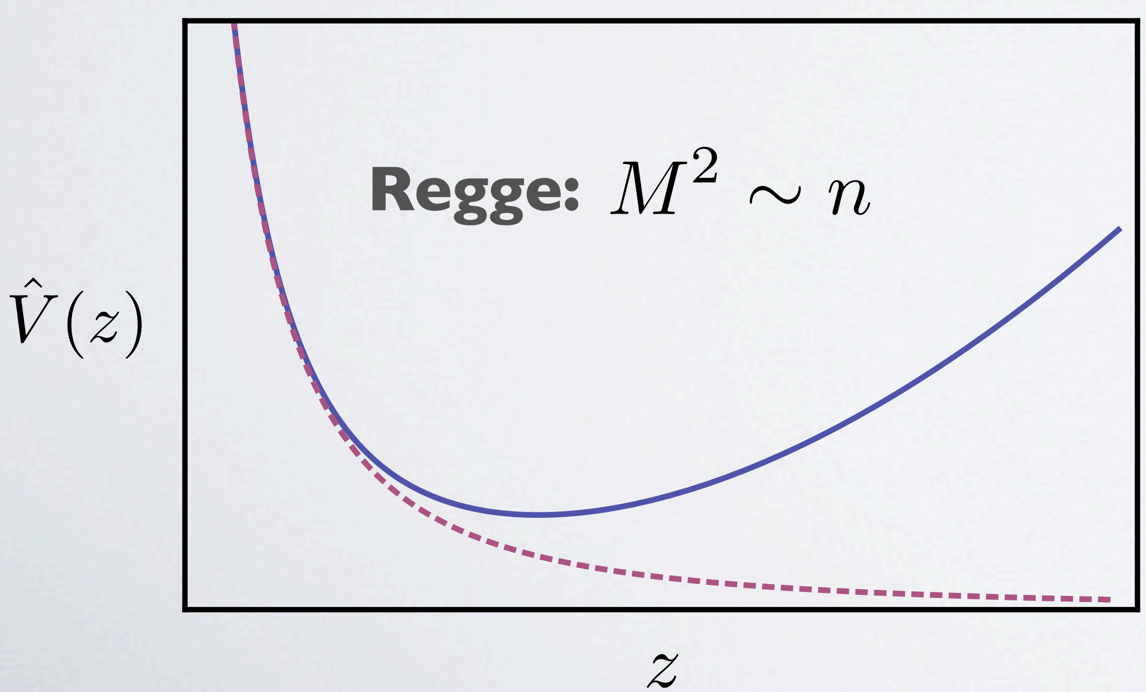
Conformal length infinite

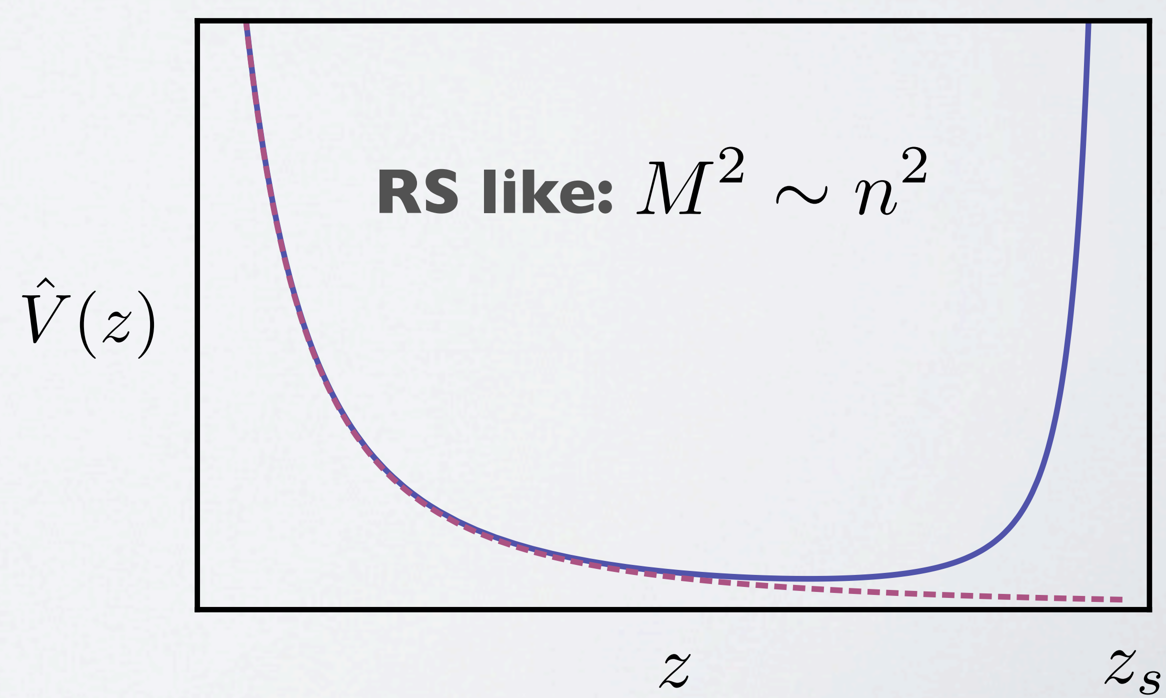




CONFINING POTENTIALS

Conformal length finite or infinite





SOFT WALL SPECTRA

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Gursoy et al '07,
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Asymptotic behaviour of W

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Asymptotic form of the spectrum

Finite Length

Gursoy et al '07,
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Asymptotic form of the spectrum

Finite Length

Mass gap appears

Gursoy et al '07,
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Asymptotic form of the spectrum

Finite Length

Mass gap appears

Spectrum discrete

Gursoy et al '07,
Cabrer, GG & Quirós '09

SOFT WALL STABILIZATION

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- Stabilization works similar as before
- Choose some suitable *W* such that

$$k y_s = \int_{\phi_0}^{\infty} \frac{1}{W'(\phi)} \approx 37$$

- Now shift superpotential $W \to W + k$

$$A(y) \to A(y) + k\, y$$

- Shift does not change position of singularity

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The Warping affects the Mass scale:
- The Unparticle mass gap
- The level spacing in the discrete case

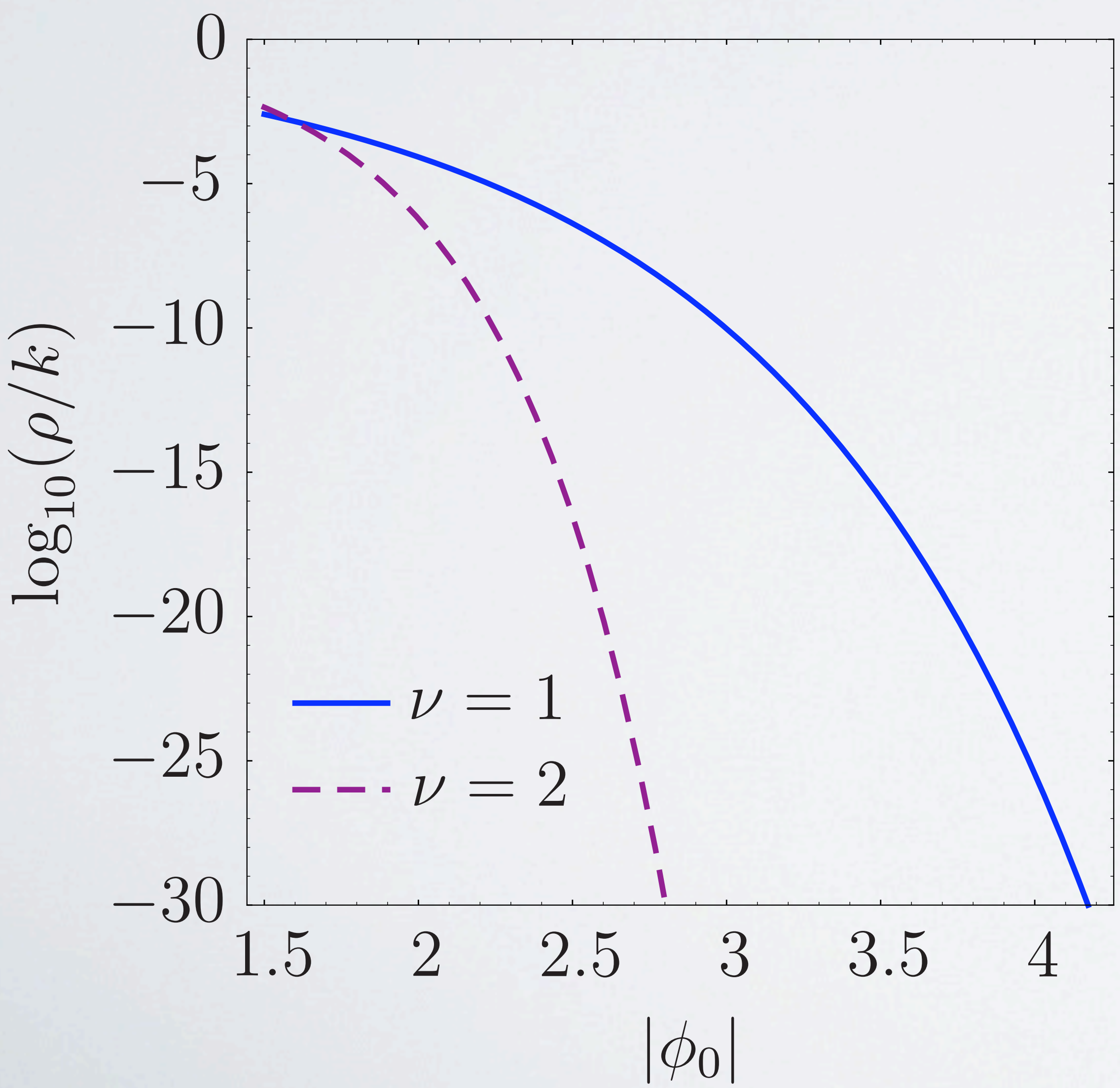
Warped down

Cabrer, GG & Quirós '09

PARTICULAR MODELS

Consider the class of models $W(\phi) = k(1 + e^{\nu\phi})$

$ky_s = \frac{1}{\nu^2} e^{-\nu\phi_0} \approx 37$ for O(1) negative values for ϕ_0



Spectrum can be
- **Continuous**
- **Continuous+gap**
- **Discrete**

THE GRAVITON SPECTRUM

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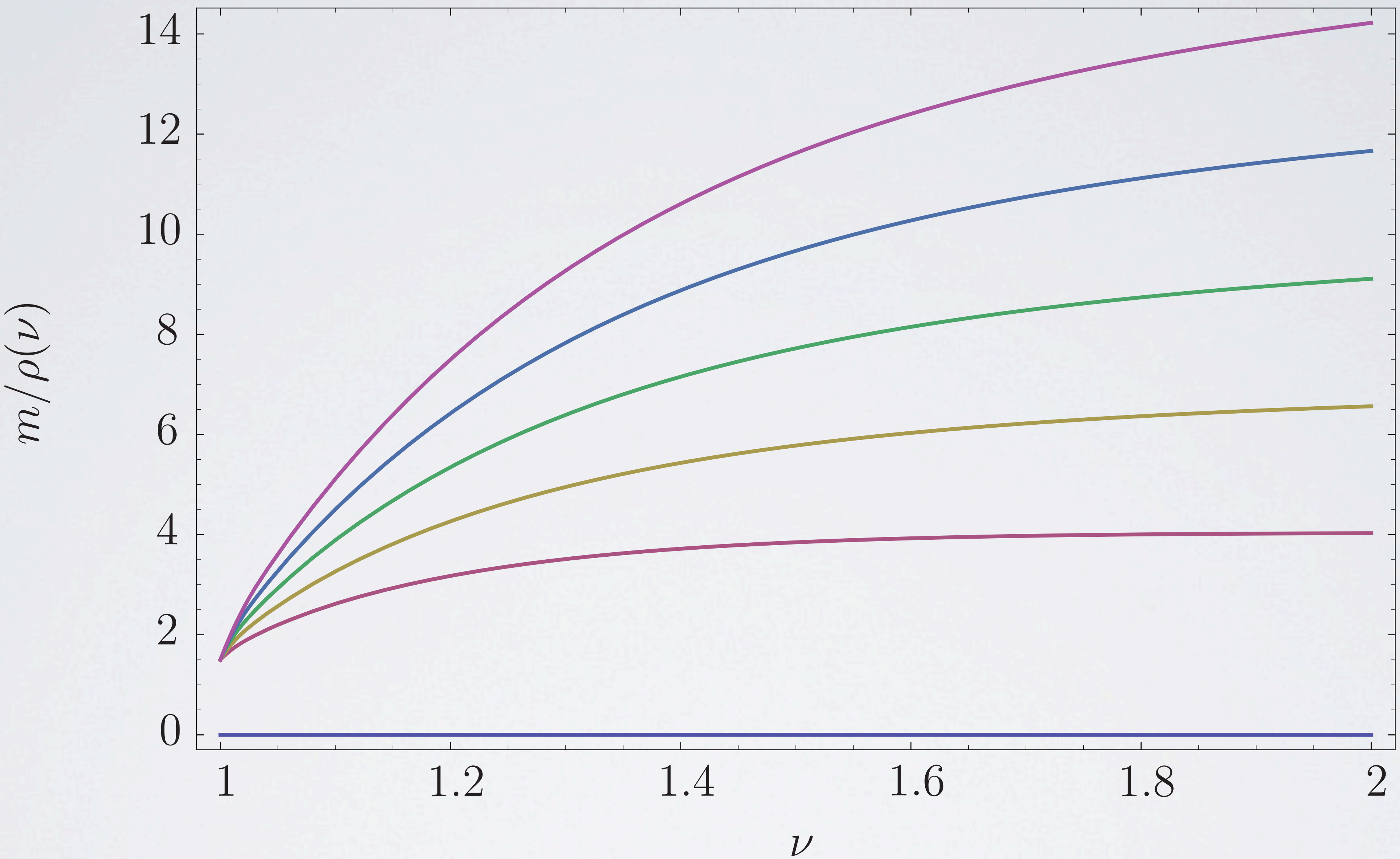

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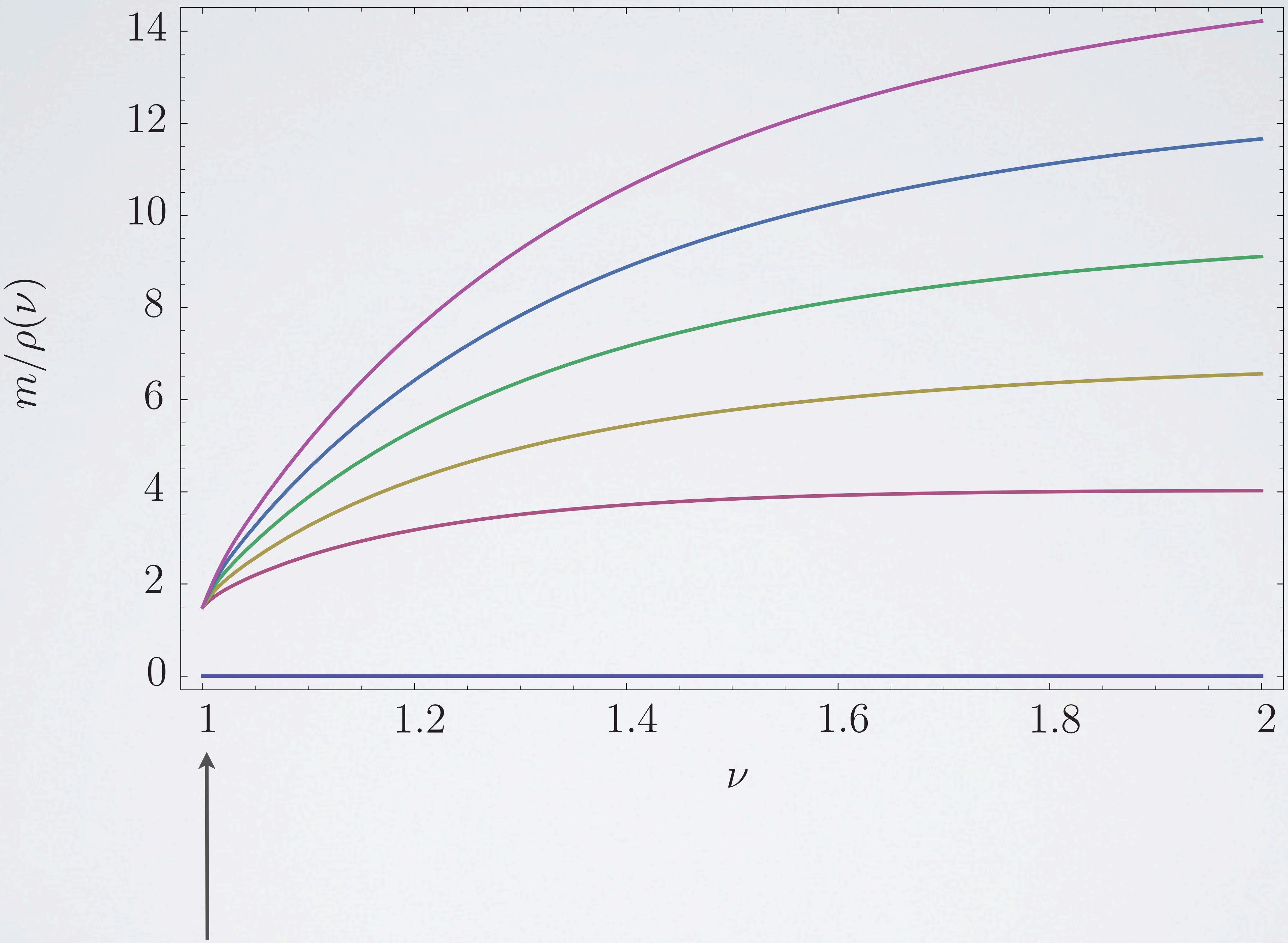
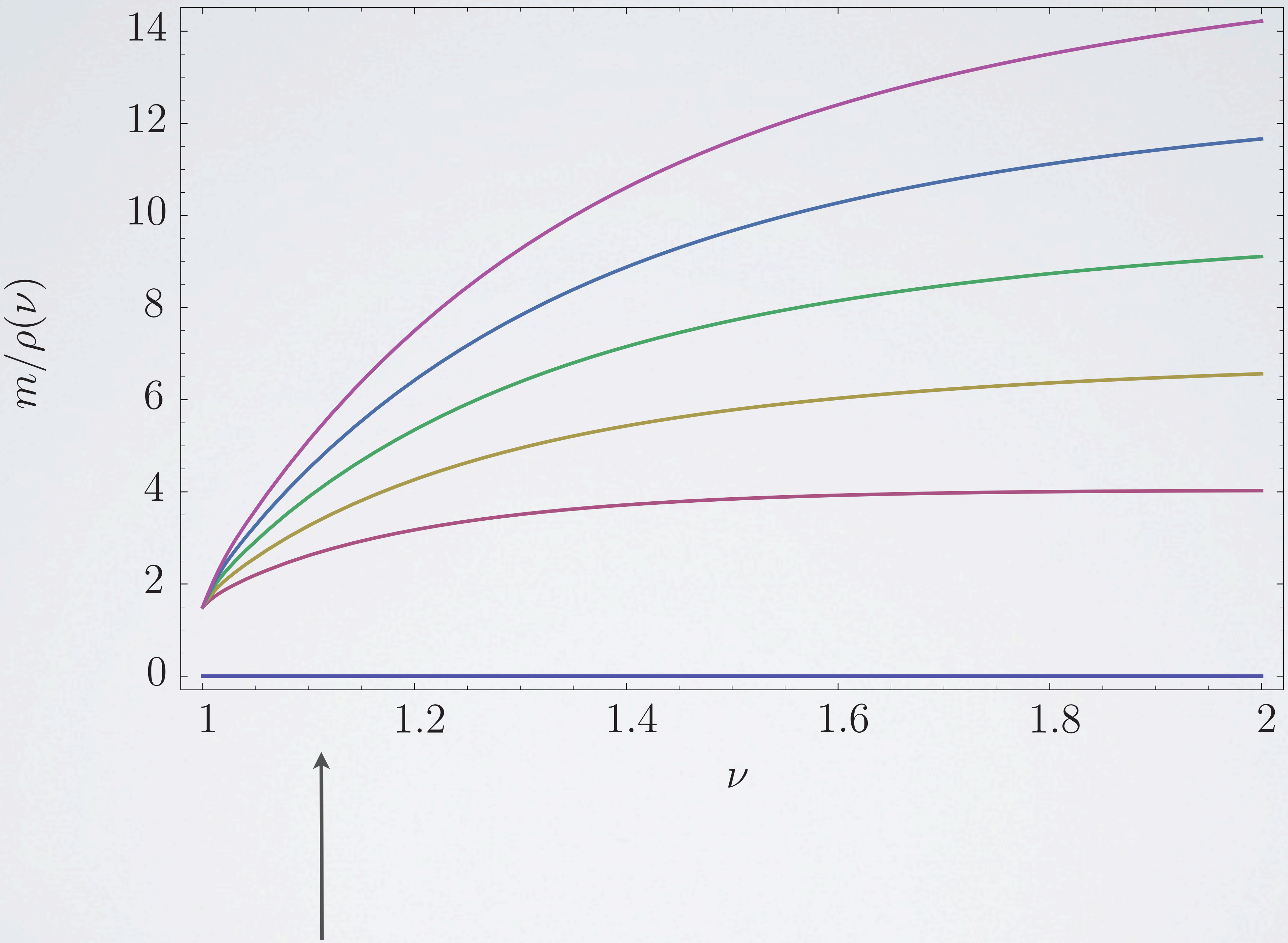


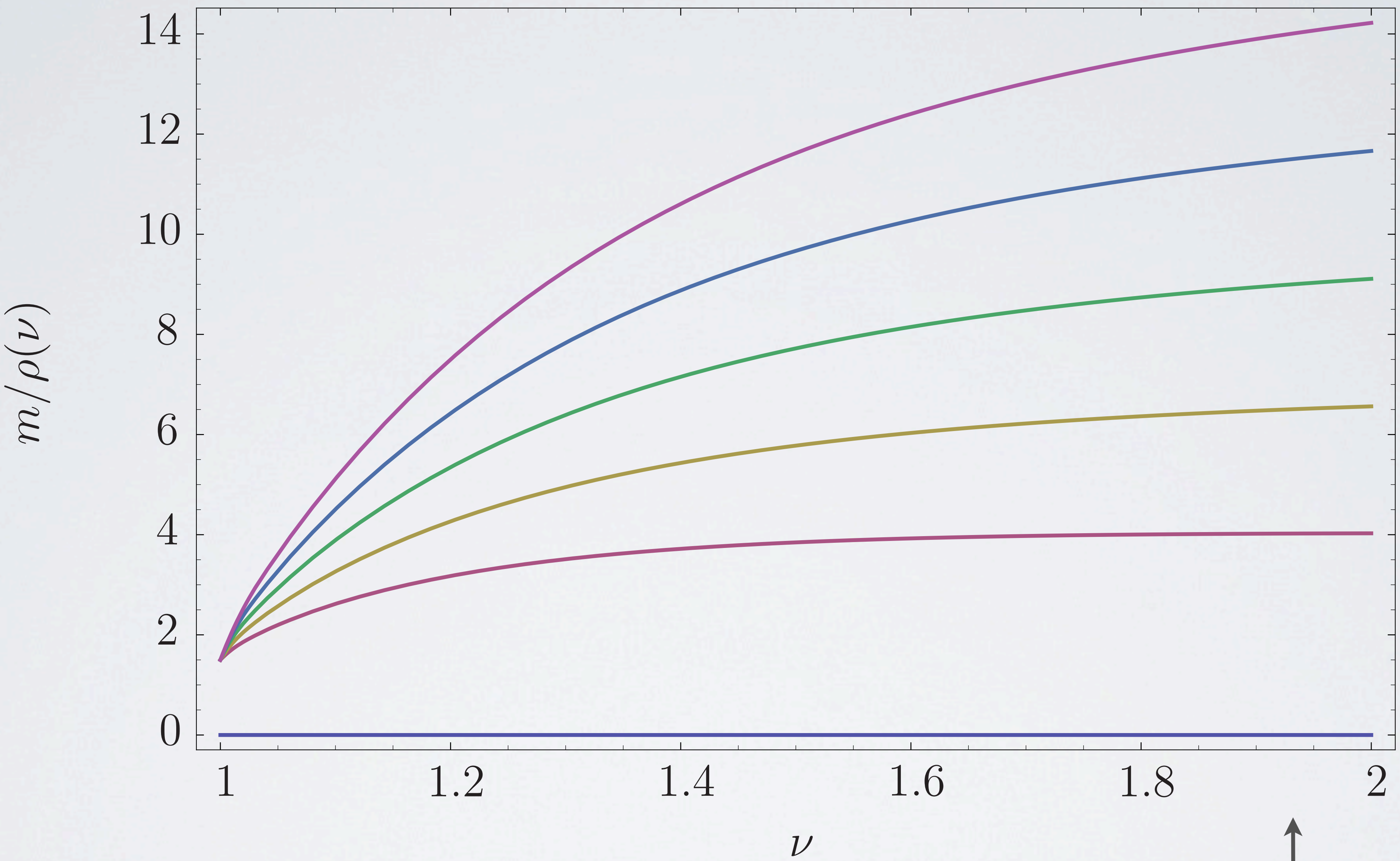
gap + continuous

THE GRAVITON SPECTRUM



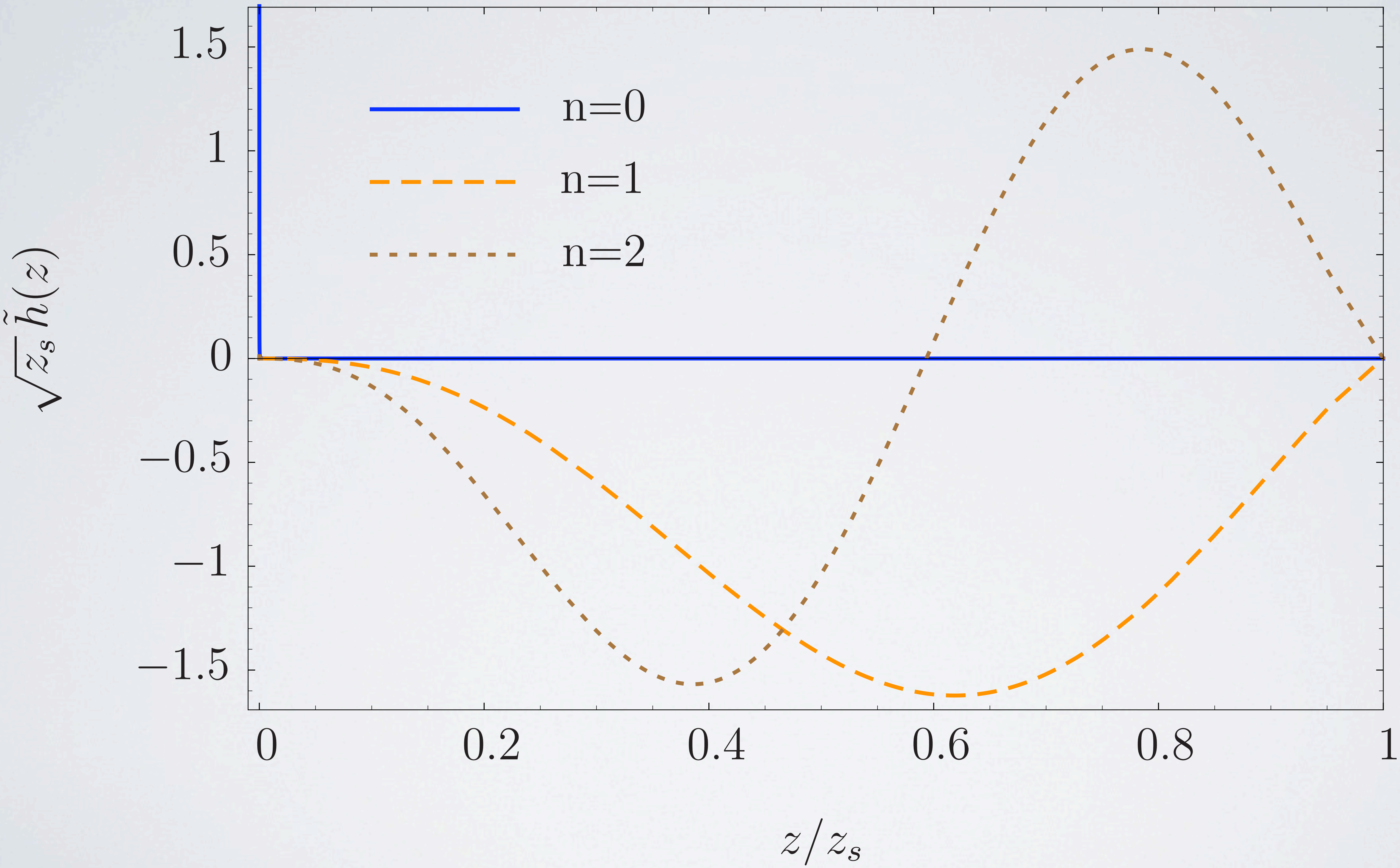
gap+very densely spaced discretuum

THE GRAVITON SPECTRUM



Discrete, hard-wall like

WAVE FUNCTIONS



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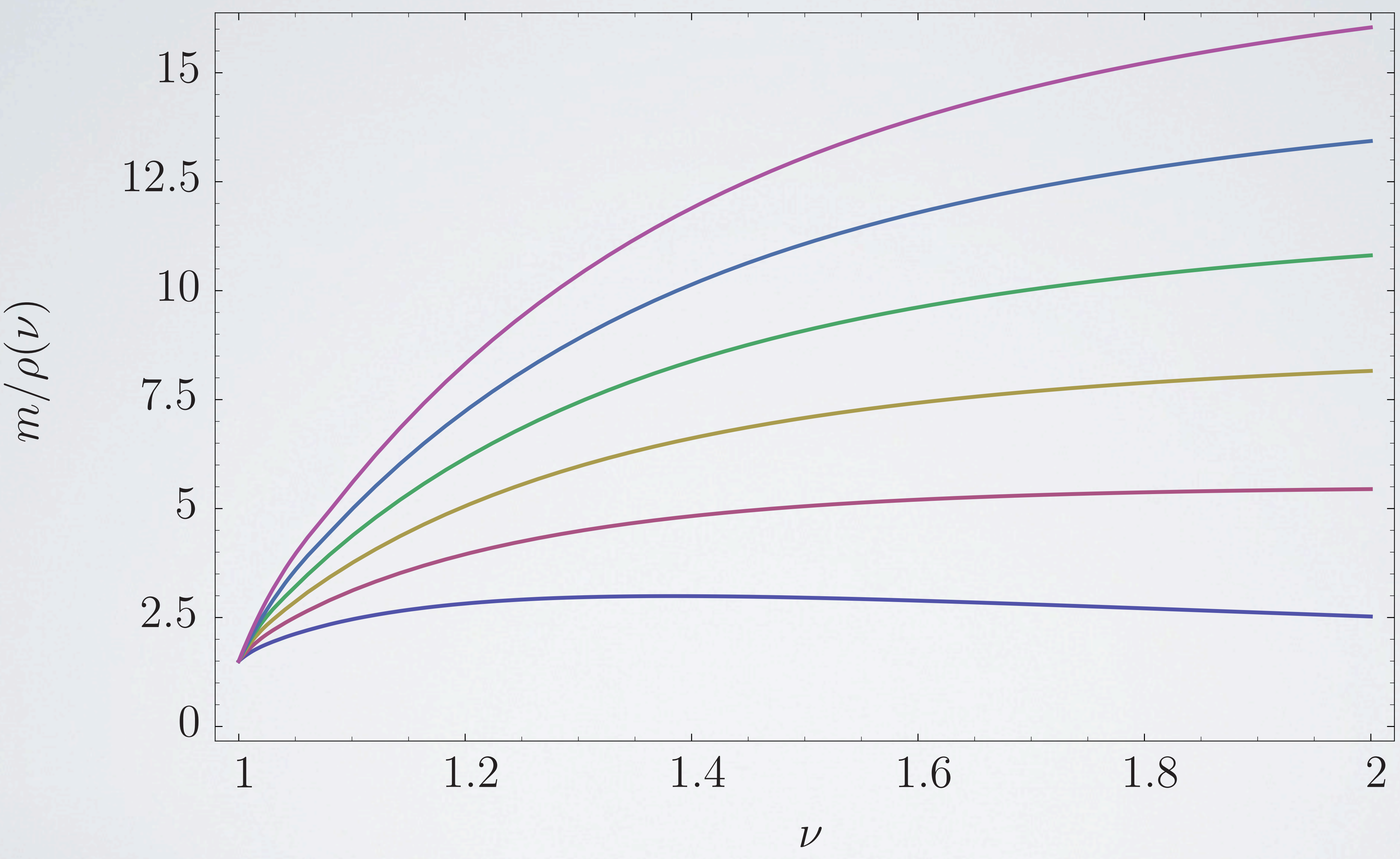

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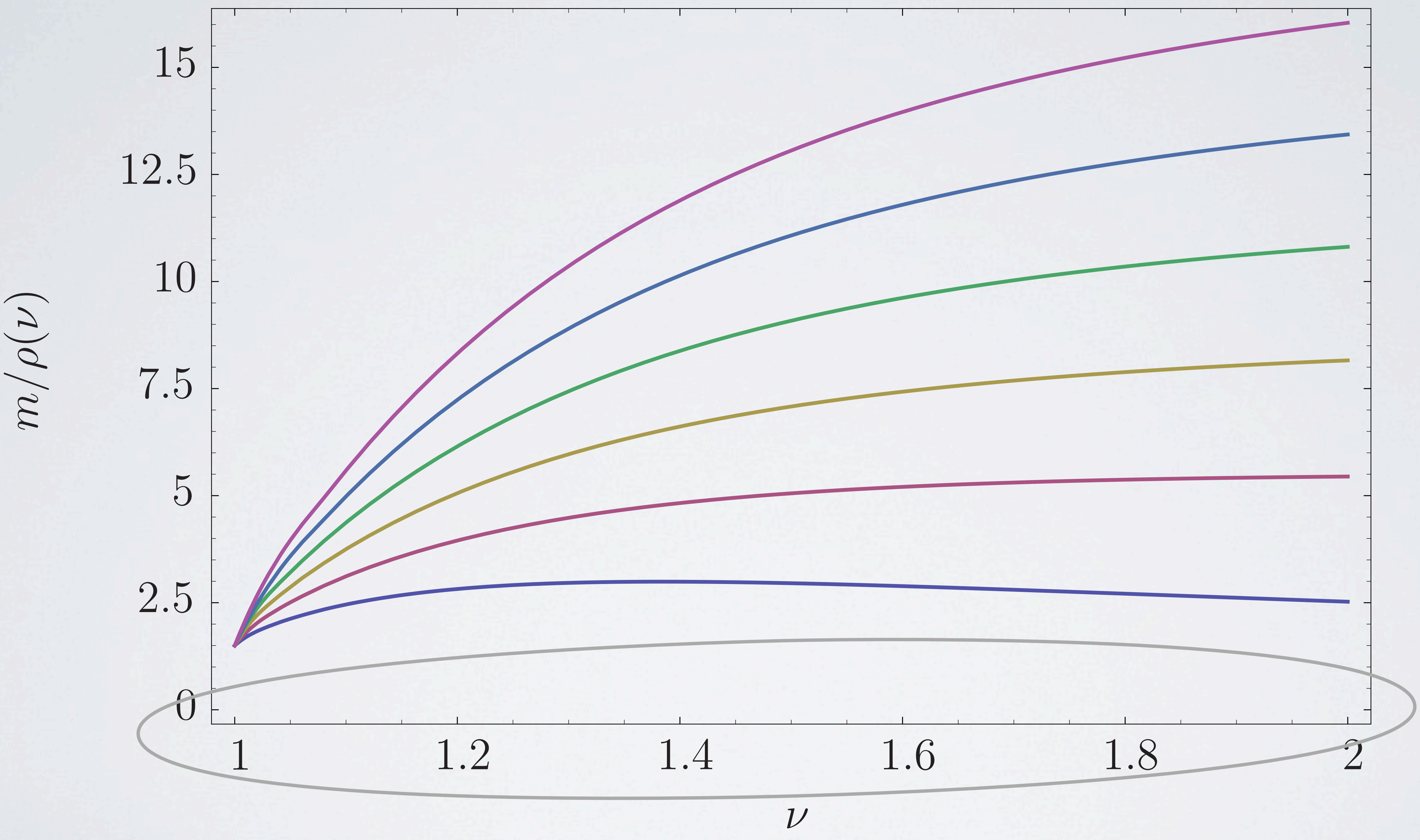

NO ZERO MODE

THE CC PROBLEM

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Forste et al ’00,
Cabrer, GG & Quirós ’09

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- Fine tuning is restored

CONCLUSIONS

- RS models provide neat way of obtaining electroweak and fermion mass hierarchy
- Stabilization can be achieved by adding extra scalar field
- IR brane can be consistently replaced by Soft Walls
- Spectra of Soft Wall models richer than in usual RS (gapped continuum, gapped, discretuum, Regge-like, etc.)
- Stabilization can be achieved without ANY fine tuning