STABILIZING EXTRA DIMENSIONS

Gero von Gersdorff (École Polytechnique)
Warsaw, October 19th 2009

Collaboration with J.A. Cabrer and M. Quirós
OUTLINE

• Features of Warped Extra Dimensions

• Stabilizing Models with 2 branes

• Soft Wall models (Models with 1 brane)

• Stabilizing the Soft Wall

• (Soft Walls and the Cosmological Constant)
OPEN QUESTIONS IN THE SM (AND BEYOND)

• What is the origin of Electroweak Symmetry Breaking?

• Why is the scale of the $Z$ and $W$ bosons $10^{17}$ times smaller than the Planck mass? (Hierarchy Problem)

• Why is there such a huge hierarchy in the masses of the Standard Model fermions?

• What is the origin of neutrino masses?

• If there is Supersymmetry, how is it broken?

• If there is a Grand Unified Theory, how is it broken to the SM, and why are there no colored Higgses?
All these issues can be addressed in models with Extra Dimensions
\[ ds^2 = e^{-2ky} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]
RS MODELS

\[ ds^2 = e^{-2k_y} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]

Warp factor

Fifth Dimension

4D Boundary

Randall & Sundrum ‘99
RS MODELS

Fifth Dimension

Warp factor

Fundamental cutoff scale is redshifted

$ds^2 = e^{-2ky} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$

Randall & Sundrum '99

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RS MODELS

Fifth Dimension

\[ ds^2 = e^{-2ky} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]

Warp factor

4D Boundary

\[ M_{5d} \]

Fundamental cutoff scale is redshifted
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\[ ds^2 = e^{-2ky} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]

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FERMIION MASSES

Arkani-Hamed & Schmalz '00, Shifman & Dvali '00, Gherghetta & Pomarol '00
FERMI\textsc{ON} \textsc{MASSES}

UV Brane

Higgs

IR brane

Arkani-Hamed & Schmalz '00, Shifman & Dvali '00, Gherghetta & Pomarol '00
FERMIION MASSES

5D fields

Wave functions

\[ q(x^\mu, y) = Q_0(y) q_0(x^\mu) \]
(+ KK modes)

4D fields

Higgs

UV Brane

IR brane

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FERMION MASSES

5D fields

Wave functions

\[ q(x^\mu, y) = Q_0(y) q_0(x^\mu) \]

(+ KK modes)

4D fields

Higgs

Fermions: 5D masses control localization of wave functions

UV Brane

IR brane

\[ Y_{4D} = Y_{5d} Q_0(y_1) U_0(y_1) \]

- Localize heavy fermions on IR brane
- Localize light fermions on UV brane

Arkani-Hamed & Schmalz '00, Shifman & Dvali '00, Gherghetta & Pomarol '00
SUMMARY RS MODELS
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• Extra Dimensions with WARPED background successful for
  • Explaining electroweak hierarchy
  • Explaining fermion mass hierarchy
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• Extra Dimensions with WARPED background successful for
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  • Explaining fermion mass hierarchy

• Other features
  • Distinctive Collider Signature (KK gravitons)
  • Dual to strongly coupled gauge theories in 4D
  • “Modelling QCD”
PROBLEMS

• Pure 5D Gravity with negative Cosmological constant (and appropriate brane tensions) has RS as a solution.

• BUT: Interbrane distance is **UNDETERMINED**

• There is an extra **massless** mode (RADION)

\[ g_{MN} = g_{MN}^{RS} + \begin{pmatrix} h_{\mu\nu} & h_{55} \end{pmatrix} \]

• Both brane tensions need to be **fine tuned**
STABILIZATION

• The question of Radius stabilization
  • What determines DISTANCE between UV and IR brane?
  • How can I generate a POTENTIAL and a MASS for the Radion?
  • What ensures that the 4D metric is FLAT?
  • How NATURAL is it?
SUPERPOTENTIAL METHOD
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- Gravity + scalar field with bulk and brane potential
- Solve Einstein equations coupled to scalar

\[ ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]

\[ \lambda_0(\phi) \quad \lambda_1(\phi) \]

\[ V(\phi) \]
SUPERPOTENTIAL METHOD

- Gravity + scalar field with bulk and brane potential
- Solve Einstein equations coupled to scalar

\[ ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 \]

- Define a “Superpotential” \( V(\phi) = 3W'(\phi)^2 - 12W^2(\phi) \)
- Einstein equations become \( \phi'(y) = W'(\phi) \quad A'(y) = W(\phi) \)

- Boundary values from Minimizing the 4D potentials \( V_i(\phi) = \lambda_i(\phi) - 6W(\phi) \)

DeWolfe et al '99, Brandhuber & Sfetsos '99
STABILIZATION
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- Solve to get bulk profiles \( \phi'(y) = W'(\phi) \quad A'(y) = W(\phi) \)
- Minimize to get brane values \( V_i(\phi_i) = \lambda_i(\phi_i) - 6 W(\phi_i) \)
STABILIZATION

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- Notice that \( e^{k y_1} = 10^{16} \quad \Rightarrow \quad k y_1 \approx 37 \)
- Choose some suitable \( W \) such that
  \[ k y_1 = \int_{\phi_0}^{\phi_1} \frac{1}{W'} \approx 37 \]
- Now shift superpotential \( W \rightarrow W + k \)
  \( \quad A(y) \rightarrow A(y) + k y \)
- Adds warping without changing the value of \( k y_1 \)

DeWolfe et al ’00, Cabrer, GG & Quirós ’09
- Take \( W'(\phi) = kb\phi \)
- Then \( ky_1 = b^{-1} \log \phi_1 / \phi_0 \approx 37 \)
- Moderate fine tuning necessary
- Metric \( A(y) = ky + \frac{1}{2} \phi_0^2 (e^{2bky} - 1) \)
- Backreaction small:
  \[
  \frac{BR}{ky} < 0.01 \quad \text{for} \quad b^{-1} = 37, \ \phi_0 = 1/e, \ \phi_1 = 1
  \]
Do we need two branes?
GAUGE/GRAVITY DUALITY
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- Gravity/Gauge theory correspondence stipulates that the 5D theory is **dual** to a strongly coupled 4D gauge theory that
  - is approximately **conformal** in the UV
  - has large number of colors
  - describes the **same physics** as 5D theory

Monday, October 19, 2009
Gauge/Gravity Duality

- Gravity/Gauge theory correspondence stipulates that the 5D theory is dual to a strongly coupled 4D gauge theory that
  - is approximately conformal in the UV
  - has large number of colors
  - describes the same physics as 5D theory
- KK modes correspond to resonances of gauge theory
  - RS with two branes: KK spectrum is roughly $m_n^2 \sim n^2$
  - 4D strongly coupled gauge theories have many more possibilities.
POSSIBLE SPECTRA

\[ \rho(M^2) \]

- **Regge** \( M^2 \sim n \)
- **RS** \( M^2 \sim n^2 \)
- **Conformal**
- **Conformal + Gap (Unparticles)**
IR brane can be replaced by SOFT WALL
SOFT WALLS
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Soft Walls models only possess a single (UV) brane, but nevertheless exhibit a finite length in the 5th dimension. The IR brane is replaced by a curvature singularity at which the metric vanishes.
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SOFT WALLS

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Profiles diverge at finite $y$ if $W(\phi) \sim \phi^2$ or faster!
APPLICATIONS
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• Things that **CAN** be done with Soft Walls
  • Electroweak Breaking
  • Strong interactions (AdS/QCD)

Batell, Gherghetta & Sword ’08, Falkowski & Perez-V. ’08, Cabrer, GG & Quiros (in progr.)
Karch et al ’06, Gursoy et al ’07, Batell & Gherghetta ’08,
APPLICATIONS

• Things that **CAN** be done with Soft Walls
  
  • Electroweak Breaking
  
  • Strong interactions (AdS/QCD)

• Things that **CANNOT** be done with Soft Walls
  
  • Solve Cosmological Constant problem

Source references:

- Batell, Gherghetta & Sword ’08, Falkowski & Perez-V. ’08, Cabrer, GG & Quiros (in progr.)
- Karch et al ’06, Gursoy et al ’07, Batell & Gherghetta ’08,
- Arkani-Hamed et al ’00, Kachru, Schulz & Silverstein ’00, Csaki et al ’00
- Forste et al ’00, Cabrer, GG & Quirós ’09
SPECTRA WITH SOFT WALLS
SPECTRA WITH SOFT WALLS

• Even though the **physical length** is finite, the **conformal length** might be either finite or infinite:

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<th>Conformally flat coordinates</th>
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• In the conformally flat frame, the KK spectrum of any bulk field follows a Schrödinger Equation

$$-\psi''(z) + \hat{V}(z)\psi(z) = m^2 \psi(z)$$

Depends on the background
NON CONFINING POTENTIALS
Conformal length infinite

Conformal (continuous)

Unparticle (continuous + gap)

Confining Potentials
Conformal length finite or infinite

Regge: $M^2 \sim n$

RS like: $M^2 \sim n^2$
### SOFT WALL SPECTRA

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<tr>
<td>mass spectrum</td>
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**Table 1:** Spectra resulting from different asymptotic forms of the superpotential.

In the first row we give the asymptotic behavior of \( W(\phi) \), with the strength of the divergence increasing from left to right (\( > \) means “diverges faster than”, etc).

Second and third row show the finiteness of \( y_s \) and \( z_s \), with the behavior changing at \( W \sim \phi^2 \) and \( W \sim e^{\phi} \) respectively. The third row shows the spectrum, while in the last one we indicate the consistency of the solution.

There are a number of phenomenological applications which are outside the scope of the present paper but which are worth of future investigations. For the range of the parameter \( 1 < \nu \leq 2 \) these applications are common with two brane models, as RS1, but with some peculiarities. In particular graviton (and radion) KK modes are at the \( \text{TeV} \) scale and they can be produced and decay at LHC by their interaction with matter \( \sim h \mu \nu T_{\mu \nu} \), so they are expected to be produced through gluon annihilation [25]. Since there is no IR brane, for soft-wall models to solve the gauge hierarchy problem the Higgs boson (either a scalar doublet or the fifth component of a gauge field in a gauge-Higgs unified model) has to propagate in the bulk and it has to be localized near the singularity for its mass to feel the warping. On the other hand fermions with sizable Yukawa couplings (third generation fermions) should be localized near the singularity as well while first and second generation fermions can propagate at (or near the) UV brane. As we have seen that the first graviton KK mode is localized near the singularity, once produced it is expected to decay into either Higgs or \( t \bar{t} \) pairs. For \( \nu = 1 \) the mass spectrum of fields propagating in the bulk is a continuum above an \( O(\text{TeV}) \) mass gap. This continuum (endowed with a given conformal dimension) can interact with SM fields propagating in the UV brane as operators of a CFT, where the conformal invariance is explicitly broken at a scale given by the mass gap, and can model and describe the unparticle phenomenology. In particular the Higgs embedded into such 5D background can describe the unHiggs theory of Ref. [26].

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Conjectured by Gursoy et al '07, Cabrera, GG & Quirós '09
### Soft Wall Spectra

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Asymptotic behaviour of $W$

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Gursoy et al '07, Cabrer, GG & Quirós '09
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Asymptotic behaviour of $W$

Singularity in “proper distance”

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Gursoy et al '07, Cabrera, GG & Quirós '09
**SOFT WALL SPECTRA**

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Singularity in “proper distance”

Singularity in “conformal distance”

Gursoy et al ’07, Cabrera, GG & Quirós ’09
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Asymptotic behaviour of $W$
Singularity in “proper distance”
Singularity in “conformal distance”
Asymptotic form of the spectrum

Gursoy et al ’07, Cabrera, GG & Quirós ’09
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Asymptotic behaviour of $W$

Singularity in “proper distance”

Singularity in “conformal distance”

Asymptotic form of the spectrum

Gursoy et al ’07, Cabrer, GG & Quirós ’09
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Asymptotic behaviour of $W$
- Singularity in “proper distance”
- Singularity in “conformal distance”
- Asymptotic form of the spectrum

Finite Length
Mass gap appears

Gursoy et al ’07, Cabrer, GG & Quirós ’09
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<th>$&gt; e^\phi \phi^{\frac{1}{2}}$</th>
<th>$\geq e^{2\phi}$</th>
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<td>$y_s$</td>
<td>$\infty$</td>
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<td>finite</td>
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<tr>
<td>$z_s$</td>
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<td>$\infty$</td>
<td></td>
<td>finite</td>
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<td>mass</td>
<td>continuous</td>
<td>continuous</td>
<td>w/ mass gap</td>
<td>discrete</td>
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<td>spectrum</td>
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<tr>
<td>consistent</td>
<td>yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>no</td>
</tr>
<tr>
<td>solution</td>
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Asymptotic behaviour of $W$
- Singularity in “proper distance”
- Singularity in “conformal distance”
- Asymptotic form of the spectrum

Finite Length
- Mass gap appears
- Spectrum discrete

Gursoy et al ’07, Cabrer, GG & Quirós ’09
SOFT WALL STABILIZATION
- Stabilization works similar as before
- Choose some suitable $W$ such that

$$ky_s = \int_{\phi_0}^{\infty} \frac{1}{W'(\phi)} \approx 37$$

- Now shift superpotential $W \rightarrow W + k$

$$A(y) \rightarrow A(y) + ky$$

- Shift does not change position of singularity
SOFT WALL STABILIZATION

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  $A(y) \rightarrow A(y) + k y$

- Shift does not change position of singularity

The Warping affects the Mass scale:
- The Unparticle mass gap
- The level spacing in the discrete case

Warped down

Cabrera, GG & Quirós ’09
Consider the class of models \( W(\phi) = k(1 + e^{\nu \phi}) \)

\[
k y_s = \frac{1}{\nu^2} e^{-\nu \phi_0} \approx 37 \quad \text{for } O(1) \text{ negative values for } \phi_0
\]

Spectrum can be
- Continuous
- Continuous+gap
- Discrete
THE GRAVITON SPECTRUM
Figure 3: Mass modes for the graviton, computed for \( k_x > 48 \). The massless \((n = 0)\) and the first 5 massive modes \((n = 1, \ldots, 5)\) are shown.

Moreover one can find an expression for the spacing of the mass eigenstates by approximating the potential as an infinite well, which is valid for \( m^2 \gg V_h \). The result of this approximation is

\[
\Delta m \approx \rho \pi \Gamma \left(1 - \frac{1}{\nu^2}\right) = \pi \zeta s.
\] (4.26)

Note that the mass spectrum is linear \((m_n \sim n)\), and that as one approaches \( \nu = 1 \), \( \Delta m \to 0 \), recovering the expected continuous spectrum at this value \((for \nu < 1 the spectrum is continuous too, since (4.26) is only valid for \nu > 1)\). The numerical result for the mass eigenvalues is shown in Fig. 3 where these behaviours can be observed.

Some profiles for the graviton computed numerically using the equation of motion (4.6) and the boundary conditions (4.7) are shown in Fig. 4. Numerically one finds that the scaling property in Eq. (4.25) ceases to be valid for \( k_x \lesssim 3 \), as discrepancies from this behaviour become greater than 1%.
THE GRAVITON SPECTRUM

Figure 3: Mass modes for the graviton, computed for $k_y s > 4.8$. The massless ($n = 0$) and the first 5 massive modes ($n = 1, \ldots, 5$) are shown.

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$$\Delta m \approx \rho \pi \Gamma (1 - 1/\nu^2) = \pi z_s.$$

(4.26)

Note that the mass spectrum is linear ($m_n \sim n$), and that as one approaches $\nu = 1$, $\Delta m \rightarrow 0$, recovering the expected continuous spectrum at this value. For $\nu < 1$ the spectrum is continuous too, since (4.26) is only valid for $\nu > 1$.

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THE GRAVITON SPECTRUM

Figure 3: Mass modes for the graviton, computed for $k_{y}s > 4$. The massless ($n=0$) and the first 5 massive modes ($n=1, \ldots, 5$) are shown.

Moreover one can find an expression for the spacing of the mass eigenstates by approximating the potential as an infinite well, which is valid for $m^2 \gg V_h$. The result of this approximation is

$$\Delta m \simeq \frac{\rho \pi}{\Gamma (1 - 1/\nu^2)} = \frac{\pi z_s}{\nu}.$$  \hspace{1cm} (4.26)

Note that the mass spectrum is linear ($m_n \sim n$), and that as one approaches $\nu = 1$, $\Delta m \to 0$, \hspace{1cm} (4.27)

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gap+very densely spaced discretuum
THE GRAVITON SPECTRUM

Figure 3: Mass modes for the graviton, computed for $k_y > 4.8$. The massless ($n = 0$) and the first 5 massive modes ($n = 1, \ldots, 5$) are shown.

Moreover one can find an expression for the spacing of the mass eigenstates by approximating the potential as an infinite well, which is valid for $m^2 \gg V_h$. The result of this approximation is

$$\Delta m \simeq \frac{\rho \pi \Gamma(1 - 1/\nu^2)}{\sqrt{\nu}} = \frac{\pi \nu}{z_s}.$$ (4.26)

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Discrete, hard-wall like
The boundary equations on the brane depend on the brane tension $\lambda(\phi)$. The precise form of the dependence can be found in [21]. At the singularity, similarly to the graviton case, one gets the boundary equation

$$e^{-4A(y)}\phi'(y)|_{y_s=0},$$

(4.33)
Figure 5: Mass modes for the radion, computed for values of $k_{ys} > 4.8$. The first 6 massive modes ($n = 0, \ldots, 5$) are shown.

Next to the singularity, using (4.19) the potential is approximated by

$$V_F|y \approx y_s \approx 6 \nu^2 + 3\nu^4 \rho^2 \left[ k(y_s - y) \right]^2 - \frac{2}{\nu^2} \sim 6 \nu^2 + 3\nu^4 \left( 1 - \nu^2 \right)^2 \frac{1}{(z_s - z)^2},$$

(4.42)

The behaviour of this solution near the singularity is $\tilde{F}(z) \sim c_1 J_\nu(\Delta z) (2\nu^2 + 1) / (2\nu^2 - 2) + c_2 J_\nu(\Delta z) (6\nu^2 - 3) / (2\nu^2 - 2) + c_3 Y_\nu(\Delta z) - 3 / (2\nu^2 - 2)$.

(4.45)

Using (4.37) we can compute the behaviour of the field and apply the normalizability condition (4.34),

$$\tilde{\phi}(z) \sim c_1' J_\nu(\Delta z) 3 / (2\nu^2 - 2) + c_2' J_\nu(\Delta z) (\nu^2 + 2) / (\nu^2 - 1) + c_3' Y_\nu(\Delta z),$$

(4.46)

and the boundary condition (4.33),

$$e^{-A}(z) \dot{\phi}(z) \sim c_1'' J_\nu(\Delta z) (\nu^2 + 2) / (\nu^2 - 1) + c_2'' Y_\nu(\Delta z) (\nu^2 - 2) / (\nu^2 - 1).$$

(4.47)
THE RADION SPECTRUM

Figure 5: Mass modes for the radion, computed for values of $k_y > 4.8$. The first 6 massive modes ($n = 0, \ldots, 5$) are shown.

Next to the singularity, using (4.19) the potential is approximated by

$$V_F | \nu_s | \approx \nu_s \approx 6 \nu^2 + 3 \nu^4 \rho^2 \left[ k(y_s - y) \right]^2 - 2 / \nu^2 \sim 6 \nu^2 + 3 (1 - \nu^2)^2 \left[ z_s - z \right]^2,$$

(4.42)

The behaviour of this solution near the singularity is

$$\tilde{F}(z) \sim c(1) J(\Delta z) (2 \nu^2 + 1) / (2 \nu^2 - 2) + c(2) J(\Delta z) (6 \nu^2 - 3) / (2 \nu^2 - 2) + c(1) Y(\Delta z) - 3 / (2 \nu^2 - 2),$$

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Using (4.37) we can compute the behaviour of the field and apply the normalizability condition (4.34),

$$\tilde{\phi}(z) \sim c'(1) J(\Delta z) 3 / (2 \nu^2 - 2) + c'(1) Y(\Delta z) - 3 / (2 \nu^2 - 2),$$

(4.46)

and the boundary condition (4.33),

$$e^{-3 A(z)} \dot{\phi}(z) \sim c''(1) J(\Delta z) (\nu^2 + 2) / (\nu^2 - 1) + c''(1) Y(\Delta z) (\nu^2 - 2) / (\nu^2 - 1).$$

(4.47)

NO ZERO MODE

Monday, October 19, 2009
THE CC PROBLEM
THE CC PROBLEM

\[ V(\phi) = 3W'(\phi)^2 - 12W^2(\phi) \]
\[ \phi'(y) = W'(\phi) \]
\[ A'(y) = W(\phi) \]

\{ 3 \text{ constants of integration} \}
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\{ 3 constants of integration \}

2 Branes:

\[ V_0'(\phi_0) = 0 \]
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\[ V_1'(\phi_1) = 0 \]
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\{ 4 boundary conditions \}

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1 fine tuning
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No fine tuning??
THE CC PROBLEM

This is incorrect!!!

Forste et al ’00, Cabrer, GG & Quirós ’09
THE CC PROBLEM

This is incorrect!!!

\[ 4D \text{ CC} = \int dy \mathcal{L}^{\text{on-shell}} = 0 \]

Forste et al ’00, Cabrer, GG & Quirós ’09
THE CC PROBLEM

This is incorrect!!!

\[ 4D \text{ CC} = \int dy \mathcal{L}^{\text{on-shell}} = \lim_{y \to y_s} W[\phi(y)]e^{-4A(y)} \]

Forste et al ’00, Cabrero, GG & Quirós ’09
THE CC PROBLEM

This is incorrect!!!

\[
4D \text{ CC } = \int dy \mathcal{L}^{\text{on-shell}} = \lim_{y \to y_s} W[\phi(y)] e^{-4A(y)}
\]

- There is a contribution to the CC from the singularity!

Forste et al ’00, Cabrер, GG & Quirós ’09
This is incorrect!!!

- There is a contribution to the CC from the singularity!
- Equations of motion are NOT satisfied at the singularity

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- Can be fixed by choosing the integration constant for W

Forste et al '00, Cabrer, GG & Quirós '09
THE CC PROBLEM

This is incorrect!!!

- There is a contribution to the CC from the singularity!
- Equations of motion are NOT satisfied at the singularity
- Can be fixed by choosing the integration constant for W
- Fine tuning is restored

\[ 4D \text{ CC} = \int dy \mathcal{L}^{\text{on-shell}} = \lim_{y \to y_s} W[\phi(y)] e^{-4A(y)} \]

Forste et al '00, Cabrer, GG & Quirós '09
CONCLUSIONS

• RS models provide neat way of obtaining electroweak and fermion mass hierarchy

• Stabilization can be achieved by adding extra scalar field

• IR brane can be consistently replaced by Soft Walls

• Spectra of Soft Wall models richer than in usual RS (gapped continuum, gapped, discretuum, Regge-like, etc.)

• Stabilization can be achieved without ANY fine tuning