Dark matter(s), MSSM Higgs and Beyond

Dumitru Ghilencea

CERN-TH and IFIN-HH Bucharest

arXiv:1001.3884 [hep-ph], 0911.1134 [hep-ph] (with G. Ross, S. Cassel)

arXiv:0910.1100 [hep-ph] (with I. Antoniadis, E. Dudas, P. Tziveloglou) Warsaw, 04.Feb.2010

- Outline:

- Fine tuning in the CMSSM: a two-loop result.
- Going beyond MSSM Higgs: an effective approach with d=5 & d=6 operators.
- Conclusions.

- The Standard Model (SM): our best model so far. Too many unanswered questions.

  (hierarchy problem, EWSB, unification of interactions,....)

- MSSM: best framework for SUSY searches. Not without problems of its own. Testable at the LHC.

- MSSM scalar potential:

$$V = m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (m_3^2\;H_1\cdot H_2 + h.c.) + \frac{1}{2}\lambda_1\,|H_1|^4 + \frac{1}{2}\lambda_2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2$$

$$+\;\lambda_4\,|H_1\cdot H_2|^2 + \left[\frac{1}{2}\lambda_5\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\right]$$

$$\lambda_{1,2} = (g_1^2+g_2^2)/4, \quad \lambda_3 = (g_2^2-g_1^2)/4, \quad \lambda_4 = -g_2^2/2, \quad \lambda_{5,6,7} = 0$$

$$m^2 \equiv m_1^2\cos^2\beta + m_2^2\sin^2\beta - m_3^2\sin 2\beta, \quad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\left(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\right)$$

- The problem:

$$v^2 = -m^2/\lambda, \qquad v = \mathcal{O}(100\ \mathrm{GeV}), \quad \lambda < 1, \qquad \text{but} \quad m,\, m_{1,2,3} \sim \mathcal{O}(1\,\mathrm{TeV}).$$

- “little hierarchy”, “residual” fine-tuning. Also $m_h < m_Z$ but LEPII bound: 114.4 GeV

$\Rightarrow$ large QC needed $\Rightarrow$ larger m_0 needed (so that $m_h > 114.4$ GeV). QC can increase λ.

- A measure of "fine" tuning:

$$v^2 = -m^2/\lambda, \quad 2\lambda\frac{\partial m^2}{\partial\beta} = m^2\frac{\partial\lambda}{\partial\beta}, \quad \Rightarrow \quad m^2, \lambda = F(p, \beta(p)),$$

$$\Delta \equiv \max\left|\Delta_p\right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

[J. Ellis et al 1986]

[Barbieri, Giudice 1988]

- Most general formula: [Casas et al]

$$\Delta_p = -\frac{p}{z}\left[\left(2\frac{\partial^2 m^2}{\partial\beta^2} + v^2\frac{\partial^2\lambda}{\partial\beta^2}\right)\left(\frac{\partial\lambda}{\partial p} + \frac{1}{v^2}\frac{\partial m^2}{\partial p}\right) + \frac{\partial m^2}{\partial\beta}\frac{\partial^2\lambda}{\partial\beta\partial p} - \frac{\partial\lambda}{\partial\beta}\frac{\partial^2 m^2}{\partial\beta\partial p}\right]$$

$$z \equiv \lambda\left(2\frac{\partial^2 m^2}{\partial\beta^2} + v^2\frac{\partial^2\lambda}{\partial\beta^2}\right) - \frac{v^2}{2}\left(\frac{\partial\lambda}{\partial\beta}\right)^2$$

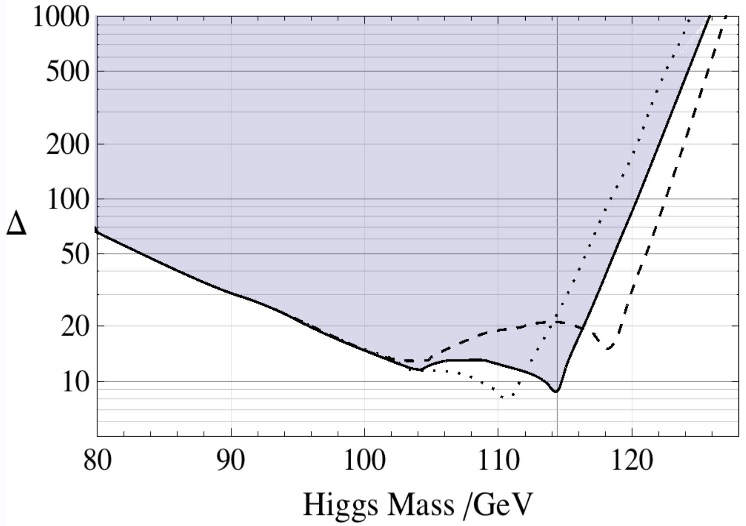
(1). Preferable: minimise Δ, for a low sensitivity to UV parameters.

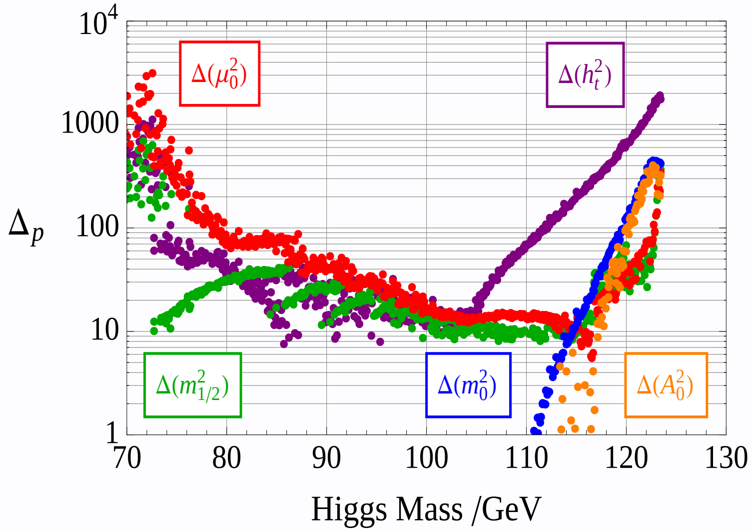
(2). At present: Δ: 1-loop. $\Delta \sim 50$ but $\Delta \sim m_0^2 \sim \exp(m_h^2/v^2)\,!$. [Pokorski, Ellis et al]

$\Rightarrow$ Two-loop calculation: SOFTSUSY (micrOMEGAs). Run time: 6y (on 30 $\times$ 3 GHz).

(3). constraints: Theor + Exp: SUSY masses, $b \rightarrow s\,\gamma$, $\delta a_\mu, \delta\rho$,... but NO LEPII bound on m_h!

- Two-loop results in CMSSM: [Cassel, DG, Ross]





$(\alpha_3, m_t) = (0.1176, 173.1)$.

dashed line: $(0.1156, 174.4)$.

dotted line: $(0.1196, 171.8)$.

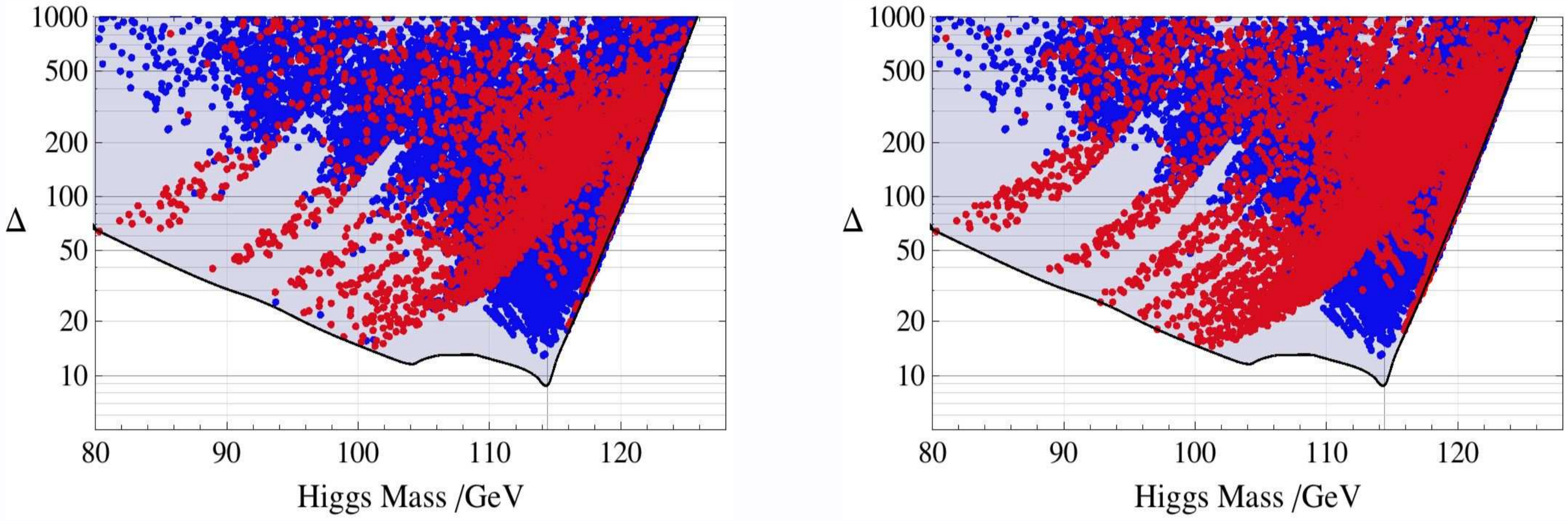
$\Delta \equiv \max|\Delta_p|, \quad p = \mu_0^2, m_0^2, A_0^2, B_0^2, m_{1/2}^2,$

$\Delta_{\mu_0^2}$, $\Delta_{m_0^2}$ dominant.

- Δ has exponential dependence on m_h, $\Delta \propto \exp(m_h^2/v^2) \Rightarrow$ Two-loop effects important!
- without imposing LEPII bound, minimising electroweak Δ gives a prediction:

$\Delta = 8.8 \Rightarrow m_h = 114 \pm 2$ GeV ie just above the LEPII bound!

• Two-loop results in CMSSM: [Cassel, DG, Ross]



LSP: good dark matter candidate ⇒ dark matter constraint:

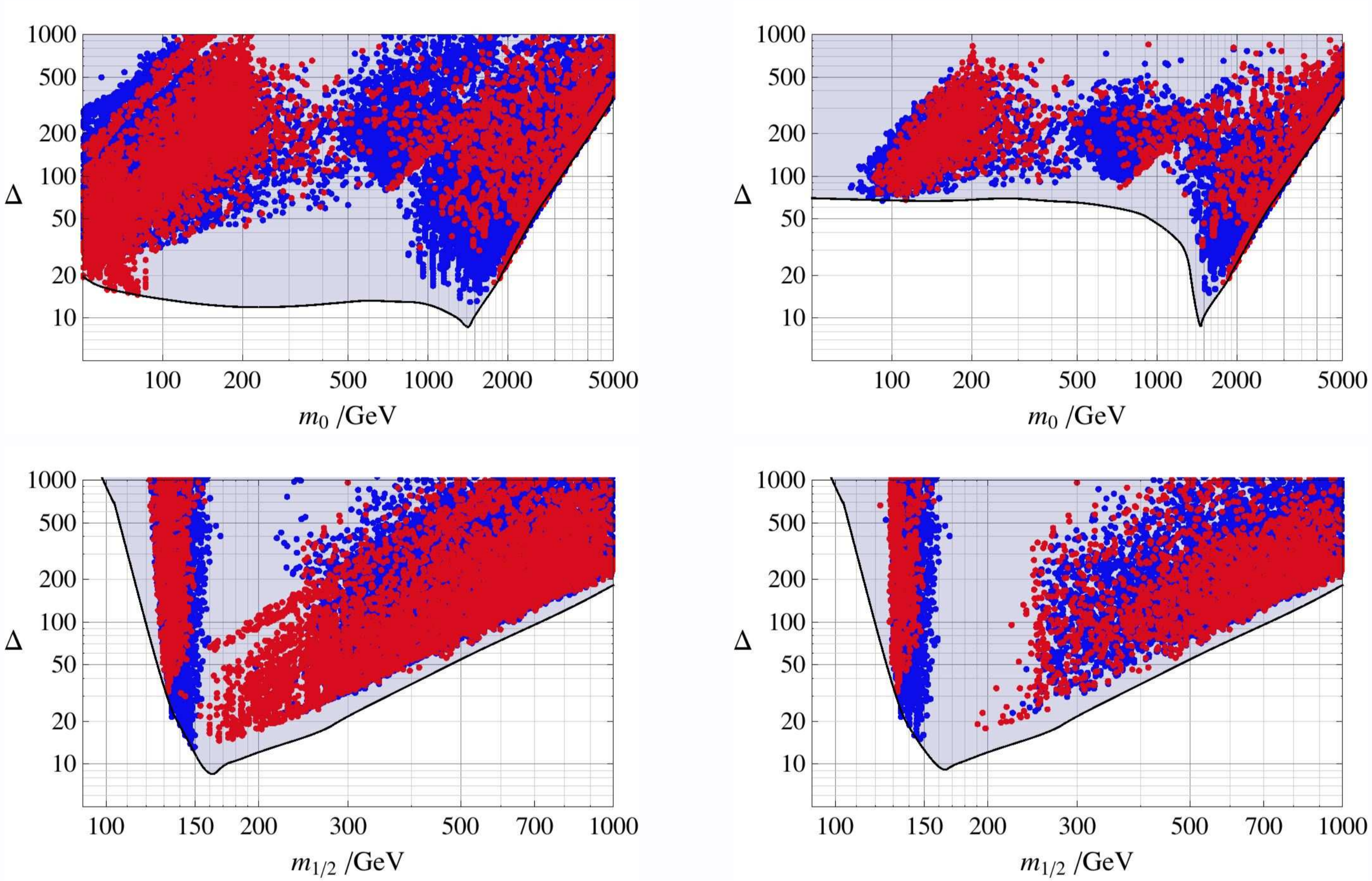
WMAP: $\Omega h^2 = 0.1099 \pm 0.0062$; blue: Ωh^2 not-saturated; red: Ω saturated: 1σ (left); 3σ (right).

• Prediction from: Min Δ + "right" dark matter abundance (no LEPII bound):

$$m_h = 114.7 \pm 2 \text{ GeV}, \quad \Delta = 15.0, \quad \text{(sub-saturating WMAP bound)}.$$
$$m_h = 115.9 \pm 2 \text{ GeV}, \quad \Delta = 17.8, \quad \text{(saturating WMAP within } 3\sigma).$$

• setting an upper bound on Δ ⇒ bounds on SUSY spectrum.

• Two-loop results in CMSSM: [Cassel, DG, Ross]



WMAP: $\Omega h^2 = 0.1099 \pm 0.0062$ blue: Ωh^2 not-saturated; red: Ω saturated.

Right figures: $m_h > 114.4$ GeV, brings further restrictions on $m_{1/2}, m_0$ values.

• The favoured CMSSM spectrum of minimal $\Delta = 15$ with a sub-saturation of Ω.

h^0	114.7	$\tilde{\chi}_1^0$	79	$\tilde{b}_1$	1147	$\tilde{u}_L$	1444
H^0	1264	$\tilde{\chi}_2^0$	142	$\tilde{b}_2$	1369	$\tilde{u}_R$	1446
$H^\pm$	1267	$\tilde{\chi}_3^0$	255	$\tilde{\tau}_1$	1328	$\tilde{d}_L$	1448
A^0	1264	$\tilde{\chi}_4^0$	280	$\tilde{\tau}_2$	1368	$\tilde{d}_R$	1446
$\tilde{g}$	549	$\tilde{\chi}_1^\pm$	142	$\tilde{\mu}_L$	1406	$\tilde{s}_L$	1448
$\tilde{\nu}_\tau$	1366	$\tilde{\chi}_2^\pm$	280	$\tilde{\mu}_R$	1406	$\tilde{s}_R$	1446
$\tilde{\nu}_\mu$	1404	$\tilde{t}_1$	873	$\tilde{e}_L$	1406	$\tilde{c}_L$	1444
$\tilde{\nu}_e$	1404	$\tilde{t}_2$	1158	$\tilde{e}_R$	1406	$\tilde{c}_R$	1446

$\Rightarrow$ light gluino, charginos, neutralino; heavy squarks, sleptons.

• If $\Delta < 100$: the upper limit beyond which we consider SUSY failed to solve hierarchy problem, then

$$\begin{array}{rcl} m_h &<& 121 \text{ GeV} \\ \mu &<& 680 \text{ GeV} \\ m_0 &<& 3.2 \text{ TeV} \end{array} \qquad \begin{array}{rcccl} 5.5 &<& \tan\beta &<& 55 \\ 120 \text{ GeV} &<& m_{1/2} &<& 720 \text{ GeV} \\ -2.0 \text{ TeV} &<& A_0 &<& 2.5 \text{ TeV} \end{array}$$

and

$\tilde{g}$	χ_1^0	χ_2^0	χ_3^0	χ_4^0	$\chi_1^\pm$	$\chi_2^\pm$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$
1720	305	550	660	665	550	670	2080	2660	2660	3140

• These values scale as $\sqrt{\Delta}$.

• What if minimal Δ scenario is not realised? Can one still have low Δ and large m_h?

(1). relax some of CMSSM constraints ...

(2). "New physics", beyond MSSM Higgs sector, can reduce Δ at larger m_h.

⇒ (2). Effective approach: MSSM+(d=5), (d=6) operators:

$$\mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_i(\propto 1/M_*) + \mathcal{O}_i(\propto 1/M_*^2)$$

• What if minimal Δ scenario is not realised? Can one still have low Δ and large m_h?

(1). relax some of CMSSM constraints ...

(2). "New physics", beyond MSSM Higgs sector, can reduce Δ at larger m_h.

$\Rightarrow$ (2). Effective approach: MSSM+(d=5), (d=6) operators:

$$\mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_i(\propto 1/M_*) + \mathcal{O}_i(\propto 1/M_*^2)$$

• d=5 operators beyond MSSM Higgs: [Seiberg, Dine]

[DG, Antoniadis, Dudas]

$$\mathcal{L}_1 = \frac{1}{M_*}\int d^2\theta\, \zeta(S)\,(H_2.H_1)^2 + h.c., \qquad [S,T]$$

$$\mathcal{L}_2 = \frac{1}{M_*}\int d^4\theta\, \Big\{ A(S,S^\dagger) D^\alpha \Big[B(S,S^\dagger)\, H_2\, e^{-V_1}\Big] D_\alpha \Big[C(S,S^\dagger)\, e^{V_1}\, H_1\Big] + h.c.\Big\}, \quad [D]$$

$$\frac{1}{M_*}\zeta(S) = \zeta_0 + \zeta_1\, m_0\, \theta\theta, \qquad \zeta_0,\ \zeta_1 \sim 1/M_*,$$

$$A(S,S^\dagger) = a_0 + a_1\, S + a_1^*\, S^\dagger + a_2\, S\, S^\dagger, \qquad S = \theta\theta\, m_0,$$

• $\mathcal{L}_2$ can be removed by general non-linear, field redefinitions. $\Rightarrow$ only $\mathcal{L}_1$ left (d=5).

• d=6 operators beyond MSSM Higgs: [DG, Antoniadis, Dudas,Tziveloglou]

- not always related to d=5 ones, if be generated by different physics/scale.
- are $\tan\beta$ enhanced, compensating the $1/M_*$ suppression relative to d=5 op's.

$\Rightarrow$ effects comparable to $d = 5$ operators!

$$\begin{aligned}
\mathcal{O}_i &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_i(S,S^\dagger)\ (H_i^\dagger\, e^{V_i}\, H_i)^2, \qquad i=1,2. \qquad (T, U(1)) \\
\mathcal{O}_3 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_3(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2), \qquad (T, U(1)) \\
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_4(S,S^\dagger)\ (H_2\, H_1)\,(H_2\, H_1)^\dagger, \qquad (S) \\
\mathcal{O}_5 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_5(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2\, H_1 + h.c.), \qquad (2\,D, S) \\
\mathcal{O}_6 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_6(S,S^\dagger)\ (H_2^\dagger\, e^{V_2}\, H_2)\,(H_2\, H_1 + h.c.), \qquad (2\,D, S) \\
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\ \frac{1}{16\kappa\, g_i^2}\mathcal{Z}_7(S,0)\ \mathrm{Tr}\, W_i^\alpha\, W_{i,\alpha}\,(H_2\, H_1) + h.c., \\
\mathcal{O}_8 &= \frac{1}{M_*^2}\int d^4\theta\ \Big[\mathcal{Z}_8(0,S^\dagger)\ (H_2\, H_1)^2 + h.c.\Big]
\end{aligned}$$

There are also operators involving extra derivatives [DG, Antoniadis, Dudas]

$$\begin{aligned}
\mathcal{O}_9 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_9(S,S^\dagger)\ H_1^\dagger\,\overline{\nabla}^2\,e^{V_1}\,\nabla^2\,H_1 \\
\mathcal{O}_{10} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{10}(S,S^\dagger)\ H_2^\dagger\,\overline{\nabla}^2\,e^{V_2}\,\nabla^2\,H_2 \\
\mathcal{O}_{11} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{11}(S,S^\dagger)\ H_1^\dagger\,e^{V_1}\,\nabla^\alpha\,W_\alpha^{(1)}\,H_1 \\
\mathcal{O}_{12} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{12}(S,S^\dagger)\ H_2^\dagger\,e^{V_2}\,\nabla^\alpha\,W_\alpha^{(2)}\,H_2 \\
\mathcal{O}_{13} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{13}(S,S^\dagger)\ H_1^\dagger\,e^{V_1}\,W_\alpha^{(1)}\nabla^\alpha\,H_1 \\
\mathcal{O}_{14} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{14}(S,S^\dagger)\ H_2^\dagger\,e^{V_2}\,W_\alpha^{(2)}\nabla^\alpha\,H_2 \\
\mathcal{O}_{15} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{15}(S,S^\dagger)\ \mathrm{Tr}\,e^V\,W^\alpha\,e^{-V}\,D^2\,e^V\,W_\alpha\,e^{-V}
\end{aligned}$$

where $\nabla_\alpha H_i \equiv e^{-V_i}\,D_\alpha\,e^{V_i}\,H_i,\quad V_i = V_W^a\,\sigma^i/2 + (\mp 1/2)\,V_Y;\quad i=1,2.$ and

$$\frac{1}{M_*^2}\,\mathcal{Z}_j(S,S^\dagger) = \alpha_{j0} + \alpha_{j1}\,m_0\,\theta\theta + \alpha_{j1}^*\,m_0\,\overline{\theta\theta} + \alpha_{j2}\,m_0^2\,\theta\theta\overline{\theta\theta}, \qquad \alpha_{jk} \sim 1/M_*^2.$$

- $\mathcal{O}_{9,\ldots,15}$ can be removed by field redefinitions or eqs motion.$\Rightarrow \mathcal{L}_{total} = \mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_1 + \sum_{i=1}^{8}\mathcal{O}_i$.

- Corrections to m_h of order $\mathcal{O}(1/M_*)$: [DG, Antoniadis, Dudas, Tziveloglou]

$$m^2_{h,H} \;=\; \Big[m_h^2\Big]^{1-loop}_{MSSM} + 2\,\zeta_0\,\mu_0\; v^2\,\sin 2\beta\left[1\pm\frac{m_A^2+m_Z^2}{\sqrt{w}}\right] + \zeta_1\, m_0\; v^2\left[1\mp\frac{(m_A^2-m_Z^2)\,\cos^2 2\beta}{\sqrt{w}}\right]$$

$$+\;\delta\, m_h^2, \quad \text{where} \quad \zeta_0,\,\zeta_1\sim\mathcal{O}(1/M_*), \quad \delta m_h^2=\mathcal{O}(1/M_*^2), \quad w\equiv[m_A^2+m_Z^2]^2-4m_A^2 m_Z^2\cos^2 2\beta$$

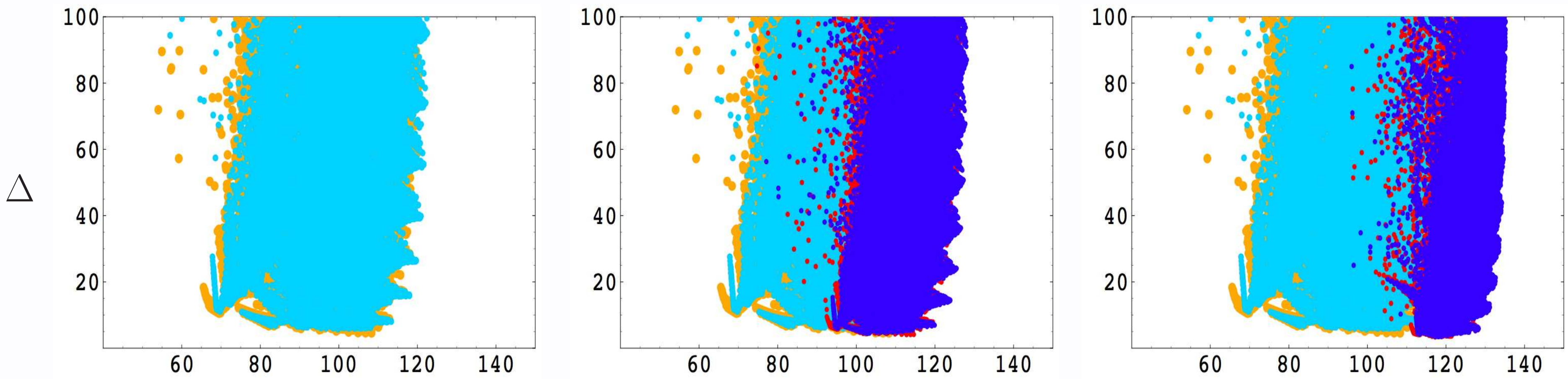
- Corrections to m_h of order $\mathcal{O}(1/M_*^2)$:

$$\delta\, m_h^2 = \; -2\,v^2\Big[\alpha_{22}\,m_0^2+(\alpha_{30}+\alpha_{40})\mu_0^2+2\alpha_{61}\,m_0\,\mu_0-\alpha_{20}\,m_Z^2\Big]-\frac{(2\,\zeta_0\,\mu_0)^2\;v^4}{m_A^2-m_Z^2}$$
$$+\frac{v^2}{\tan\beta}\bigg[\frac{1}{(m_A^2-m_Z^2)}\Big(4\,m_A^2\,\big(\,(2\alpha_{21}+\alpha_{31}+\alpha_{41}+2\alpha_{81})\,m_0\,\mu_0+(2\alpha_{50}+\alpha_{60})\,\mu_0^2+\alpha_{62}\,m_0^2\big)$$
$$-(2\alpha_{60}-3\alpha_{70})\,m_A^2\,m_Z^2-(2\alpha_{60}+\alpha_{70})\,m_Z^4\Big)+\frac{8\,(m_A^2+m_Z^2)\;(\mu_0\,m_0\,\zeta_0\,\zeta_1)\,v^2}{(m_A^2-m_Z^2)^2}\bigg]+\mathcal{O}(\tilde m^2/(M_*^2\tan^2\beta))$$

- Leading $\mathcal{O}(1/M_*^2)$ corrections from: $\mathcal{O}_{2,3,4}$ (recall $\alpha_{ij}\sim 1/M_*^2$, $\zeta_{10},\zeta_{11}\sim 1/M_*$).

- Fine tuning in MSSM+(d=5) operators: [Cassel, DG, Ross]

- Δ computed analytically at one-loop (also MSSM).



MSSM: m_h | MSSM + (d=5) operators $\mu_0\,\zeta_0 = m_0\,\zeta_1 = 0.015$ | MSSM + (d=5) operators $\mu_0\,\zeta_0 = 0.035,\ \zeta_1 = 0$

$$171.2 \leq m_t \leq 174\,\text{GeV};\ \ 1 < \tan\beta < 6;\ \ 50 \leq m_0, \mu_0, m_{12} \leq 10^3\,\text{GeV},$$

$$\Rightarrow\ \ \Delta < 10, \qquad 114.4 \leq m_h \leq 130\,\text{GeV}, \qquad M_* \approx 1/\zeta_0 \approx 65 \times \mu_0 = 5 \text{ to } 10\,\text{TeV}$$

- (d=5) operator generated: massive gauge singlet or SU(2) triplet.

- At large $\tan\beta$: d=6 operators relevant: $\lambda \propto (2\mu_0\,\zeta_0)^2 \sim (2\,\zeta_0\mu_0)/\tan\beta \Rightarrow (2\mu_0\zeta_0) < 1/\tan\beta$

- (d=6) effective operators: Leading correction:

$$\delta\, m_h^2 = -2\, v^2 \Big[\alpha_{22}\, m_0^2 + (\alpha_{30} + \alpha_{40})\mu_0^2 + 2\alpha_{61}\, m_0\, \mu_0 - \alpha_{20}\, m_Z^2\Big] + \mathcal{O}(1/\tan\beta)$$

maximised for $\alpha_{22} = \alpha_{61} = \alpha_{30} = \alpha_{40} = -\alpha_{20} < 0$ then:

$$\delta\, m_h^2 = -2\, v^2\, \alpha_{20}\Big[m_0^2 + 2\,\mu_0^2 + 2\, m_0\, \mu_0 + m_Z^2\Big] + \mathcal{O}(1/\tan\beta)$$

$\Rightarrow \delta m_h \approx 10 - 30$ GeV $\;(m_0 = 1\,$TeV$,\ \mu_0 = 0.35\,$TeV$,\ M_* \sim \alpha_{20}^{-1/2} \sim 10\,TeV)$

- ρ constraints:

$$\rho - 1 = -\frac{v^2}{M_*^2}\Big[\alpha_{10}\, \cos^4\beta + \alpha_{20}\, \sin^4\beta - \alpha_{30}\, \sin^2\beta \cos^2\beta\Big] + \mathcal{O}(v^4/M_*^4)$$

Large $\tan\beta$: α_{20} constrained; larger α_{30}, α_{10} allowed $\Rightarrow$ α_{30}, α_{40} largest SUSY contribution to m_h

$$\mathcal{O}_3 \sim \alpha_{30}\int d^4\theta\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2),\ [T, U(1)] \quad \mathcal{O}_4 \sim \alpha_{40}\int d^4\theta\ (H_2\, H_1)\,(H_2\, H_1)^\dagger,\ [S]$$

- difficult to generate α_{30}, α_{40}, α_{20} with the “right” signs from integrating massive $T, U(1), S$!

- Conclusions:

- CMSSM:

Minimising two-loop fine-tuning, no LEPII bound $\Rightarrow m_h = 114 \pm 2$ GeV, ie at LEPII bound!

- At larger m_h, since $\Delta \sim \exp(m_h^2)$ ....

  $\Rightarrow$ 1. relax CMSSM constraints or

  $\Rightarrow$ 2. "new physics" beyond CMSSM Higgs sector, parametrized by $d = 5, 6$ effective operators.

  (d=5) operators: small $\Delta < 10$ still possible for $114.4 \leq m_h \leq 130$ GeV.

  (d=6) operators: $\delta m_h \big|_{(d=6)} = 10 - 30$ GeV, for $M_* \sim 8 - 10$ TeV!. Extra U(1) or S ?

- Last bet for the LHC....min Δ + dark matter constraint in CMSSM but no LEPII bound:

$m_h = 114 \pm 2$ GeV, $\Delta = 8.8$, (no dark matter constraint).
$m_h = 114.7 \pm 2$ GeV, $\Delta = 15.0$, (sub-saturating WMAP bound).
$m_h = 115.9 \pm 2$ GeV, $\Delta = 17.8$, (saturating WMAP within 3σ).

- Δ at one-loop MSSM+(d=5) operators:

$$\Delta_{\mu_0^2} = -\frac{1}{v^2 D}\Big\{v^2 \cos 2\beta \Big[\sin 2\beta \Big(\zeta_0 \,(2\,\gamma_2 - \delta\, g^2\, v^2/8) + 2\,\gamma_1 \left[\delta g^2/8 - \Big((4+\delta)\, g^2/8 - \zeta_1\Big) \cos 2\beta\,\right]\Big)\Big] + 2\Big[2\mu_0^2\,\sigma_8^2 + (\zeta_0\, v^2 - \gamma_1)\sin 2\beta\Big]\,\gamma_4\Big\}$$

$$\Delta_{m_0^2} = -\frac{1}{4\,v^2\,D}\Big\{-v^2\zeta_1 \sin 4\beta\,\Big[4\gamma_1 \cos 2\beta + \big(\delta\, g^2\, v^2/8 - 2\gamma_2\big)\sin 2\beta\Big] + 2\,v^2\Big[2\,(\gamma_1 - \mu_0\, m_{12}\sigma_2)\cos 2\beta + \gamma_3 \sin 2\beta\Big]\Big[4\zeta_0 \cos 2\beta + \delta g^2 \cos\beta \sin^3\beta + (\zeta_1 - g^2/2)\sin 4\beta\Big] + 8\,\gamma_4\Big[2m_0^2 - \gamma_3 \sin^2\beta + \zeta_1\, v^2 \sin^2\beta\cos^2\beta - (\gamma_1 - m_{12}\,\mu_0\,\sigma_2)\sin 2\beta\Big]\Big\}$$

$$\Delta_{m_{12}^2} = -\frac{m_{12}}{v^2\,D}\,\Big\{\frac{v^2}{2}\Big[2\mu_0\sigma_2\cos 2\beta - \Big(A_t\,\sigma_5\; m_0 + 2\,m_{12}\,(\sigma_4 - \sigma_1)\Big)\sin 2\beta\Big] \times \Big[4\zeta_0 \cos 2\beta + \delta\, g^2 \cos\beta \sin^3\beta + (\zeta_1 - g^2/2)\sin 4\beta\Big] + 2\Big[2\,m_{12}\,\sigma_1 - \mu_0\,\sigma_2 \sin 2\beta + \Big(A_t\,\sigma_5\, m_0 + 2\,m_{12}(\sigma_4 - \sigma_1)\Big)\sin^2\beta\Big]\gamma_4\Big\}$$

$$\Delta_{B_0^2} = -\frac{2\,B_0\,m_0\,\mu_0\,\sigma_8}{v^2\,D}\Big\{v^2\Big[2\zeta_0 + (\zeta_1 - (4+\delta)g^2/8)\sin^3 2\beta\Big] + (\delta\, g^2\, v^2/16 - \gamma_2)\sin 4\beta - 4\gamma_1 \sin^2 2\beta\Big\}$$

$$\Delta_{A_0^2} = -\frac{A_0}{v^2\,D}\Big\{2\,m_0\sin\beta\left[2\mu_0\,\sigma_3\,\cos\beta + (2\,A_t\,\sigma_6\,m_0 - \sigma_5\,m_{12})\sin\beta\right](-\gamma_4)$$
$$+\;m_0\,v^2\left[\mu_0\sigma_3\cos 2\beta - (1/2)\sigma_5\,m_{12}\,\sin 2\beta + A_t\,\sigma_6\,m_0\sin 2\beta\right]$$
$$\times\;\left[4\,\zeta_0\cos 2\beta + \delta\,g^2\cos\beta\sin^3\beta + (\zeta_1 - g^2/2)\,\sin 4\beta\right]\Big\}$$

$$D \equiv 2\Big\{-\frac{1}{8}\,v^2\left[4\zeta_0\cos 2\beta + \zeta_1\sin 4\beta + g^2\,(\delta\cos\beta\sin^3\beta - 1/2\,\sin 4\beta)\right]^2$$
$$-\;2\left[\zeta_0\sin 2\beta + \zeta_1/4\sin^2 2\beta + g^2/8\,(\cos^2 2\beta + \delta\sin^4\beta)\right](-\gamma_4)\Big\}$$

$$\gamma_1 \equiv \mu_0\,(B_0\,m_0\,\sigma_8 + m_{12}\,\sigma_2 + A_t\,m_0\,\sigma_3)$$
$$\gamma_2 \equiv (-1 + \sigma_7 - A_t^2\,\sigma_6)\,m_0^2 + A_t\,\sigma_5\,m_0\,m_{12} + m_{12}^2\,(\sigma_4 - \sigma_1)\;+ \delta\,g^2\,v^2/16$$
$$\gamma_3 \equiv 2\,(1 - \sigma_7 + A_t^2\,\sigma_6)\,m_0^2 - A_t\,\sigma_5\,m_{12}\,m_0$$
$$\gamma_4 = 2\gamma_2\,\cos 2\beta + 4\gamma_1\,\sin 2\beta - (4+\delta)(g^2v^2/8)\cos 4\beta - v^2\,\big(2\zeta_0\sin 2\beta - \zeta_1\cos 4\beta\big)$$

• Soft Masses at one-loop:

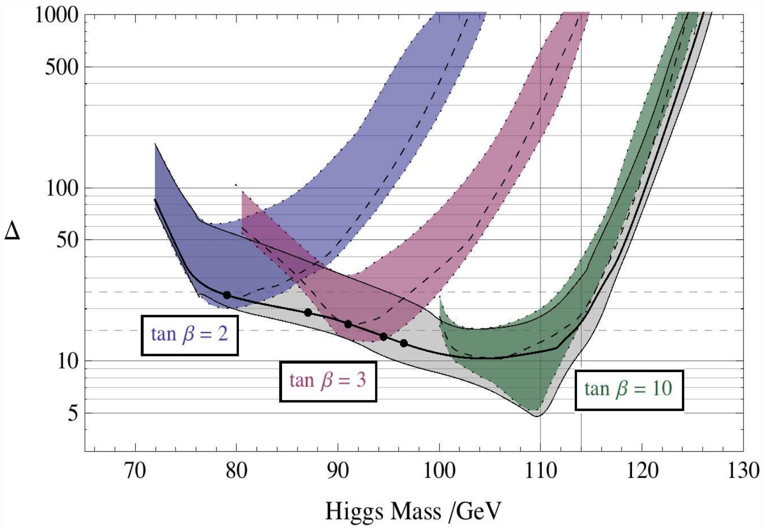
$$\begin{aligned} m_1^2(t) &= m_0^2 + \mu_0^2\,\sigma_8^2(t) + m_{12}^2\,\sigma_1(t) \\ m_2^2(t) &= \mu_0^2\,\sigma_8^2(t) + m_{12}^2\,\sigma_4(t) + A_t\,m_0\,m_{12}\,\sigma_5(t) + m_0^2\,\sigma_7(t) - m_0^2\,A_t^2\,\sigma_6(t) \\ m_3^2(t) &= \mu_0\,m_{12}\,\sigma_2(t) + B_0\,m_0\,\mu_0\,\sigma_8(t) + \mu_0\,m_0\,A_t\,\sigma_3(t) \end{aligned}$$

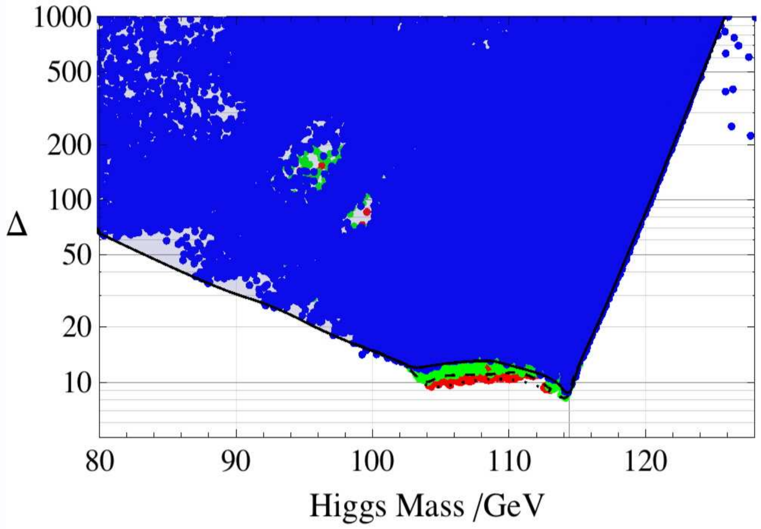
$$\delta = \frac{3\,h_t^4}{g^2\,\pi^2}\left[\ln\frac{M_{\tilde t}}{m_t} + \frac{X_t}{4} + \frac{1}{32\pi^2}\left(3\,h_t^2 - 16\,g_3^2\right)\left(X_t + 2\ln\frac{M_{\tilde t}}{m_t}\right)\ln\frac{M_{\tilde t}}{m_t}\right],$$

where $M_{\tilde t}^2 \equiv m_{\tilde t_1}\,m_{\tilde t_2}$, g_3 is the strong coupling and

$$X_t \equiv \frac{2\,(A_t\,m_0 - \mu\cot\beta)^2}{M_{\tilde t}^2}\left(1 - \frac{(A_t\,m_0 - \mu\cot\beta)^2}{12\;M_{\tilde t}^2}\right).$$

- Loop effects, $\tan\beta$ and $b \to s\gamma$ impact on Δ





- massive $SU(2)$ triplets: $\vec{T} = T_i \sigma^i$: $T_{1,2,3}$ of $Y = \pm 1, 0$.

$$\mathcal{L} \supset \int d^4\theta \left[T_1^\dagger e^V T_1 + T_2^\dagger e^V T_2 \right] + \int d^2\theta \left[\mu H_1 H_2 + M_* T_1 T_2 + \lambda_1 H_1 T_1 H_1 + \lambda_2 H_2 T_2 H_2 \right] + h.c$$

$$\Rightarrow \quad \mathcal{L}_{eff} = \frac{-\lambda_1 \lambda_2}{2\, M_*} \int d^2\theta \, (H_1 H_2)^2 + h.c.$$