Cosmology of Two Higgs Doublet Model

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Seventh Workshop on Particle Physics and Cosmology
Warsaw, 3-6.02.2010

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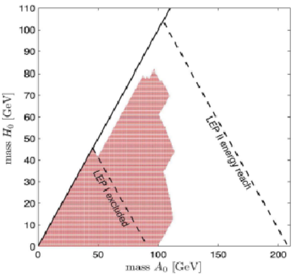
Motivation

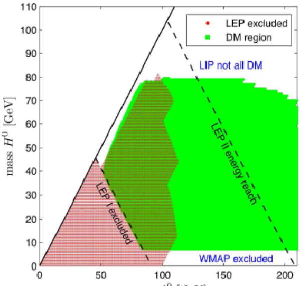
T.D. Lee, '73

Deshpande, Ma, '78

Two Higgs Doublet Model (2HDM):

- two scalar $SU(2)_W$ doublets Φ_1, Φ_2 with the same hypercharge $Y = 1$
- CP violation in the scalar sector (explicit or spontaneous violation)
- different types of extrema (possible violation of $U(1)_{EM}$)
- 2HDM with an exact Z_2 symmetry
  $\rightarrow$ candidate for the dark matter (Inert Model)





Lundstrom, '08

2HDM

T.D. Lee, '73

Higgs potential V with an explicit Z_2 symmetry:

$$Z_2: \quad \Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2$$

$$V = -\frac{1}{2}\left[m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2\right] + \frac{1}{2}\left[\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2\right]$$

$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{1}{2}\lambda_5 \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right]$$

with:

Ginzburg, Ivanov, Kanishev, '09

$$\lambda_2/\lambda_1 = k^4, \quad m_{11}^2 = m^2(1-\delta), \quad m_{22}^2 = k^2 m^2 (1+\delta),$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad \tilde{\lambda}_{345} = \lambda_3 + \lambda_4 - \lambda_5,$$

$$\Lambda_{345\pm} = \sqrt{\lambda_1 \lambda_2} \pm \lambda_{345}, \ \tilde{\Lambda}_{345\pm} = \sqrt{\lambda_1 \lambda_2} \pm \tilde{\lambda}_{345}, \ \Lambda_{3\pm} = \sqrt{\lambda_1 \lambda_2} \pm \lambda_3.$$

- All parameters $\in \mathbf{R}$ (and $m^2, k^2 > 0$) – no CP violation
- Model I – only Φ_1 couples to fermions.

Extremum conditions

The positivity constrains are required to have a stable vacuum:

$$\lambda_1 > 0\,, \quad \lambda_2 > 0,\ \Lambda_{3+} > 0,\ \Lambda_{345+} > 0,\ \tilde{\Lambda}_{345+} > 0.$$

Positivity constrains → extremum with the lowest energy is the global minimum (vacuum).

Extremum conditions:

$$\partial V/\partial\Phi_i|_{\Phi_i=\langle\Phi_i\rangle} = 0\,, \qquad \partial V/\partial\Phi_i^\dagger|_{\Phi_i=\langle\Phi_i\rangle} = 0$$

The EW symmetric extremum:

$$\langle\Phi_1\rangle = \langle\Phi_2\rangle = 0$$

- a local minimum if $m_{11,22}^2 < 0$
- a local maximum if $m_{11,22}^2 > 0$

Spontaneous Symmetry Breaking

The general type of EWSB VEV:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_2 \end{pmatrix}$$

Barroso et al., '05

$u \neq 0 \Longrightarrow U(1)_{EM}$ broken:

- charge breaking (Ch)

$u = 0 \Longrightarrow U(1)_{EM}$ conserved:

- $v_{1,2} \neq 0$ neutral normal (N)
- $v_2 = 0$ neutral Inert (I)

  Deshpande, Ma, '78; Barbieri, Hall, Rychkov, '06
- $v_1 = 0$ neutral Phase B (B)

Charge breaking and Normal extremum

Charge breaking extremum (Ch):

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \end{pmatrix}$$

- $U(1)_{EM}$ symmetry broken by $u \neq 0$ – massive photon
- not a case that is realized now, a possible vacuum in the past if

$$\Lambda_{3-} > 0, \quad \lambda_4 + \lambda_5 > 0, \quad \lambda_4 - \lambda_5 < 0$$

Normal extremum (N):

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

- Normal = CP conserving
- massive $Z^0, W^{\pm}$, massless photon, 5 physical Higgs bosons $H^{\pm}, A, H, h$

$$\Lambda_{345-} > 0, \quad \lambda_4 + \lambda_5 < 0, \quad \lambda_5 < 0$$

Inert and B

Deshpande, Ma, '78, Barbieri et al., '06

Inert extremum:

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_I \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- exact Z_2 symmetry – both in Lagrangian and in the extremum
- only Φ_2 has odd Z_2-parity

  $\rightarrow$ the lightest scalar is a candidate for the dark matter
- Φ_1 as in SM (SM-like Higgs boson h)

  Φ_2 – "dark" or inert doublet with 4 dark scalars $(H, A, H^{\pm})$, no interaction with fermions

"Phase B" extremum:

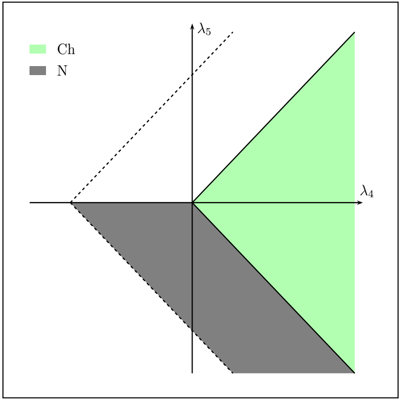
$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_B \end{pmatrix}$$

- Φ_1 and Φ_2 change roles
- fermions massless at tree-level (Model I)

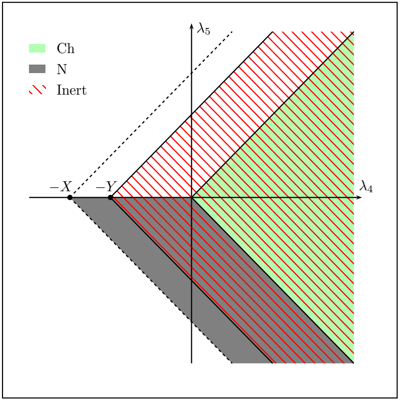
Local minimum

Values of λ_i → existence of (local) minimum of given properties:

N and Ch in separate regions:



Inert (or B) overlaps N and Ch.

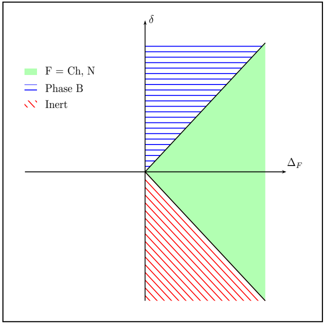


Energy $\mathcal{E}$ of extremum

The extremum with the lowest energy is the vacuum:

$$\mathcal{E}_I - \mathcal{E}_N = \frac{m^4}{8}\,\frac{\left(\Lambda_{345-}+\delta\Lambda_{345+}\right)^2}{\lambda_1\Lambda_{345-}\Lambda_{345+}},\quad \mathcal{E}_I - \mathcal{E}_{Ch} = \frac{m^4}{8}\,\frac{\left(\Lambda_{3-}+\delta\Lambda_{3+}\right)^2}{\lambda_1\Lambda_{3-}\Lambda_{3+}}$$

$$\mathcal{E}_I - \mathcal{E}_B = \frac{m^4\delta}{2\lambda_1},\quad \mathcal{E}_N - \mathcal{E}_{Ch} = \frac{k^2m^2}{4}\,\lambda_{45}\left[\frac{1}{\Lambda_{3+}\Lambda_{345+}} - \frac{\delta^2}{\Lambda_{3-}\Lambda_{345-}}\right]$$



$m_{11}^2 = m^2(1-\delta)$

$m_{22}^2 = m^2k^2(1+\delta)$

Charge breaking vacuum:

$\mathcal{S}_{Ch}$: $\Lambda_{3-}>0,\quad \lambda_4+\lambda_5>0,\quad \lambda_4-\lambda_5<0,$

$|\delta|<\Delta_{Ch}$ for $\Delta_{Ch}=\Lambda_{3-}/\Lambda_{3+}$

Normal vacuum:

$\mathcal{S}_N$: $\Lambda_{345-}>0,\quad \lambda_4+\lambda_5<0,\quad \lambda_5<0,$

$|\delta|<\Delta_N$ for $\Delta_N=\Lambda_{345-}/\Lambda_{345+}$

Inert Model:

$\delta<-\Delta_{Ch,N}$ for $\mathcal{S}_{Ch,N}$, $\quad\delta<0$ otherwise

Phase B:

$\delta>\Delta_{Ch,N}$ for $\mathcal{S}_{Ch,N}$, $\quad\delta>0$ otherwise

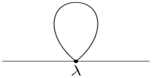
Evolution of the Universe

Ivanov '08

- We assume that today Inert Model is realized, however, in the past some other extrema could have been lower.
- We consider evolution of the Universe due to the thermal corrections to the potential.
- At finite T ground state is given by minimum of Gibbs potential:

$$V_G(T) = Tr(Ve^{-H/T})/Tr(e^{-H/T}) \equiv V(T=0) + \Delta V(T)$$

- $\Delta V(T)$ – leading corrections $\propto T^2$ given by diagrams:



$\Longrightarrow$ fixed quartic terms, quadratic (mass) terms change with T

$$\Delta V(T) = c_1 \frac{T^2}{12} \Phi_1^\dagger \Phi_1 + c_2 \frac{T^2}{12} \Phi_2^\dagger \Phi_2$$

Evolution of the Universe

From scalar contributions to ΔV:

$$m_{11}^2(T) = m_{11}^2 - 2c_1 m^2 w\,, \quad m_{22}^2(T) = m_{22}^2 - 2k^2 c_2 m^2 w$$

$$c_1 = \tfrac{3\lambda_1+2\lambda_3+\lambda_4}{2}\,, \qquad c_2 = \tfrac{3\lambda_2+2\lambda_3+\lambda_4}{2k^2}\,; \quad w = \tfrac{T^2}{12m^2}\,.$$

$$m^2(T) = m^2\left(1-(c_2+c_1)w\right)$$

$$\delta(T) = \tfrac{m^2}{m^2(T)}\left(\delta - \tfrac{c_2-c_1}{c_2+c_1}\right) + \tfrac{c_2-c_1}{c_2+c_1}$$

- c_1 and c_2 positive to restore EW symmetry in the past
- $m_{11}^2(T), m_{22}^2(T)$ decrease with T

for today ($T=0$) we use parameters m_{ii}^2, m^2, δ

The evolution of vacuum states and phase transitions in 2HDM during cooling of Universe, I.F. Ginzburg, I.P. Ivanov, K.A. Kanishev, arXiv:0911.2383v2 – 2HDM with soft Z_2 violation

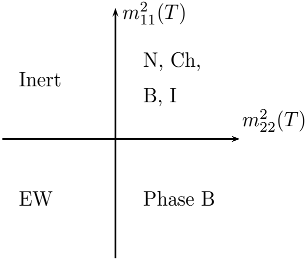
Possible phase transitions

For a given T we determine:

- sign of m_{ii}^2 $\rightarrow$ possible existence of given extremum
- values of λ_i (fixed) $\rightarrow$ existence of a given local minimum
- value of δ $\rightarrow$ global minimum

Possible sequences that lead to Inert:

- EW $\xrightarrow{II}$ Phase B $\xrightarrow{I}$ Inert
- EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ N $\xrightarrow{II}$ Inert
- EW $\xrightarrow{II}$ Phase B $\xrightarrow{II}$ Ch $\xrightarrow{II}$ Inert
- EW $\xrightarrow{II}$ Inert

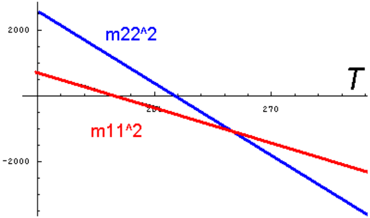


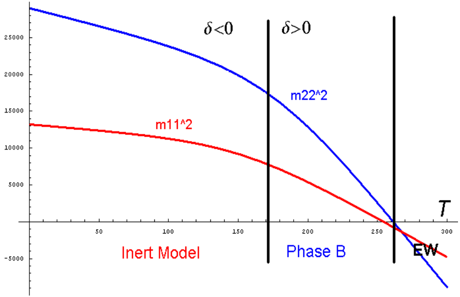
Note:

- $EW \rightarrow B \rightarrow \cdots \rightarrow I$: $m_{11}^2, m_{22}^2 > 0$
- $EW \rightarrow I$: $m_{11}^2 > 0$

$EW \to B \to I$

- $m_{11}^2(T) < 0, m_{22}^2(T) < 0$: EW symmetric phase
- $m_{11}^2(T) < 0, m_{22}^2(T) > 0$: Phase B becomes a vacuum, Inert is not an extremum
- $m_{11}^2(T) > 0, m_{22}^2(T) > 0$ $\delta(T) > 0$: Phase B still a vacuum, Inert becomes an extremum
- $m_{11}^2(T) > 0, m_{22}^2(T) > 0$ $\delta(T) < 0$: Inert becomes a vacuum, Phase B is an extremum

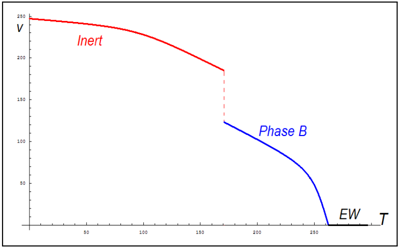




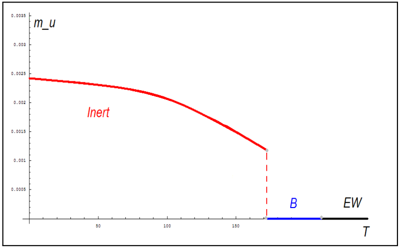
$EW \to B \to I$

$$M_h = 115 \text{ GeV}, \quad M_H = 60 \text{ GeV}, \quad M_A = 80 \text{ GeV}, \quad M_{H^\pm} = 140 \text{ GeV}$$
$$\lambda_1 = 0.217, \quad \lambda_4 = -0.48, \quad \lambda_5 = -0.046$$
$$\lambda_3 = 1.116, \quad k^2 = \frac{9}{4}, \quad \lambda_2 = 1.1, \quad \delta = -0.016$$

Value of v



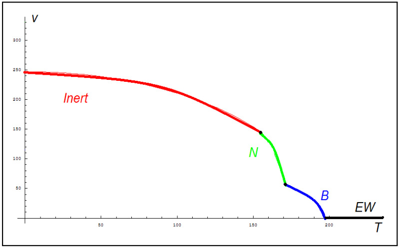
Value of m_u



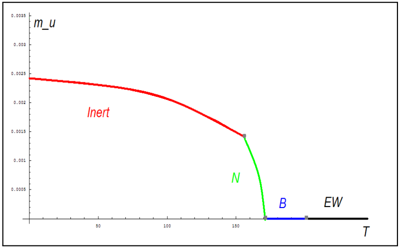
$$EW \xrightarrow{II} B \text{ at } T_{EW,B} = 261 \text{ GeV}$$
$$B \xrightarrow{I} I \text{ at } T_{B,I} = 174.5 \text{ GeV}$$

$EW \to B \to N \to I$

Value of v



Value of m_u



Transitions for $T = 197$ GeV, 170 GeV, 156 GeV.

$$M_h = 115 \text{ GeV}, \quad M_H = 50 \text{ GeV}, \quad M_A = 70 \text{ GeV}, \quad M_{H^\pm} = 250 \text{ GeV},$$
$$\lambda_3 = 2.80, \quad \lambda_2 = 3.47.$$

$EW \to B \to Ch \to I$

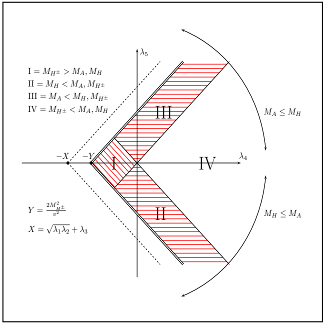
In S_{Ch} region we go from Phase B into Ch at:

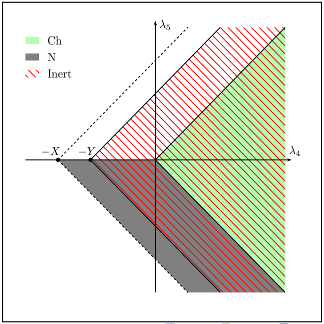
$$\delta(T_{Ch+}) = \Delta_{Ch} = \Lambda_{3-}/\Lambda_{3+}$$

and then from Ch to Inert at:

$$\delta(T_{Ch-}) = -\Delta_{Ch}$$

Note, that we must be in the region in which $M_{H\pm}^2 < M_A^2, M_H^2$.





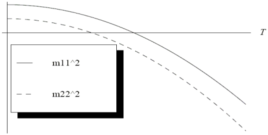
$EW \to I$

Inert vacuum exist during an entire history of the Universe after SSB if only one phase transition took place at

$$w_{EW,I} = \frac{1-\delta}{c_1}$$

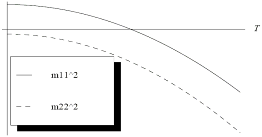
- Phase B can be an extremum (and so $m_{22}^2 > 0$) but not a vacuum if:

$$-1 < \delta < \frac{c_2 - c_2}{c_2 + c_1} < 0$$



- Phase B can not be an extremum (and so $m_{22}^2 < 0$) if:

$$\delta < -1$$



Note, that $m_{11}^2 > 0$ (Inert)

Conclusions

- Different types of extrema can be realized in the past.
- Today – Inert Model (dark matter).
- Possible sequences of phase transitions:

$$EW \to B \to N \to I$$
$$EW \to B \to Ch \to I$$
$$EW \to B \to I$$
$$EW \to I$$

- $EW \to B \to Ch \to I$ requires charged "dark matter".
- It is possible to have no DM for hight T (going through Phase B).
- Need for further analysis for $m_{11}^2, m_{22}^2 > 0$.