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Finite Unified Theories and the prediction of the Higgs mass

Standard Model very successful

But contains > 20 free parameters

ad hoc Higgs sector

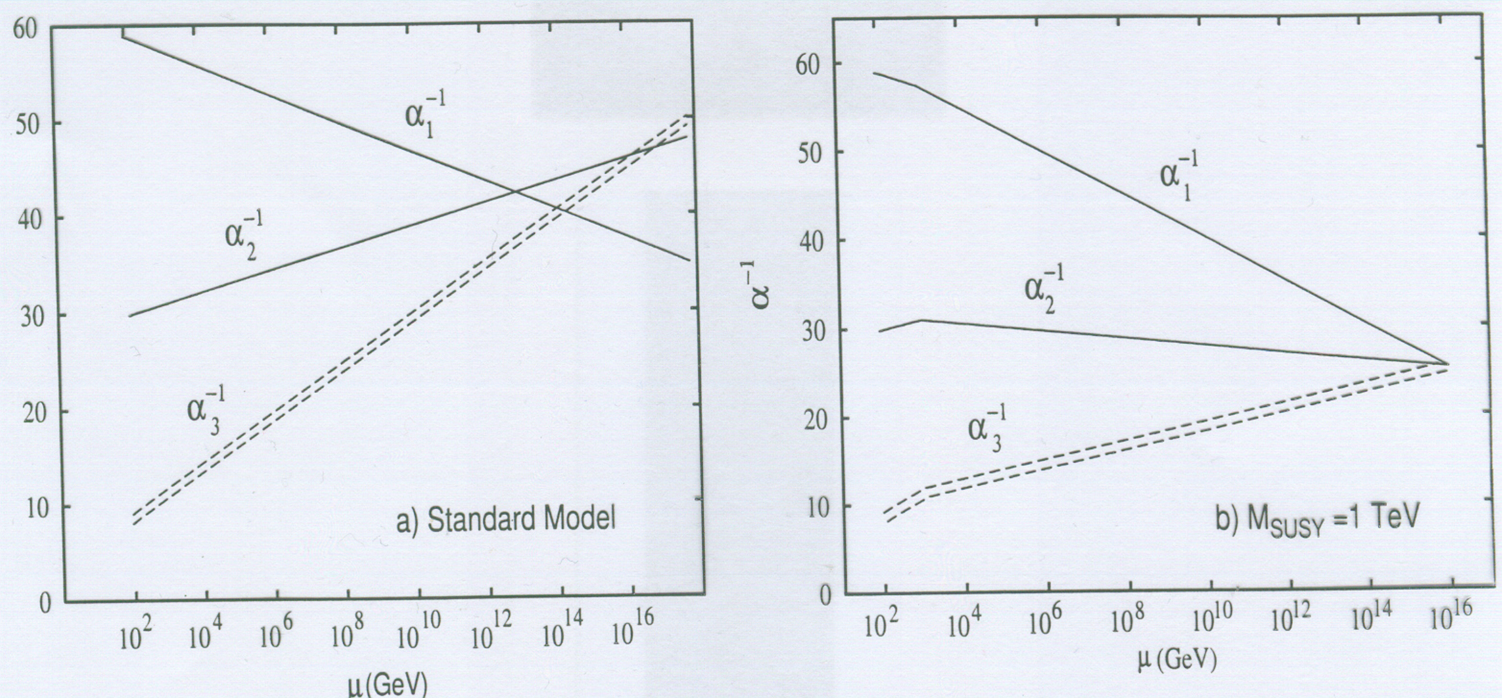
ad hoc Yukawa couplings

Best candidate for Physics Beyond SM,

MSSM with > 100 ! free parameters

mostly in its SSB sector.

- cures problem of quadratic divergencies of the SM (hierarchy problem)
- restricts the Higgs sector



60
50
40
30
20
10
0
α_1^{-1}
α_2^{-1}
α_3^{-1}
a) Standard Model
10^2 10^4 10^6 10^8 10^{10} 10^{12} 10^{14} 10^{16}
μ (GeV)
α^{-1}
b) M_{SUSY} =1 TeV

• SM with two-Higgs doublet

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + (m_3^2 H_1 H_2 + h.c.)$$

$$+ \frac{1}{2}\lambda_1 (H_1^+ H_1)^2 + \frac{1}{2}\lambda_2 (H_2^+ H_2)^2$$

$$+ \lambda_3 (H_1^+ H_1)(H_2^+ H_2) + \lambda_4 (H_1 H_2)(H_1^+ H_2^+)$$

$$+ \left\{ \frac{1}{2}\lambda_5 (H_1 H_2)^2 + \left[\lambda_6 (H_1^+ H_1) + \lambda_7 (H_1^+ H_2^+)\right](H_1 H_2) + h.c. \right]$$

Supersymmetry provides tree level relations among couplings

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2)$$

$$\lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = -\frac{1}{4} g^2$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle \mathrm{Re}\, H_1^0 \rangle$, $v_2 = \langle \mathrm{Re}\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2$, $\frac{v_2}{v_1} \equiv \tan\beta$

$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Rightarrow \begin{cases} M_{h^0} < M_Z \,|\cos 2\beta| \\ M_{H^0} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^0} \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}^2_{t_1}\tilde{m}^2_{t_2}}{m_t^4}$$

- Finite Unified Theories (from Quantum Reduction of Couplings)
- Higher Dimensional Unified Theories and Coset Space Dimensional Reduction (Classical Reduction of Couplings)
- Fuzzy Extra Dimensions and Renormalisable Unified Theories

Quantum
Reduction of Couplings

Consider a GUT with

g – gauge coupling

g_i – other couplings (Yukawas, self couplings)

Any relation among the couplings

$$\Phi(g, g_1, \dots) = \text{const}$$

which is RGI should satisfy

$$\frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d}{dt}\Phi = \frac{\partial \Phi}{\partial g}\overset{\beta_g}{\overbrace{\frac{dg}{dt}}} + \sum_i \frac{\partial \Phi}{\partial g_i}\overset{\beta_i}{\overbrace{\frac{dg_i}{dt}}} = 0$$

which is equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \cdots$$ characteristic system

$$\Longrightarrow \quad \beta_g \frac{d g_i}{d g} = \beta_i$$

Reduction eqs

Oehme

Zimmermann

Demand power series solution to the RE

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l} \beta_i^{(1)\,jkl} g_j g_k g_l + \sum_{j\neq g} \beta_i^{(1)\,j} g_i g^2\right] + \ldots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$, insert g_i in REs and collect terms of $O\left(g^{2r+1}\right)$

$$\rightsquigarrow \sum_{l \neq g} M(r)_i^{\ l} \rho_l^{(r+1)} = \text{lower order quantities Known by assumption}$$

where

$$M(r)_i^{\ l} = 3 \sum_{j,k \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} + b_i^{(1)l} - (2r+1) b_g^{(1)} \delta_i^l$$

$$0 = \sum_{j,k,l \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} \rho_l^{(1)} + \sum_{l \neq g} b_i^{(1)l} \rho_l^{(1)} - b_g^{(1)} \rho_i^{(1)}$$

$\Longrightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n > 1$ can be uniquely determined if

$$\det M(n)_i^{\ l} \neq 0$$

for all n

Kubo et. al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars;

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

$\lambda^a \, (a = 1, \dots, N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\sqrt{2}\left[g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + h.c. - V(\phi, \hat{\phi}),\right.$$

$$V(\phi, \hat{\phi}) = \frac{1}{4} \lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4} \lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g; \lambda_1 = \lambda_2 = \frac{N-1}{N} g^2; \lambda_3 = \frac{1}{2N} g^2; \lambda_4 = -\frac{1}{2} g^2$$

i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly free $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} – gauge invariant tensors

ϕ^i – matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij} m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k'}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} Z^{1/2\, k'}_{k''} = \delta^i_{(i''} \delta^j_{j''} \delta^k_{k'')}$$

$$Z^{ij}_{i'j'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} = \delta^i_{(i''} \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightarrow$ Only surviving infinities are $Z^i_j \, (Z_V)$

i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of the non-renormalization thm, are related with the anomalous dim. matrix γ_i^j of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma_k^l + C_{ikl}\gamma_j^l + C_{jkl}\gamma_i^l$$

$$\gamma_i^{(1)j} = Z^{-\frac{1}{2}}{}_i^k \frac{d}{dt} Z^{\frac{1}{2}}{}_k^j$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta_i^j\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2}\, 2\, g^5 \left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{g^3}{r}\, C_2(R_i)\left[C^{jkl} C_{ikl} - 2g^2 C_2(R_i)\delta^j_i\right]$$

$$r : \mathrm{tr}\,\delta^{ab}$$

Parkes, West, Jones
Mezincescu, Yau
Machacek, Vaughn

$$\gamma^{(2)i}{}_j = \frac{1}{(16\pi^2)^2}\, 2\, g^4\, C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{1}{2}\left[C^{ikl} C_{jkm} + 2g^2 (R^a)^i{}_m (R^a)^l{}_j\right]$$

$$\cdot \left[C^{mpq} C_{lpq} - 2\delta^m_l\, g^2 C_2(R_i)\right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_i \ell(R_i)(1 - 2\gamma_i) - 3C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right]$$

Novikov – Shifman – Vainstein – Zakharov

Finite Unification

Old days...

...divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years ...

...divergences reflect existence of a higher scale where new degrees of freedom are excited.

Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unkown physics at higher scales.

⇝ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⇝ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4$ ⇝ finite to all orders in pert.
- $N=2$ ⇝ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

Spin \ N	1	1	2	2	4
1	—	1	—	1	1
1/2	1	1	2	2	4
0	2	—	4	2	6

$$N=2 \; : \; \beta(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental

reps $\rightsquigarrow \beta(g) = 0$

$SU(5)$: $p\,(5+\bar{5})$; $q\,(10+\overline{10})$; $r\,(15+\overline{15})$

with $p + 3q + 7r = 10$

$SO(10)$: $p\,(10+\overline{10})$; $q\,(16+\overline{16})$

with $p + 2q = 8$

E_6 : $4\,(27+\overline{27})$

Finite Unified Theories

$N=1$

- 1-loop finiteness conditions

$$b_g^{(1)} = 0$$

$$\gamma^{(1)i}_{j} = 0$$ – anomalous dimensions of all chiral superfields

- • Exists complete classification of all chiral $N=1$ models with $b_g^{(1)} = 0$

Hamidi – Patera – Schwarz
Jiang – Zhou

- • • 1-loop finiteness → 2-loop finiteness

Parkes-West
Jones
Mezincescu

... Exist simple criteria that guarantee all loop finiteness (vanishing of all-loop beta functions)

Lucchesi-Piguet-Sibold
Ermushev, Kazakov, Tarasov
Leigh-Strassler

- All-loop finite SU(5) $\Longrightarrow$ top quark mass ✓

Kapetanakis, Mondragon, Z '92

~~Susy~~ sector

- 1-loop finiteness conds (require in particular universal soft ~~susy~~ scalar masses

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j \,)$$

Jones, Mezincescu, Yao

·· 1-loop finiteness

→ 2-loop finiteness

Jack
Jones

Reduction of couplings

· Extension of method in SSB sector

+ application in min susy SU(5)

Kubo
Mondragon
Z

·· 1-loop sum rule for soft scalar masses in non-finite susy ths.

Kawamura
Kobayashi
Kubo

··· 2-loop sum rule for soft scalar masses in finite ths.

Kobayashi
Kubo
Mondragon
Z

* All-loop RGI relations in finite and non-finite ths

Yamada
Hisano, Shifman
Kazakov
Jack, Jones, Pickering

* * All-loop sum rule for soft scalar masses in finite and non-finite ths

Kobayashi
Kubo
Z

• • SU(5) FUTs

Kobayashi
Kubo
Mondragon
Z

• Prediction of s-spectrum in terms of few parameters starting from several hundreds GeV.

• • The LSP is neutralino ✓ (see e.g. Kazakov et. al. Yoshioka)

• • • Radiative E-W breaking ✓ (see e.g. Brignole Ibanez, Munoz)

• • • • No funny colour, charge ✓ (see e.g. Casas et. al)

* Prediction of Higgs masses

Lightest ~ 118 - 129 GeV

Similar results also for min susy SU(5)

Consider a chiral, anomaly free, $N=1$ gauge theory with group G.

The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$\left.\begin{matrix} Y^{ijk} \\ \mu^{ij} \end{matrix}\right\}$ gauge invariant

Yukawa couplings

Φ_i – matter superfields in irreducible reps of G

Necessary and sufficient conditions for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i \ell(R_i) = 3\, C_2(G)$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – Quadratic Casimir of G (adjoint)

$\Rightarrow$ Selection of the field content (representations) of the theory

•• Vanishing of $\gamma^{(1)i}_{\ \ j}$ implies

$$\underset{\substack{\uparrow \\ \text{Yukawa}}}{Y^{ikl}}\, Y_{jkl} = 2\,\delta^i_j\, \underset{\substack{\uparrow \\ \text{gauge}}}{g^2}\, C_2(R_i)$$

$C_2(R_i)$ – quadratic Casimir of R_i

$Y_{ijk} = (Y_{ijk})^*$

$\Rightarrow$ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

•• Appearence of $U(1)$ is incopatible with 1^{st} condt.

••• 2^{nd} condt forbids the presence of singlets with nonvanishing couplings.

•••• $\Rightarrow$ ~~Susy~~ by G-invariant soft terms

* 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

* 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon

Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (on γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

** Necessary and sufficient condts for vanishing β_g and β_{ijk} to all orders

Lucchesi
Piguet
Sibold

1. $\beta_g^{(1)} = 0$

2. $\gamma^{(1)i}{}_j = 0$

3. $$\beta_Y^{ijk} = \beta_g \frac{dY^{ijk}}{dg}$$

admit power series solution which in lowest order is a solution of condt 2.

3. → 3! There exist solutions to $\gamma^{(1)i}{}_j = 0$ of the form
$$Y^{ijk} = \rho^{ijk} g, \quad \rho^{ijk}\text{-complex}$$

3. → 4. These solutions are isolated and non-degenerate considered as solutions of $\beta_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless $N=1$ ths

$U(1)$ chiral transformation R:

$$A_\mu \to A_\mu \;, \quad \lambda \to e^{-i\alpha}\lambda \;,$$

$$\phi \to e^{-i\frac{2}{3}\alpha}\phi \;, \quad \psi \to e^{i\frac{1}{3}\alpha}\psi \;, \quad \ldots$$

$$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\psi_D$$

Noether current $J_R^\mu = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$$\rightsquigarrow \partial_\mu J_R^\mu = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$$

$$r = b_g^{(1)} \; !$$

Only 1-loop contributions due to non-renormalization thm.

Adler, Bardeen, Jackiw, Pi, Shei, Zee

Supercurrent

$$\mathcal{J} \equiv \left\{ J'^{\mu}_{R} \; , \; Q^{\mu}_{\alpha} \; , \; T^{\mu}_{\nu} \right\} , \cdots$$

vector super multiplet

associated to R-invariance

associated to susy

associated to translation inv.

Ferrara + Zumino

supercurrent is represented as vector superfield

$$V_\mu(x,\theta,\bar\theta) = R_\mu(x) - i\,\theta^\alpha Q_{\mu\alpha}(x) + i\,\bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}_{\mu}(x) - 2(\theta\sigma^\nu\bar\theta)\,T_{\mu\nu}(x) + \cdots$$

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$

∴ $J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition

Clark
Piquet
Sibold

$$\mathcal{S} = \left\{ b_g F^{\mu\nu} F_{\mu\nu} + \cdots , \; b_g \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots , \; b_g \lambda^{\beta} \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots , \; \cdots \right\}$$

chiral super multiplet

Super trace anomaly

trace anomaly of T^{μ}_{ν}

anomaly of R-current

trace anomaly of susy current

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of β-functions) may be scheme dependent

$$r = b_g(1+x_g) + \beta_{ijk}\, x^{ijk} - \gamma_A\, r^A$$

radiative corrections

linear combinations of anomalous dims

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

Thm: If (i) no gauge anomaly

(ii) $\beta^{(1)}(g) = 0$ i.e. no R-current anomaly

(iii) $\gamma^{(1)i}{}_j = 0$ implies also $r^A = 0$

(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $c_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex

(v) these solutions are isolated + non-degenerate when considered as solutions of $b^{(1)}_{ijk} = 0$

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N=1$ Y-M models depend on the single coupling constant g with a b-function vanishing to all orders.

Proof: Inserting $b_{ijk} = b_g \dfrac{d f_{ijk}}{dg}$ in the identity and taking into account the vanishing of r, r^A

$$\rightsquigarrow \quad 0 = b_g \left(1 + O(\hbar) \right)$$

Its solution (as formal power series in $\hbar$) is: $b_g = 0$

and $b_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin-Vaughn-Yamada-Jack-Jones '94

Consider $N=1$ gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + h.c.$$

- 1-loop finiteness conditions

$$h^{ijk} = -M Y^{ijk}$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$$ universality

in addition to $\beta_g^{(1)} = \gamma^{(1)i}_j = 0$

- 1-loop finiteness

$\longrightarrow$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)}{}_i^j = 0$
- the reduction eq

$$b_Y^{ijk} = b_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}$$

- $(m^2)^i_j = m_j^2 \delta^i_j$

$$\rightarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)}$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$

where $\Delta^{(2)} = -2 \sum_l \left[(m_l^2/MM^*) - \frac{1}{3} \right] \ell(R_l)$

- $\Delta^{(2)} = 0$ for $N=4$ with STr cond
- $\Delta^{(2)} = 0$ for the $N=1$, $SU(5)$ FUTs !

The SU(5) finite model

Kapetanakis, Mondragon, Z

Kobayashi, Kubo, Mondragon, Z

Content

$$3(\bar{5}+10) + 4(\underset{H_\alpha}{5}+\underset{\bar{H}_\alpha}{\bar{5}}) + 24$$

fermion supermultiplets — scalar supermultiplets

Jones-Raby
Hamidi-Schwarz
Quiros et. al
Kazakov
Babu-Enkhbat
Gogoladze

$$\rightsquigarrow W = \sum_{i=1}^{3}\left[\frac{1}{2} g_i^u\, 10_i\, 10_i\, H_i + g_i^d\, 10_i\, \bar{5}_i\, \bar{H}_i\right] + g_{23}^u\, 10_2\, 10_3\, H_4 + g_{23}^d\, 10_2\, \bar{5}_3\, \bar{H}_4 + g_{32}^d\, 10_3\, \bar{5}_2\, \bar{H}_4 + \sum_{\alpha=1}^{4} g_\alpha^f\, H_\alpha\, 24\, \bar{H}_\alpha + g^\lambda/3\,(24)^3$$

(with enhanced discrete symmetry after reduction of couplings)

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u(1)} = \frac{1}{16\pi^2}\Bigg[-\frac{96}{5}g^2 + \sum_{\beta=1}^{4}\left(g_{i\beta}^{u}\right)^2 + 3\sum_{j=1}^{3}\left(g_{j\alpha}^{u}\right)^2 + \frac{24}{5}\left(g_{\alpha}^{f}\right)^2 + 4\sum_{\beta=1}^{4}\left(g_{i\beta}^{d}\right)^2\Bigg]g_{i\alpha}^{u}$$

$$b_{i\alpha}^{d(1)} = \frac{1}{16\pi^2}\Bigg[-\frac{84}{5}g^2 + 3\sum_{\beta=1}^{4}\left(g_{i\beta}^{u}\right)^2 + \frac{24}{5}\left(g_{\alpha}^{f}\right)^2 + 4\sum_{j=1}^{3}\left(g_{j\alpha}^{d}\right)^2 + 6\sum_{\beta=1}^{4}\left(g_{i\beta}^{d}\right)^2\Bigg]g_{i\alpha}^{d}$$

$$b^{\lambda(1)} = \frac{1}{16\pi^2}\Bigg[-30g^2 + \frac{63}{5}\left(g^{\lambda}\right)^2 + 3\sum_{\alpha=1}^{4}\left(g_{\alpha}^{f}\right)^2\Bigg]g^{\lambda}$$

$$b_{\alpha}^{f(1)} = \frac{1}{16\pi^2}\Bigg[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}\left(g_{i\alpha}^{u}\right)^2 + 4\sum_{i=1}^{3}\left(g_{i\alpha}^{d}\right)^2 + \frac{48}{5}\left(g_{\alpha}^{f}\right)^2 + \sum_{\beta=1}^{4}\left(g_{\beta}^{f}\right)^2 + \frac{21}{5}\left(g^{\lambda}\right)^2\Bigg]g_{\alpha}^{f}$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$$\Rightarrow \left(g^u_{ii}\right)^2 = \frac{8}{5} g^2 + \cdots, \left(g^d_{ii}\right)^2 = \frac{6}{5} g^2 + \cdots$$

$$\left(g^\lambda\right)^2 = \frac{15}{7} g^2 + \cdots, \left(g^f_4\right)^2 = g^2, \left(g^f_\alpha\right)^2 = 0 + \cdots \ (\alpha = 1,2,3)$$

Higher order terms can be uniquely determined.

$\Rightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_{\alpha}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{H}\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_{\alpha}\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_{\alpha}\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Longrightarrow$ Necessary and sufficient conds for finiteness to all orders are satisfied

- SU(5) breaks down to the standard model due to $\langle 24 \rangle$

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka

- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

Marciano + Senjanović
Amaldi et. al.

$\sin^2\theta_W,\ \alpha_{em} \rightsquigarrow \alpha_3(M_Z)$

2) Bottom-Tau Yukawa Unif.

SU(5)-type

Barger et. al.
Carena et. al.

$\rightsquigarrow m_t \sim 100-200$ GeV

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FUT

Similar to SO(10)

Ananthanarayan et. al.
Barger et. al.
Carena et. al.

$\rightsquigarrow m_t \sim 160-200$ GeV

*4) Gauge-Top-Bottom-Tau Unif.

e.g. FUT-SU(5): $h_t^2 = \frac{8}{5} g_U^2$; $h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

The predictions for the three models for different M_s

With theoretical corrections and uncertainties

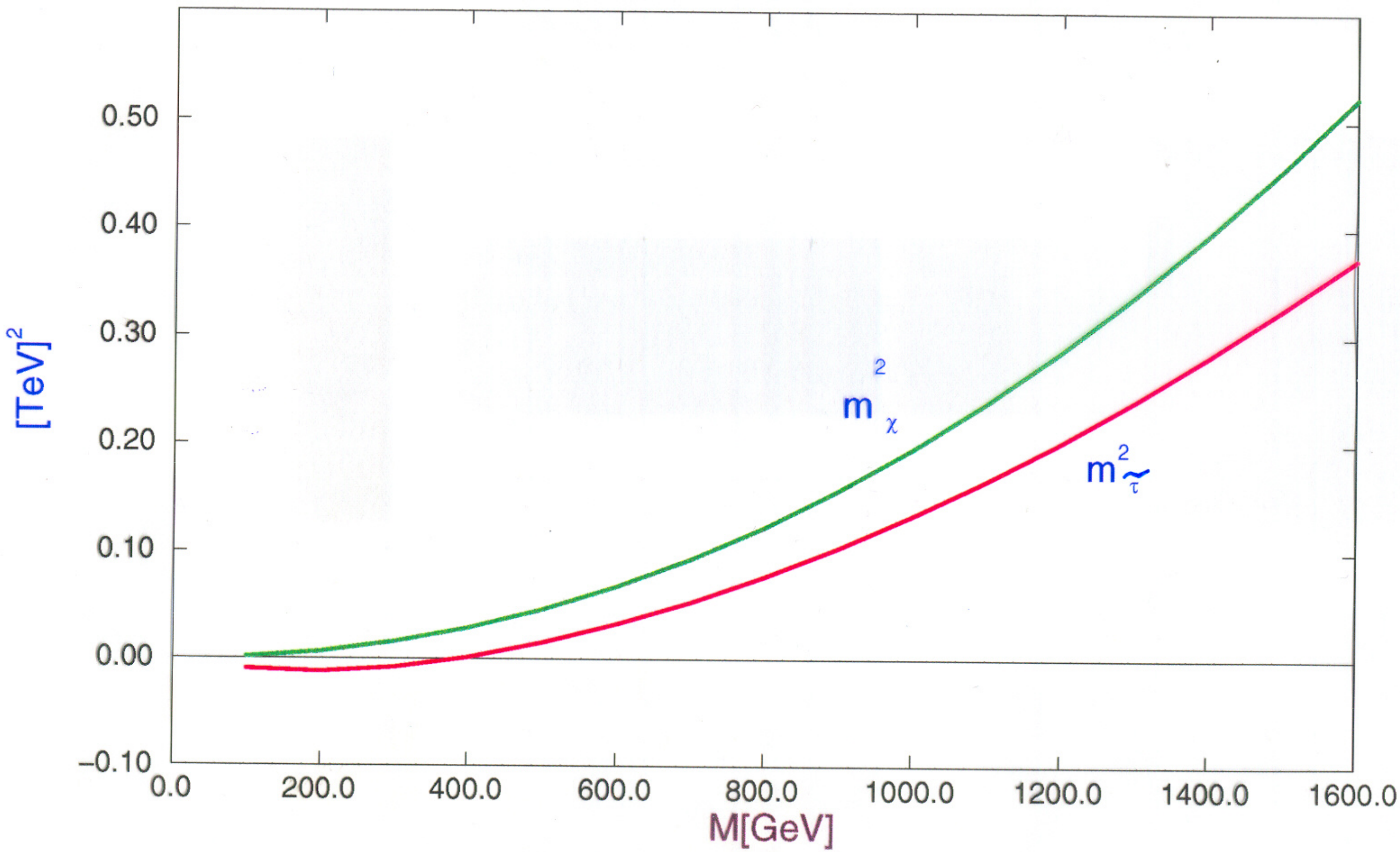
$\sim 4\%$

$M_t = 173.8 \pm 5$ GeV ; 178.0 ± 4.3 GeV

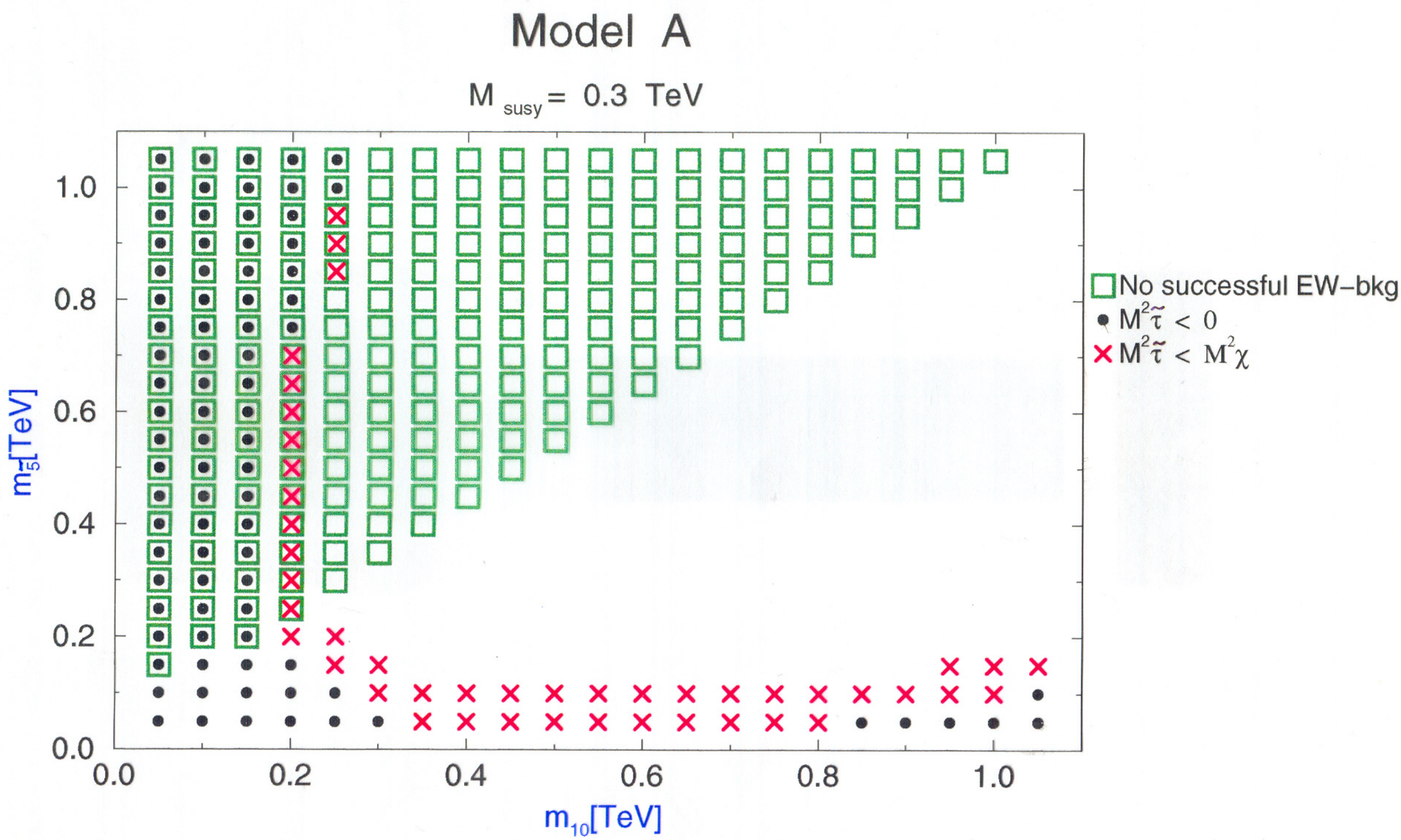
CDF + D0

Model A

Similar behaviour holds for Model B too

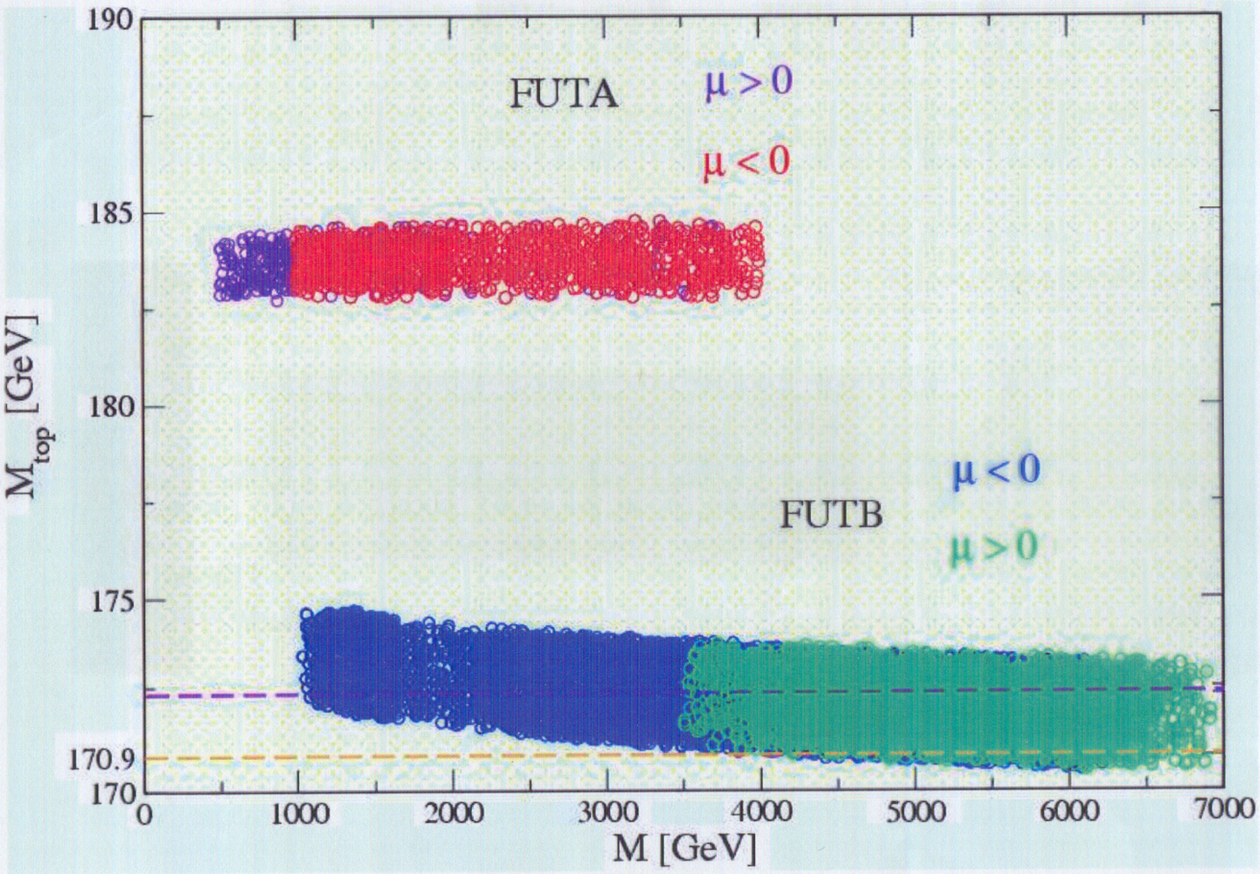


$m^2_{\tilde{\tau}}$ and m^2_χ for the universal choice of soft scalar masses



The empty region yields a neutralino as LSP

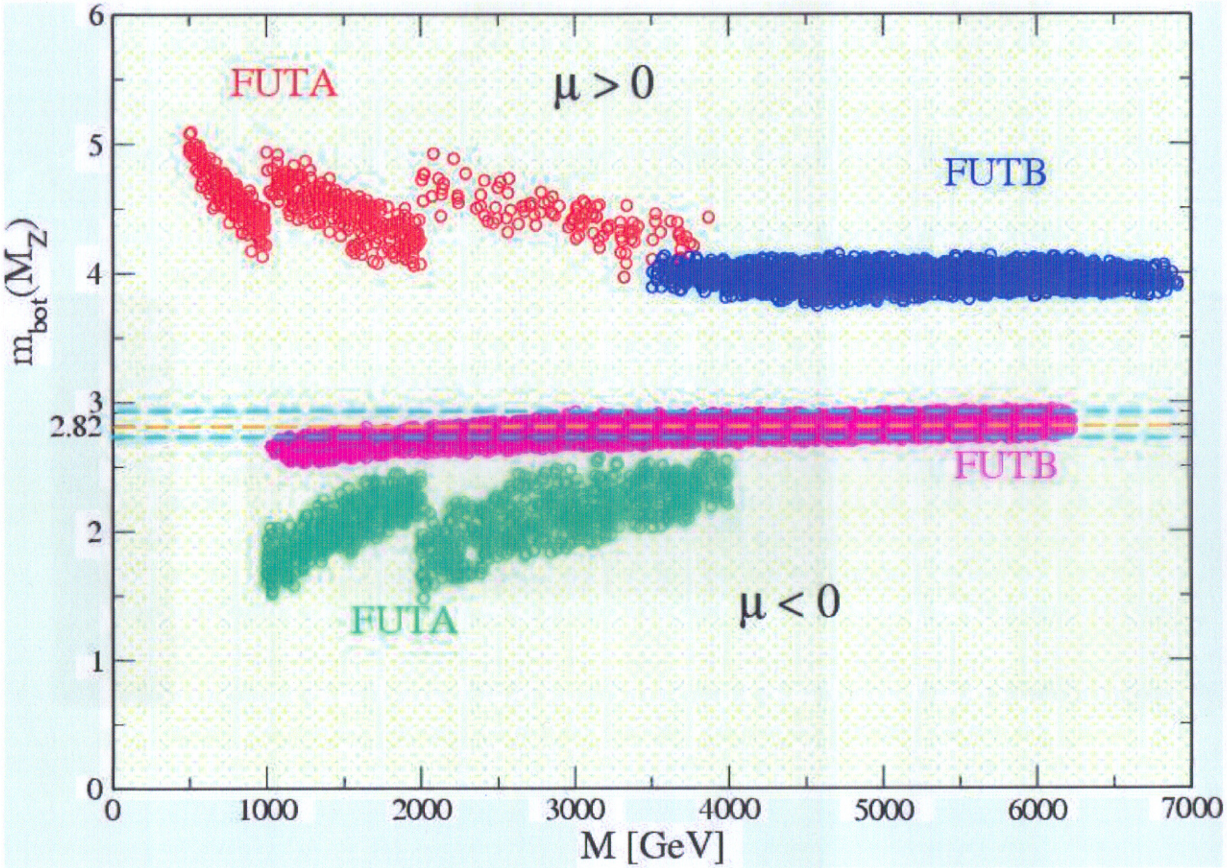
Application of m_t^{pole}:



⇒ FUTB gives the correct prediction for m_t

⇒ FUTA is ruled out experimentally

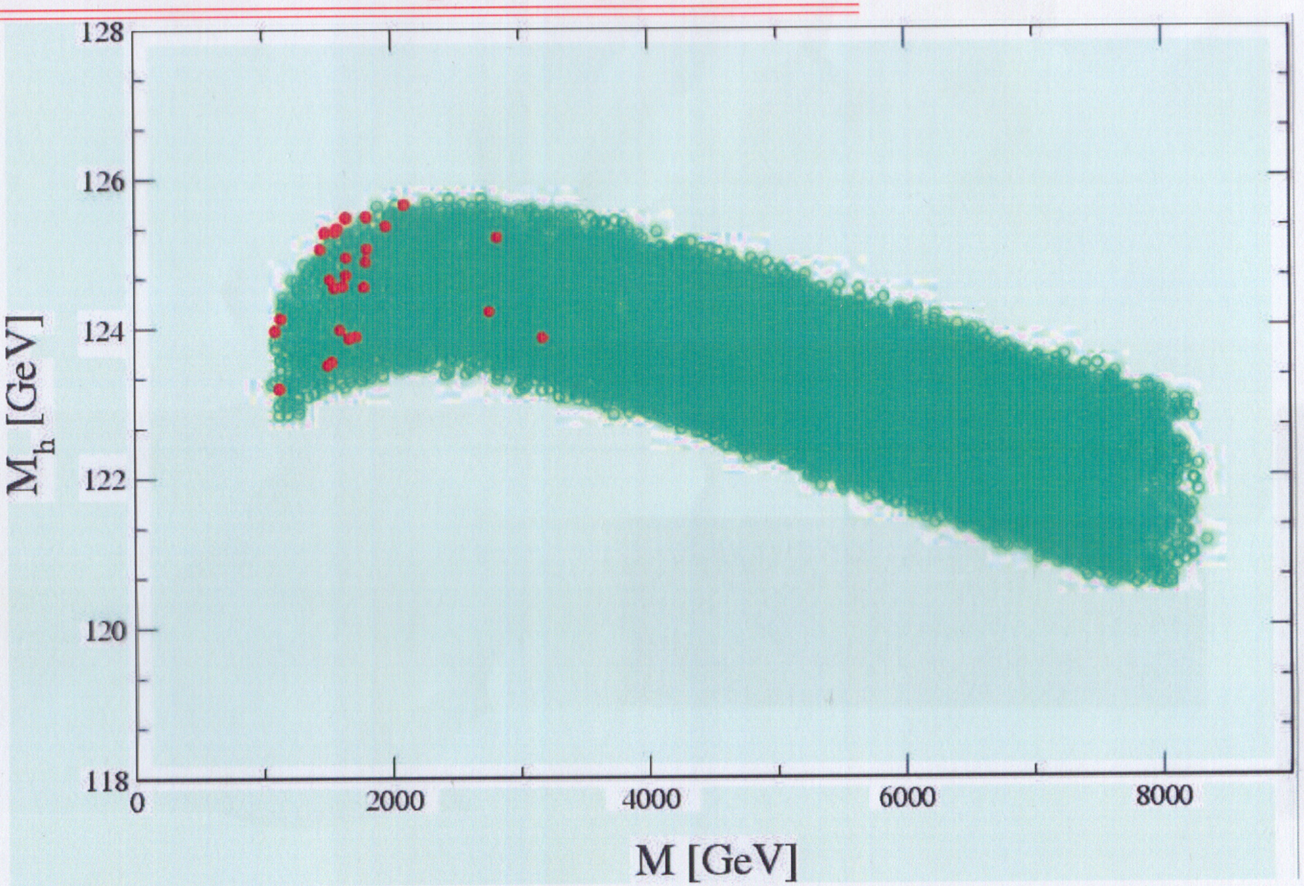
Application of $m_b(M_Z)$:



$\Rightarrow \mu < 0$ strongly favored

$\Rightarrow \mu > 0$ experimentally excluded

3D) Predictions for the light Higgs boson



green: consistent with B physics constraints
red: agreement with (loose) CDM bound

118 GeV $\leq M_h \leq$ 129 GeV (incl. theor. unc.)

$\Rightarrow$ "easy" to find for LHC (but "only" SM-like . . .)

Typical mass spectrum for FUTB- :

m_t	172	$\overline{m_b}(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}_1^0}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}_2^0}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}_3^0}$	2048	μ	-2046
$m_{\tilde{\chi}_4^0}$	2052	B	4722
$m_{\tilde{\chi}_1^\pm}$	1462	M_A	870
$m_{\tilde{\chi}_2^\pm}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

M1	580 GeV
M2	1077 GeV
Mgluino	2754 GeV
Stop1	1876 GeV
Stop2	2146 GeV
Sbot1	1849 GeV
Sbot2	2117 GeV
Mstau1	635 GeV
Mstau2	867 GeV
Char1	1072 GeV
Char2	1597 GeV
Neu1	579 GeV
Neu2	1072 GeV
Neu3	1591 GeV
Neu4	1596 GeV
Mtop	172.2 GeV
Mbot(M_Z)	2.71 GeV

FUTB, $\mu < 0$