

Generator Coordinate Method with proton-neutron pairing amplitudes

Nobuo Hinohara

Center for Computational Sciences, Univ. of Tsukuba, Japan
National Superconducting Cyclotron Laboratory, Michigan State Univ., USA



Jun. 26, 2015

The Future of multi-reference density functional theory

Outline

□ GCM with pn-pairing coordinates

NH and Engel, Phys. Rev. C90, 031301 (2014)

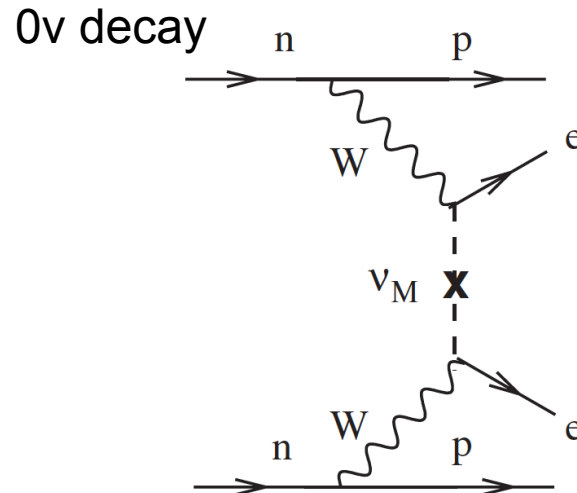
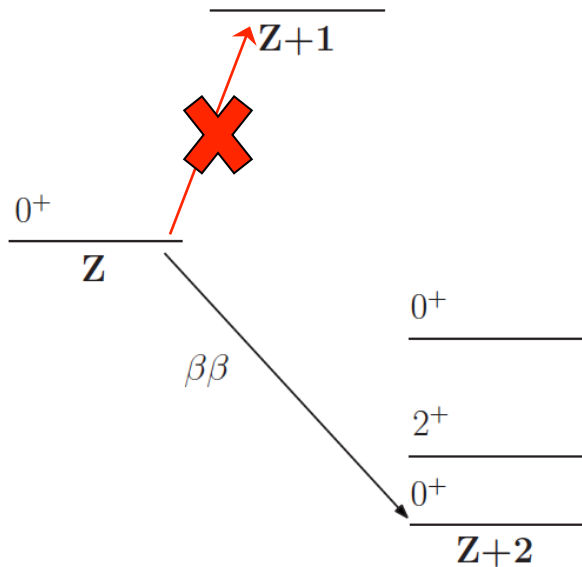
- Introduction, double-beta decay, QRPA, generator coordinate method, proton-neutron correlations
- application to SO(8) model, ^{76}Ge double-beta decay
- future plans
- summary

□ Finite-amplitude method for Nambu-Goldstone modes

NH, in preparation

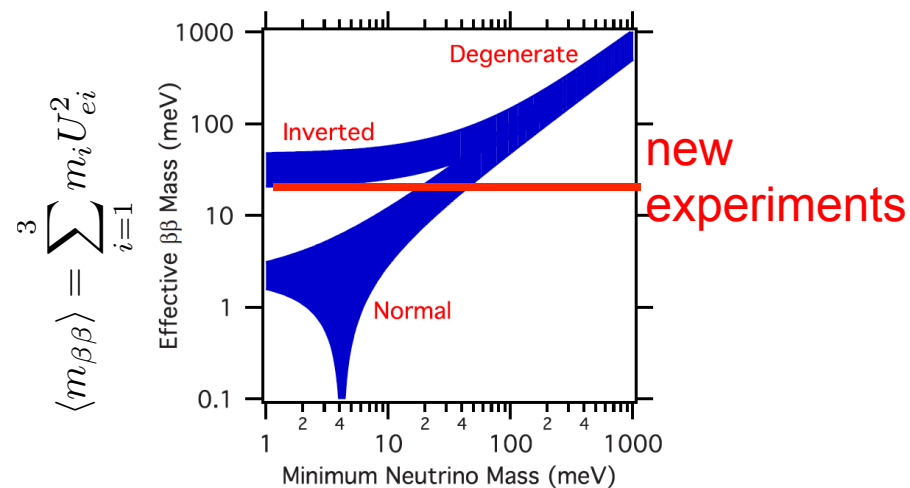
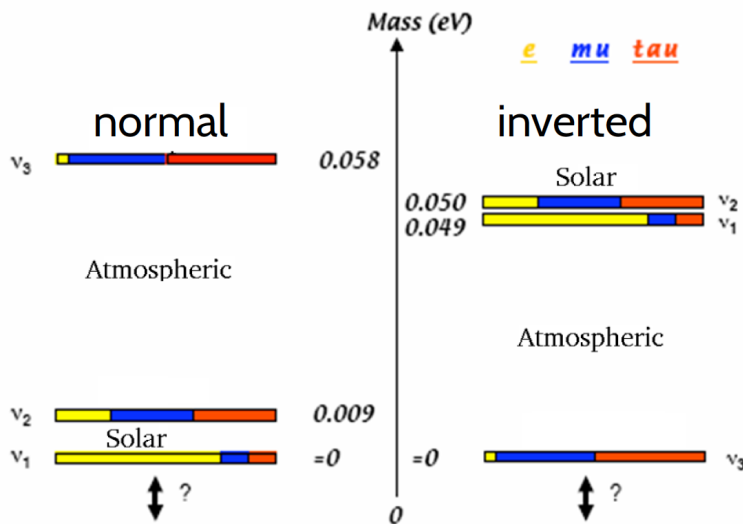
- Introduction
- formulation
- center of mass modes
- pairing rotation
- summary

Double-beta decay



0ν decay possible if neutrino is Majorana particle

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z) |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$



$$\langle m_{\beta\beta} \rangle = \sum_{i=1}^3 m_i U_{ei}^2$$

Nuclear Matrix Element

0ν half life

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z) |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$

$$M_{0\nu} \approx M_{0\nu}^{\text{GT}} - \frac{g_V^2}{g_A^2} M_{0\nu}^{\text{F}}$$

Closure approximation

$$M_{0\nu}^{\text{F}} = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \tau_a^+ \tau_b^+ | i \rangle$$

$$M_{0\nu}^{\text{GT}} = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle$$

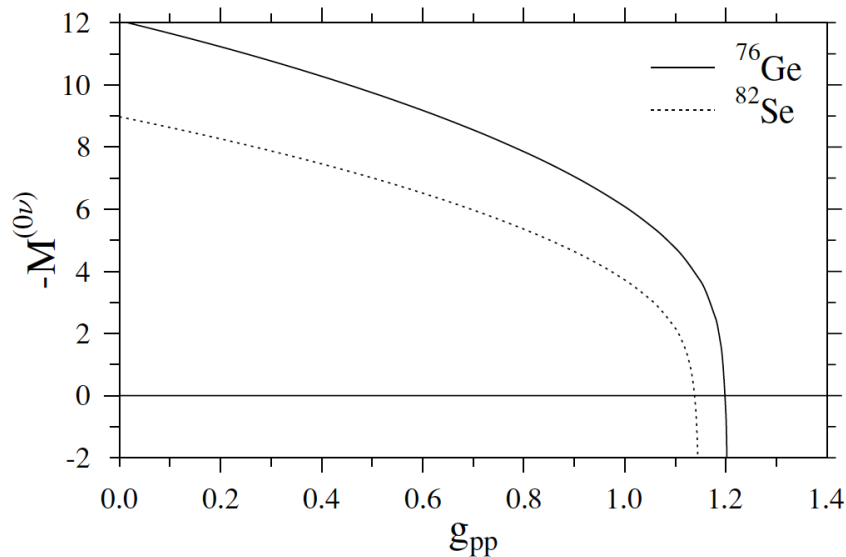
H: neutrino potential $\sim 1/r$

Neutron-proton QRPA

$$M_{0\nu}^F = \sum_{a,b,n_i,n_f} H(r_{ab}, \bar{E}) \langle f | \tau_a^+ | n_f \rangle \langle n_f | n_i \rangle \langle n_i | \tau_b^+ | i \rangle$$

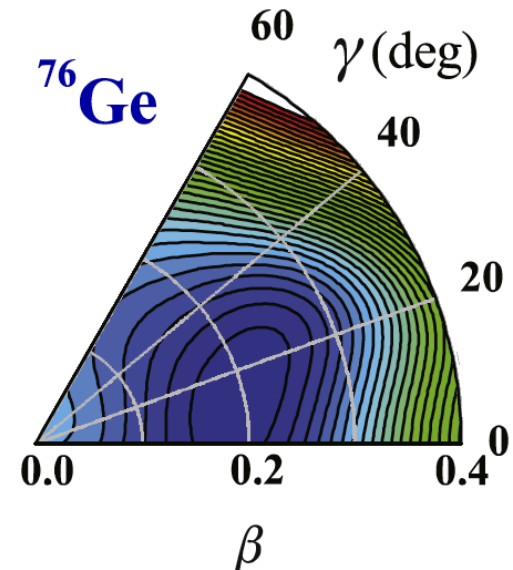
$$M_{0\nu}^{GT} = \sum_{a,b,n_i,n_f} H(r_{ab}, \bar{E}) \langle f | \vec{\sigma}_a \tau_a^+ | n_f \rangle \langle n_f | n_i \rangle \langle n_i | \vec{\sigma}_b \tau_b^+ | i \rangle$$

isoscalar neutron-proton pairing
interaction dependence



Kortelainen and Suhonen, PRC**75**, 051303(2007)

Shape fluctuation
Shape coexistence



J.J.Sun et al, PLB**734**, 308 (2014)

QRPA is valid when the mean field approximation is good for ground states and system is not too close to the phase transition

Going beyond small-amplitude approximation

Generator Coordinate Method

superposition of the projected mean fields along generator coordinates q

$$|\Psi(N, Z, I = 0, M = 0)\rangle = \sum_q f_k(q) |\phi_{I=0, M=0}^{N, Z}(q)\rangle$$

initial/final ground state weight function

- correlations along important coordinates (q : collective property: deformation, pairing...)
- weight function is determined from variational equation (Hill-Wheeler equation)
- mean field breaks symmetries and correlations are included:
 - symmetry restoration: particle number and angular momentum projection
- “shell model” using a small set of collective non-orthogonal basis only

Large model space

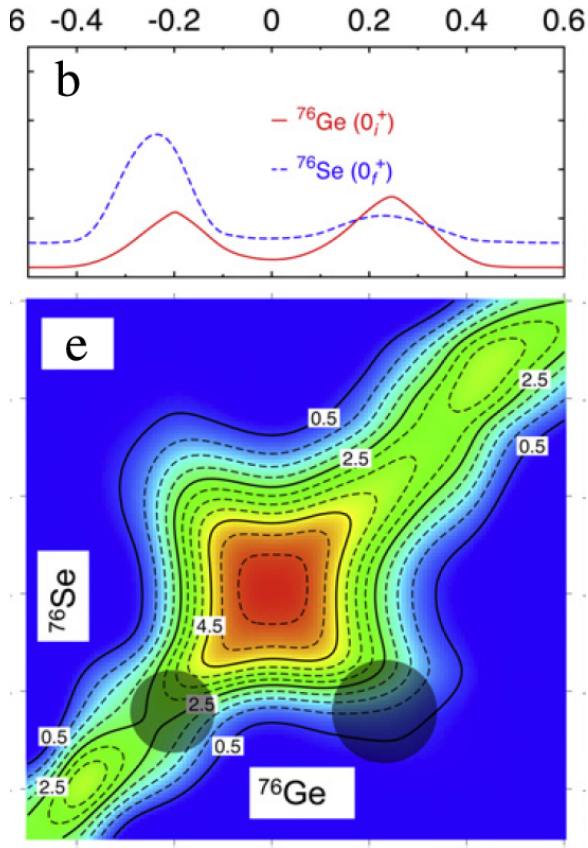
based on mean field approximation

Phase transition / fluctuation

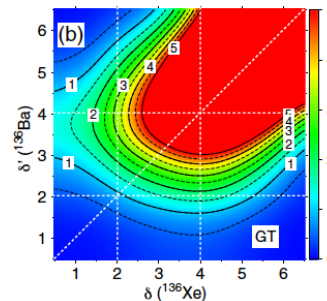
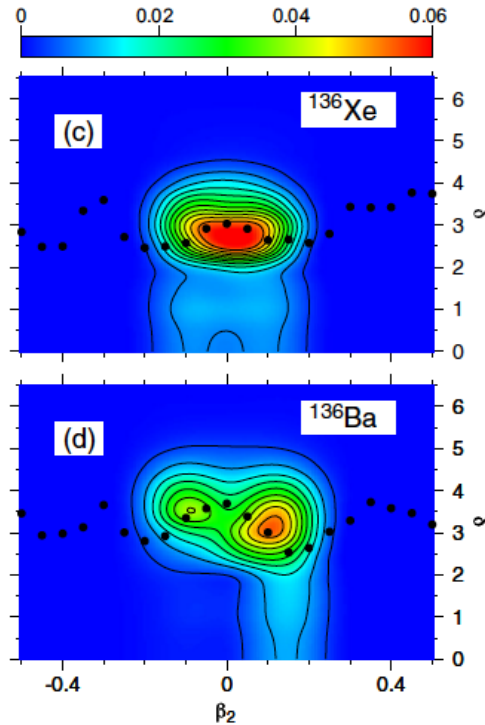
including order parameters in the generator coordinates

Generator coordinate method (Gogny D1S)

deformation



isovector pairing/deformation



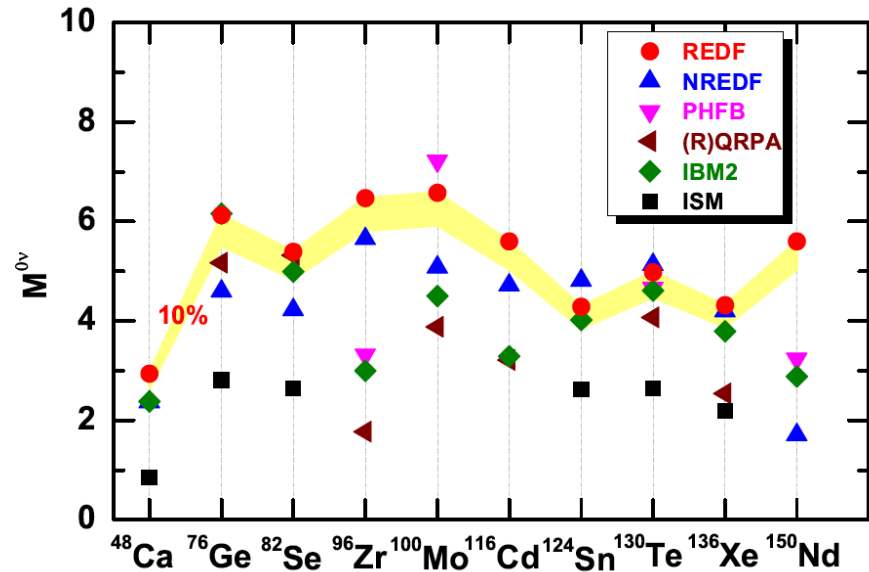
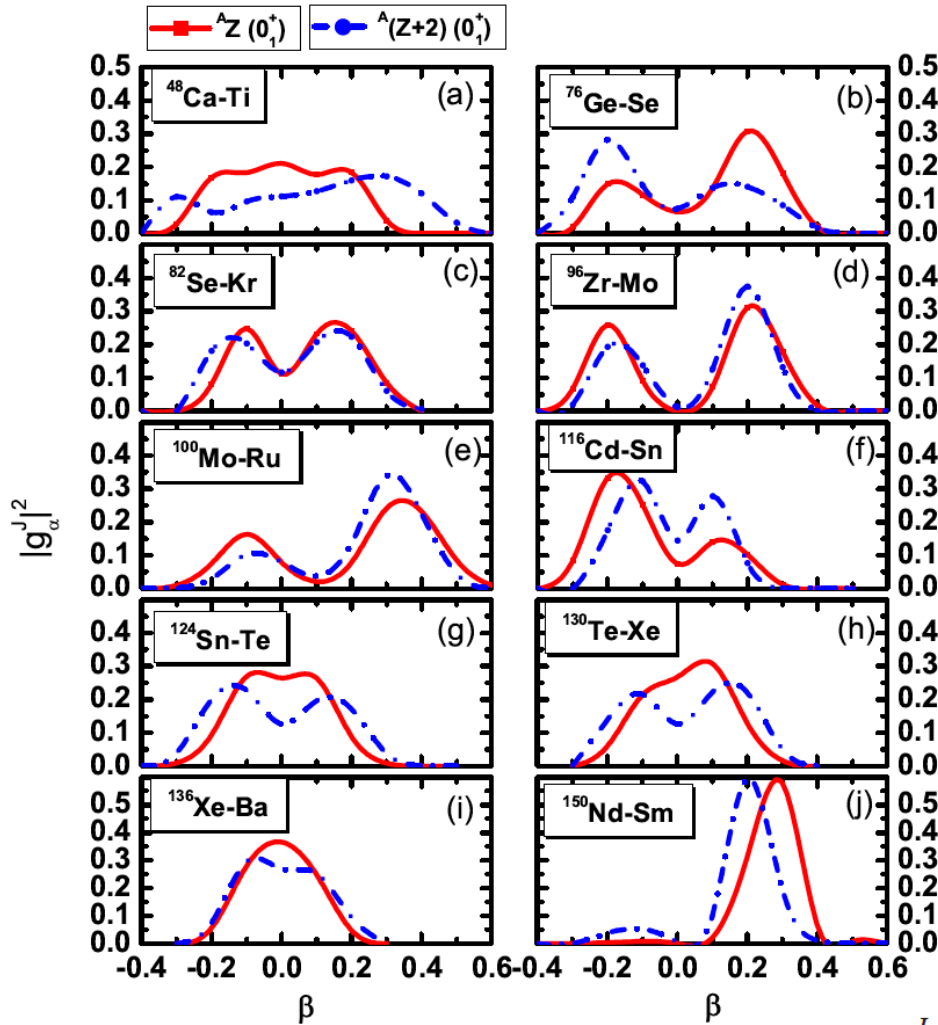
Isotope	$M^{0\nu}(\beta_2)$	$M^{0\nu}(\beta_2, \delta)$
^{48}Ca	$2.370^{1.914}_{0.456}$	$2.229^{1.797}_{0.431}$
^{48}Ti		
^{76}Ge	$4.601^{3.715}_{0.886}$	$5.551^{4.470}_{1.082}$
^{76}Se		
^{82}Se	$4.218^{3.381}_{0.837}$	$4.674^{3.743}_{0.931}$
^{82}Kr		
^{96}Zr	$5.650^{4.618}_{1.032}$	$6.498^{5.296}_{1.202}$
^{96}Mo		
^{100}Mo	$5.084^{4.149}_{0.935}$	$6.588^{5.361}_{1.227}$
^{100}Ru		
^{116}Cd	$4.795^{3.931}_{0.864}$	$5.348^{4.372}_{0.976}$
^{116}Sn		
^{124}Sn	$4.808^{3.893}_{0.916}$	$5.787^{4.680}_{1.107}$
^{124}Te		
^{128}Te	$4.107^{3.079}_{1.027}$	$5.687^{4.255}_{1.432}$
^{128}Xe		
^{130}Te	$5.130^{4.141}_{0.989}$	$6.405^{5.161}_{1.244}$
^{130}Xe		
^{136}Xe	$4.199^{3.673}_{0.526}$	$4.773^{4.170}_{0.604}$
^{136}Ba		
^{150}Nd	$1.707^{1.278}_{0.429}$	$2.190^{1.639}_{0.551}$
^{150}Sm		

$$M_{\xi}^{0\nu}(\beta_i, \beta_f) = \frac{\langle \Phi_{\beta_f} | P^{N_f} P^{Z_f} \hat{M}_{\xi}^{0\nu} P^{I=0} P^{N_i} P^{Z_i} | \Phi_{\beta_i} \rangle}{\sqrt{\langle \Phi_{\beta_f} | P^{I=0} P^{N_f} P^{Z_f} | \Phi_{\beta_f} \rangle} \sqrt{\langle \Phi_{\beta_i} | P^{I=0} P^{N_i} P^{Z_i} | \Phi_{\beta_i} \rangle}}$$

T. Rodriguez and G. Martinez-Pinedo, Prog. Part. Nucl. Phys. **66**, 436 (2011)
 Vaquero, Rodriguez, Egido Phys. Rev. Lett. 111,142501 (2013)

Generator coordinate method (RMF)

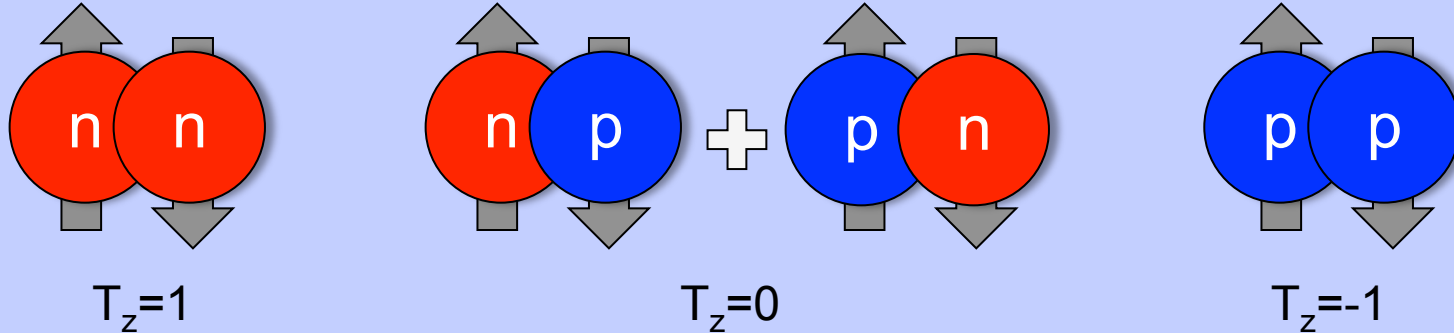
generator coordinate: axial quadrupole deformation β



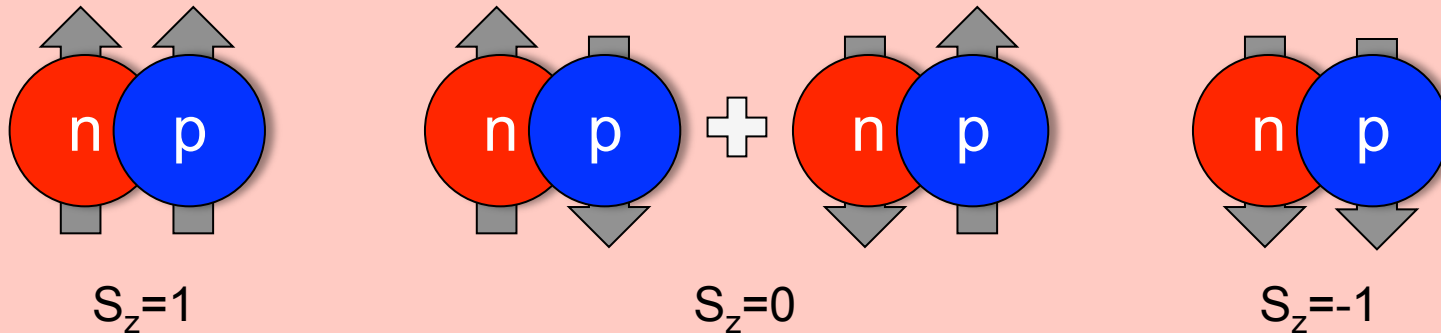
Yao et al. Phys. Rev. C **91**, 024316 (2015)
 Song et al., Phys. Rev. C **90**, 054309 (2014)

Neutron-proton correlation

Isovector ($T=1, S=0$) pairings \rightarrow Fermi matrix element



Isoscalar ($T=0, S=1$) pairings \rightarrow Gamow-Teller matrix element



$\sigma\tau$ (Gamow-Teller type) particle-hole ($T=1, S=1$)
 \rightarrow Gamow-Teller matrix element

GCM for nuclear matrix element

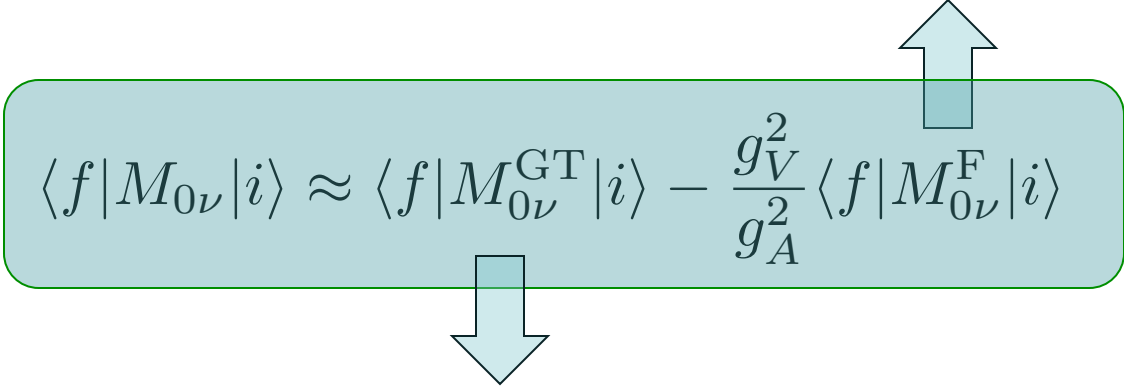
GCM with deformation and np pairing degrees of freedom
with a simple shell model interaction (P+Q model)

Generalized Hartree-Bogoliubov (spherical 3D HO basis)

$$\hat{a}_k^\dagger = \sum_l \left(U_{lk}^{(n)} \hat{c}_l^{(n)\dagger} + V_{lk}^{(n)} \hat{c}_k^{(n)} + U_{lk}^{(p)} \hat{c}_l^{(p)\dagger} + V_{lk}^{(p)} \hat{c}_k^{(p)} \right)$$

$$a_k |\phi(q)\rangle = 0$$

GCM with deformation and isovector np pairing


$$\langle f | M_{0\nu} | i \rangle \approx \langle f | M_{0\nu}^{\text{GT}} | i \rangle - \frac{g_V^2}{g_A^2} \langle f | M_{0\nu}^{\text{F}} | i \rangle$$

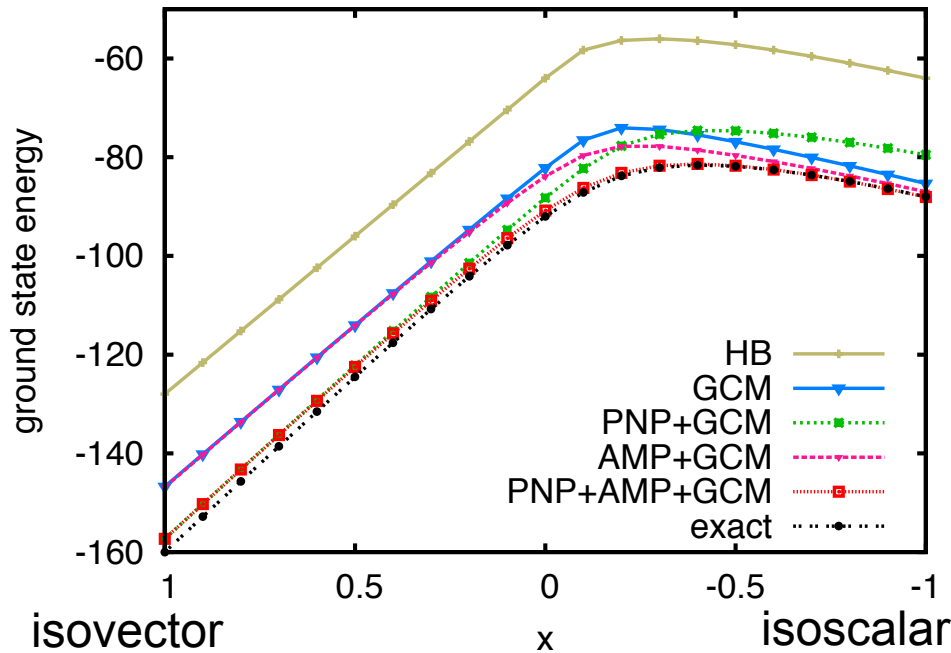
GCM with deformation and isoscalar np pairing

reliable framework for the nuclear matrix elements
with neutron-proton correlation

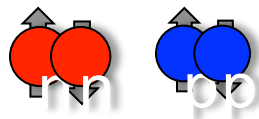
Comparison with SO(8) model

SO(8) Hamiltonian: isovector, isoscalar pairing, spin-isospin ph int.
 $\Omega=12, A=24$ 2nu GT matrix element of $T=4 \rightarrow T=2$

initial state energy: $T=4(N=16, Z=8)$

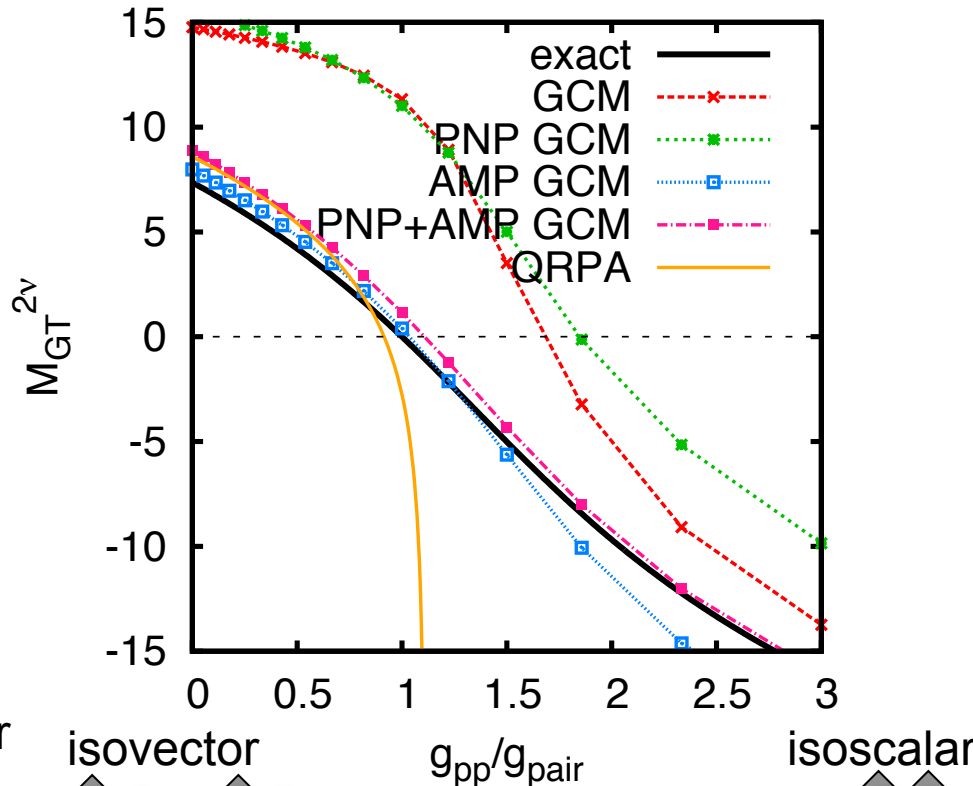
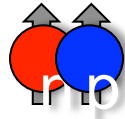


isovector

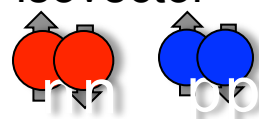


x

isoscalar



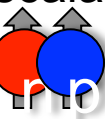
isovector



g_{pp}/g_{pair}

$$g_{pp}/g_{pair} = (1-x)/(1+x)$$

isoscalar



- Generator coordinate: isoscalar pairing P_0 ($S_z=0$, 1-dimensional GCM)
- Exact solution: isospin symmetric: GCM basis breaks isospin symmetry

$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ 0ν matrix element (1D GCM)

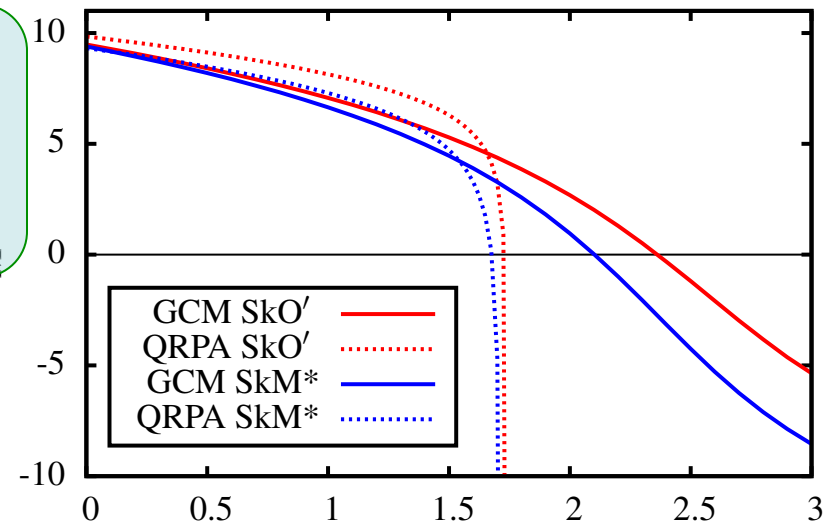
NH and J. Engel, Phys.Rev.C90, 031301(R) (2014)

Hamiltonian: sp energy + SO(8) + QQ force (three indep isovector pairing strengths)
 parameters: fitted to Skyrme SkO'/SkM* deformation, gap

g_{ph} : GT- resonance peak of ^{76}Ge , (Skyrme HFB) g_{pp} : exp B(GT+) of ^{76}Se
 pf + sdg two major shells

1D GCM
 calc without QQ force
 generator coordinate: isoscalar pairing

$$g_{\text{pp}} = 1.47(\text{SkO}'), 1.56(\text{SkM}^*)$$

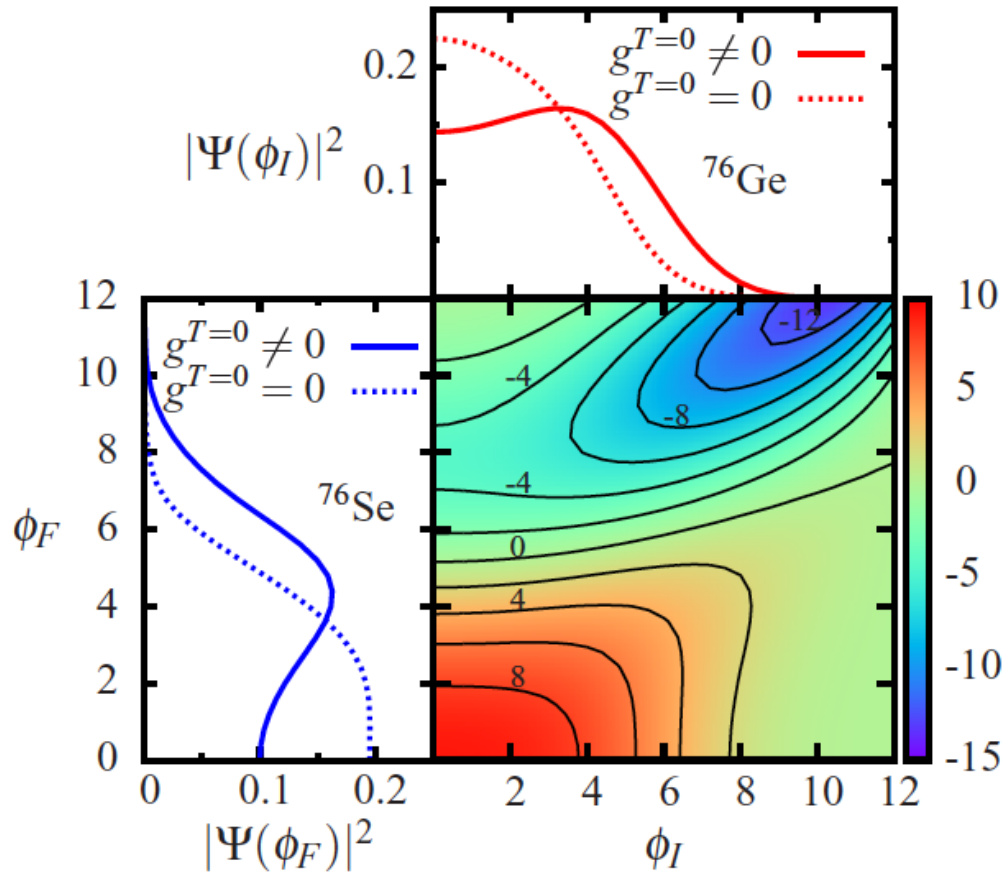


QRPA: collapse near the phase transition $g_{\text{pp}} = g^{T=0}/g^{T=1} \sim 1.6 \frac{g^{T=0}}{\bar{g}^{T=1}}$
 GCM: smooth dependence on isoscalar pairing

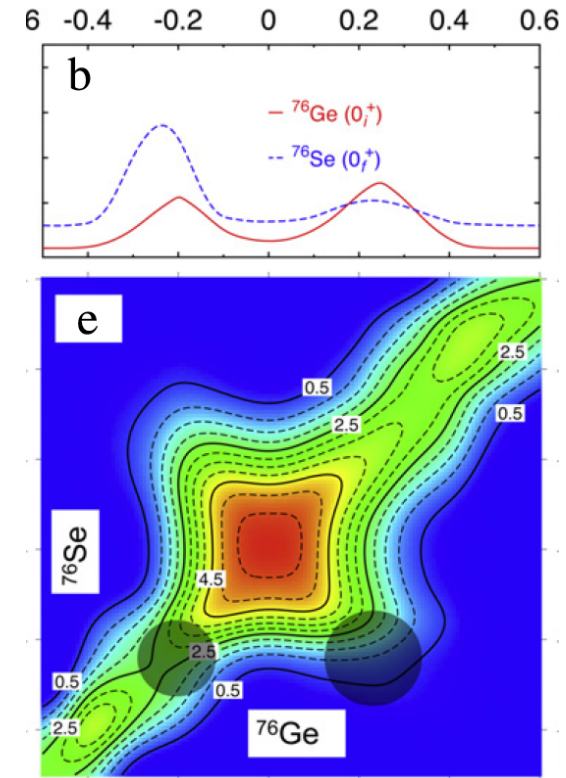
Skyrme	no $g_{\text{ph}}/$ $g^{T=0}$	no $g^{T=0}$	1D full	QRPA
SkO'	14.0	9.5	5.4	5.6
SkM*	11.8	9.4	4.1	3.5

$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ 0ν matrix element (1D GCM)

matrix element and collective wave function squared



similar plot for β
(Rodriguez)



matrix element is large at the
same deformation

generator coordinate: $\phi = \frac{\langle P_0 + P_0^\dagger \rangle}{2}$

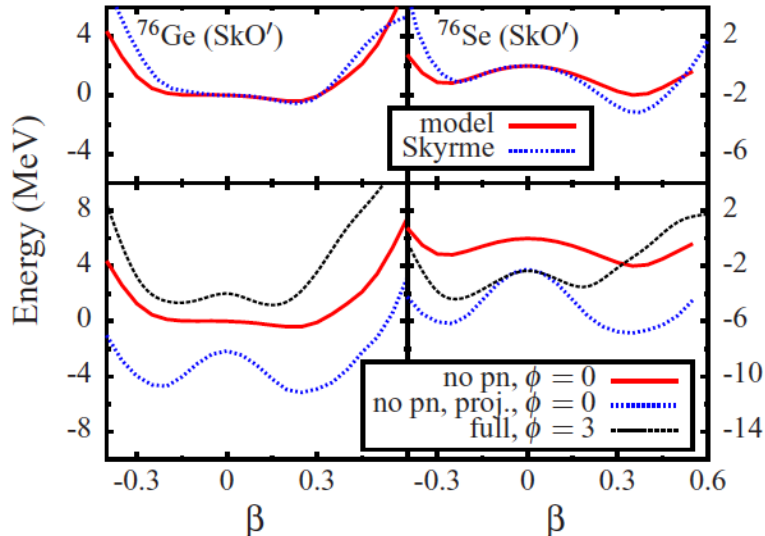
off-diagonal part of the matrix element important

negative region at large isoscalar pairing of final state

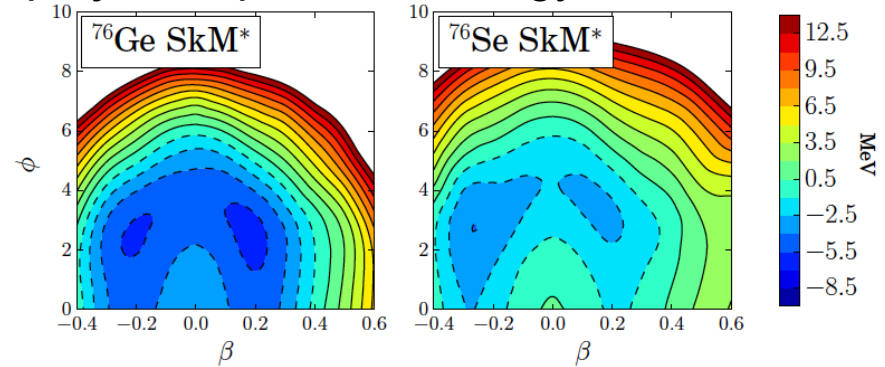
isoscalar pairing shifts the wave function to isoscalar region

Inclusion of quadrupole deformation (2D GCM)

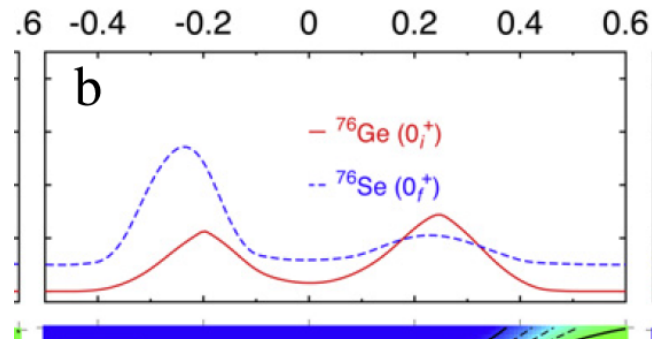
potential energy surface



$g_{pp} = 1.75(\text{SkO}'), 1.51(\text{SkM}^*)$
projected potential energy surface

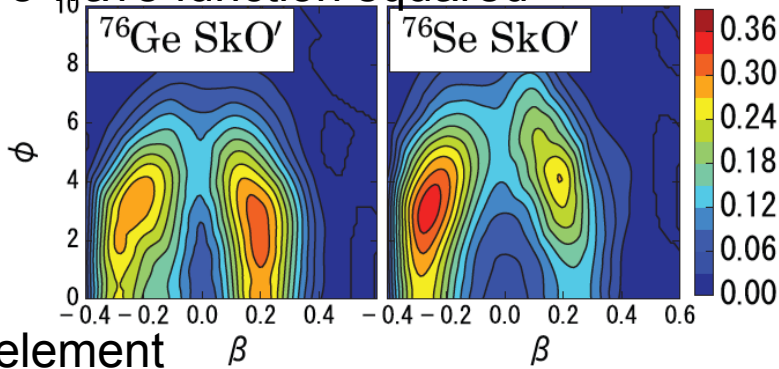


Rodríguez and Martínez-Pinedo
Prog. Part. Nucl. Phys. **66** (2011) 436.



Gogny beta-GCM: 4.6
PRL 105, 252503 (2010)
Gogny beta+delta GCM: 5.6
PRL 111, 142501 (2013)
Skyrme pnQRPA SkM*: 5.1
PRC 87, 064302 (2013)

collective wave function squared



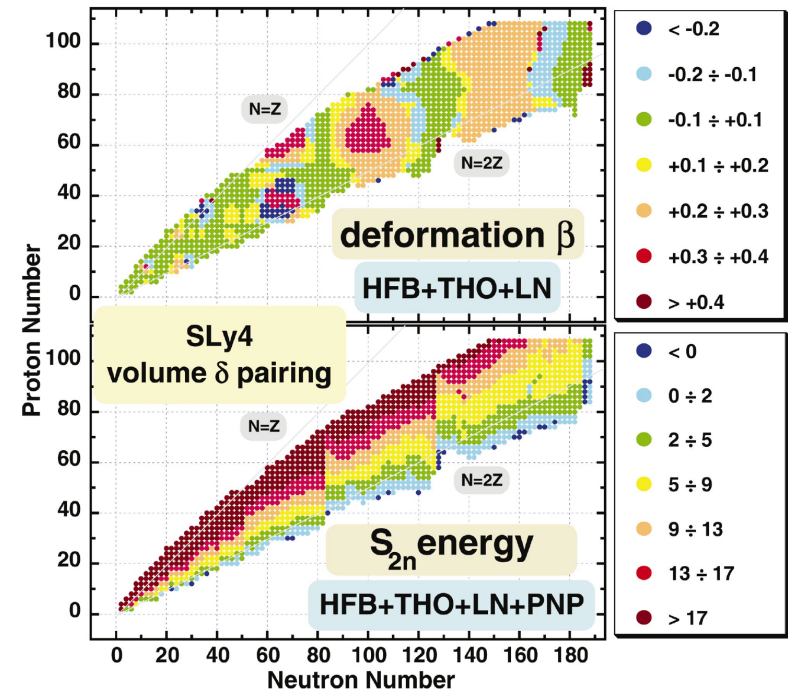
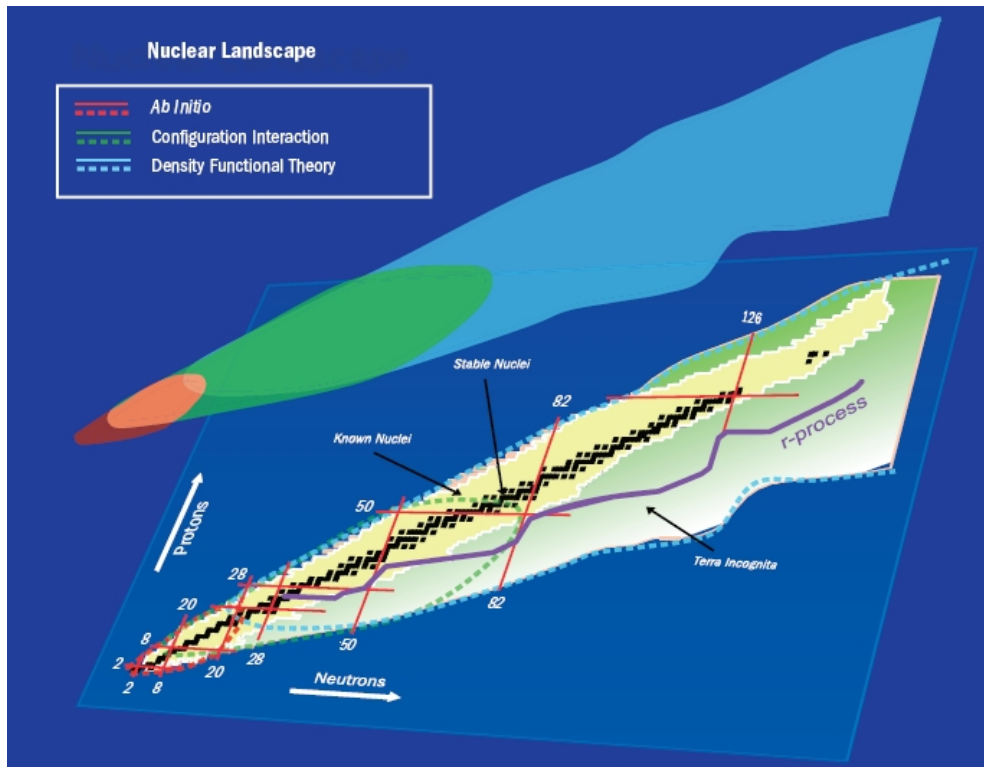
matrix element

Skyrme	1D full	2D full	spherical QRPA
SkO'	5.4	4.7	5.6
SkM*	4.1	4.7	3.5

Future plans for systematic calculation

things to be improved: **effective interaction**

- 1) Alternative approach to shell model for heavier system
- 2) Extension to Skyrme-DFT



Stoitsov et al., Phys. Rev. **C68**,054312 (2003)

Comparison with shell model in pf-shell nuclei

J. Menendez, NH, J. Engel, in preparation

Shell model: KB3G interaction (black)

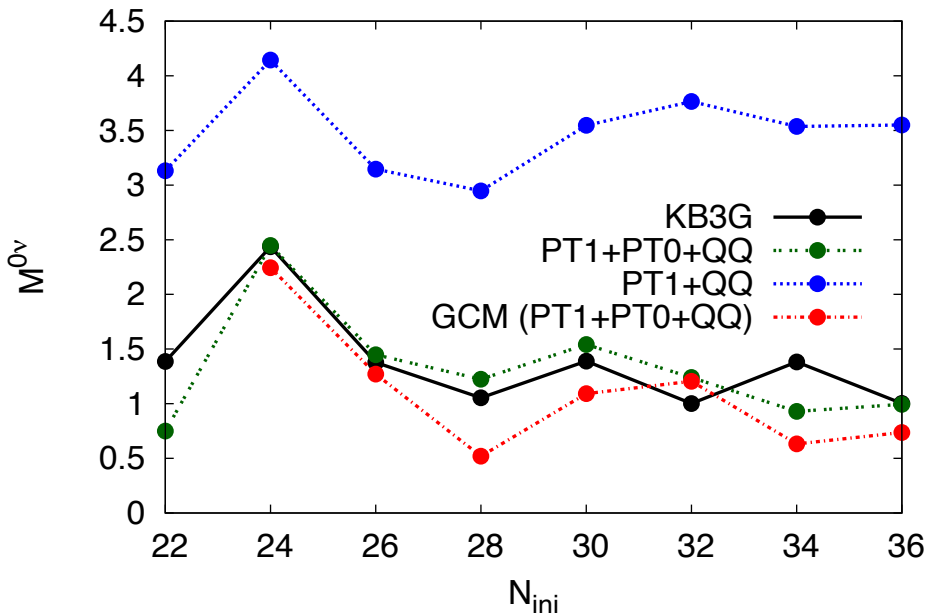
PT1+PT0+QQ P+Q derived from Dufour and Zuker prescription (green)

Shell model without isoscalar pairing (blue)

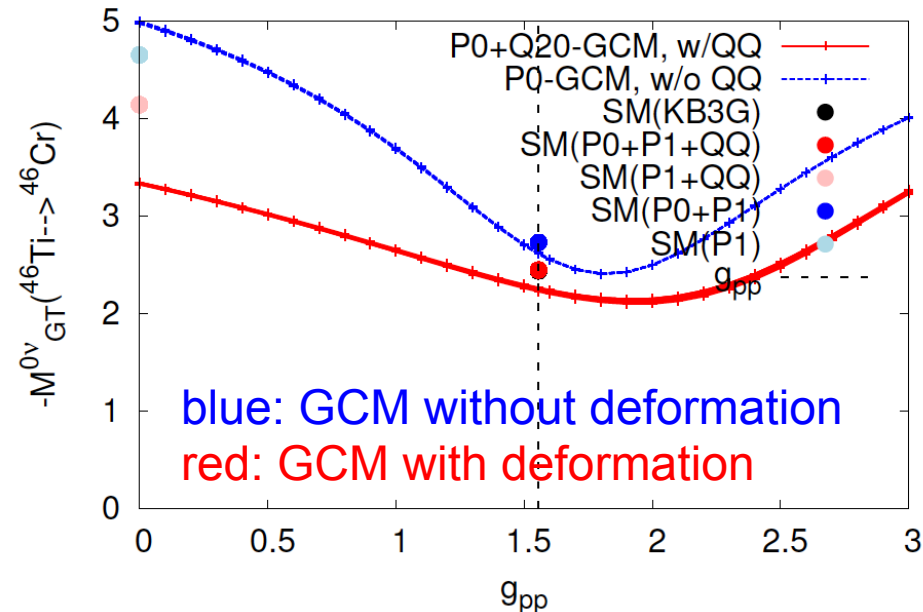
GCM: generator coordinate: isoscalar pairing and axial deformation (red)

Dufour and Zuker, Phys. Rev. C **54**, 1641 (1996)

GT matrix element Ti → Cr



gpp dependence



“collective” degrees of freedom (isoscalar pairing) play major role even in light systems.

alternative solution of a shell model Hamiltonian diagonalization for heavier system

Extension to Skyrme DFT

neutron-proton DFT for generating GCM basis

isospin-invariant DFT: Perlinska et al., Phys. Rev. C **69**, 014316 (2004)

current status:

neutron-proton mixing in particle-holes (HFODD/HFBTHO)

K. Sato, et al. Phys. Rev. C **88**, 061301 (2013)

J. Sheikh, NH, et al. Phys. Rev. C **89**, 054317 (2014)

pairing part in progress

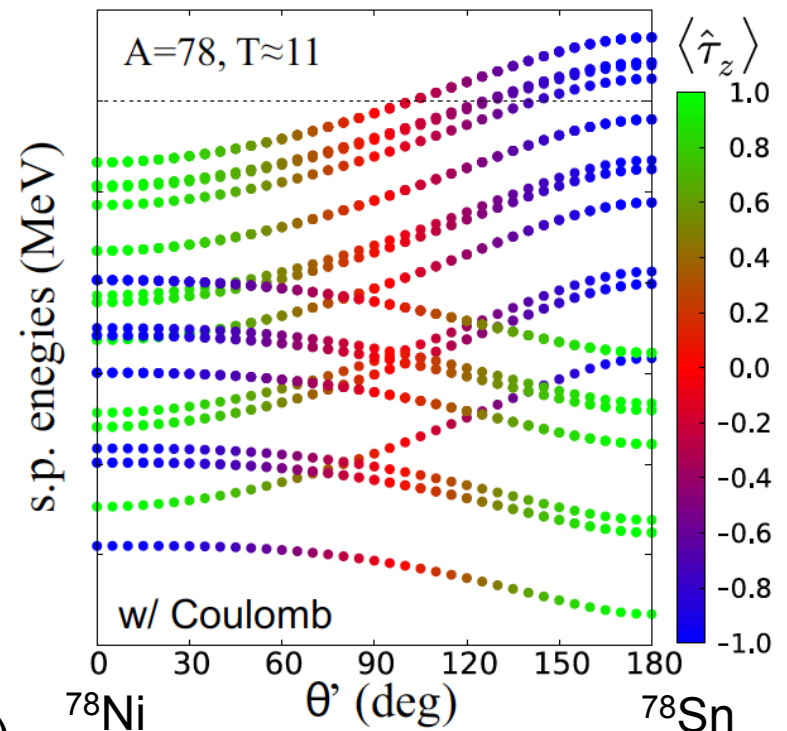
projection problem:

regularization schemes

Lacroix, Duguet, Bender Phys. Rev. C **79** (2009)

Satula and Dobaczewski Phys. Rev. C **90**, 054303 (2014) ...

T=11 isobaric analogue states



Summary

□ Double-beta decay nuclear matrix elements are calculated using generator coordinate method including both axial quadrupole deformation and isoscalar/isovector proton-neutron pairing degrees of freedom.

□ Future plans

- 1) alternative approach to shell model for heavier system
- 2) Extension to Skyrme-DFT

Finite amplitude method for Nambu-Goldstone modes

NH, in preparation

Spontaneous symmetry breaking

- Nambu-Goldstone (NG) mode appears as a solution of self-consistent QRPA when mean field (DFT) breaks continuous symmetries which the original EDF has

broken symmetry	mean field	NG mode in the QRPA	K^π
translational (Galilean invariance)	center of mass fixed to the origin	center of mass motion	$0^-, 1^-$
rotational	deformation (axial or triaxial)	rotation	$1^+, (2^+)$
particle number (gauge symmetry)	pairing condensation (BCS)	pairing rotation	0^+
neutron-proton (isospin symmetry)	neutron-proton mixing	isospin rotation	0^+

NG mode restores the broken symmetry in the QRPA level

$$[\hat{H}_{\text{QRPA}}, \hat{\mathcal{P}}_{\text{NG}}] = i\Omega_{\text{NG}}^2 M_{\text{TV}} \hat{\mathcal{Q}}_{\text{NG}} = 0$$

$$[\hat{H}_{\text{QRPA}}, \hat{\mathcal{Q}}_{\text{NG}}] = -\frac{i}{M_{\text{TV}}} \hat{\mathcal{P}}_{\text{NG}}$$

\mathcal{P}_{NG} : broken symmetry (momentum operator)

Thouless-Valatin inertia from QRPA

$$[\hat{H}_{\text{QRPA}}, \hat{Q}_{\text{NG}}] = -\frac{i}{M_{\text{TV}}} \hat{P}_{\text{NG}}$$

M_{TV} : Thouless-Valatin inertia

Q_{NG} : canonical conjugate coordinate op.

Thouless-Valatin inertia from QRPA

$$M_{\text{TV}} = 2P_{\text{NG}}(A + B)^{-1}P_{\text{NG}}$$

Conjugate coordinate operator (not known except for center of mass motion)

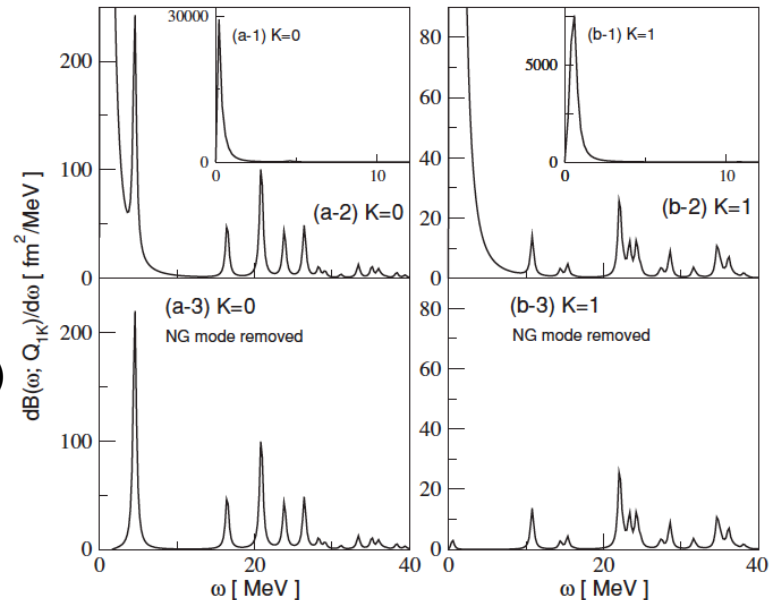
$$Q_{\text{NG}} = iM_{\text{NG}}^{-1}(A + B)^{-1}P_{\text{NG}} \quad (\text{Im } P_{\text{NG}} = 0),$$

$$Q_{\text{NG}} = -iM_{\text{NG}}^{-1}(A - B)^{-1}P_{\text{NG}} \quad (\text{Re } P_{\text{NG}} = 0).$$

Required for spurious mode removal (FAM/iterative Arnoldi)

$$\delta\rho_{\text{cal}}(\omega) = \delta\rho_{\text{phy}}(\omega) + \lambda_P \delta\rho_P + \lambda_R \delta\rho_R,$$

$$\delta\rho_R \equiv i[R, \rho_0] = \sum_i (|\bar{R}_i\rangle\langle\phi_i| + |\phi_i\rangle\langle\bar{R}_i|),$$



Thouless-Valatin inertia from FAM

Strength function in PQ representation

$$S(\hat{F}, \omega) = \sum_i \frac{1}{\omega^2 - \Omega_i^2} \left\{ \frac{1}{M_i} |\langle P_i | \hat{F} | 0 \rangle|^2 + M_i \Omega_i^2 |\langle Q_i | \hat{F} | 0 \rangle|^2 + \omega [QP]_i(\hat{F}) \right\}$$

$$[QP]_i(\hat{F}) \equiv i \left(\langle Q_i | \hat{F} | 0 \rangle^* \langle P_i | \hat{F} | 0 \rangle - \langle P_i | \hat{F} | 0 \rangle^* \langle Q_i | \hat{F} | 0 \rangle \right)$$

FAM for NG modes: (Blaizot and Ripka)

$$S(\hat{F}, \omega)_{\text{NG}} = \sum_{i, \Omega_i=0} \frac{1}{\omega^2} \left\{ \frac{1}{M_i} |\langle P_i | \hat{F} | 0 \rangle|^2 + \omega [QP]_i(\hat{F}) \right\}$$

external field $F = Q_{\text{NG}}$

Thouless-Valatin inertia from
the energy-weighted sum rule of the conjugate coordinate operator

$$M_{\text{TV}}^{-1} = 2m_1(\hat{Q}_{\text{NG}}) = \frac{2}{2\pi i} \int_D \omega S(\hat{Q}_{\text{NG}}, \omega) d\omega$$

Thouless-Valatin inertia from FAM

external field $F = P_{\text{NG}}$

Thouless-Valatin inertia from a linear response calculation at zero energy, using a broken-symmetry operator (related to inverse energy-weighted sum rule)

$$S(\hat{\mathcal{P}}_{\text{NG}}, \omega) = \begin{cases} 0 & (\omega \neq \pm\Omega_i) \\ -M_{\text{NG}} & (\omega = 0) \end{cases}$$

Coordinate operator is computed from the amplitudes

$$Q_{\text{NG}} = iM_{\text{NG}}^{-1}(A + B)^{-1}P_{\text{NG}} = i\frac{X(0) + Y(0)}{2S(\hat{\mathcal{P}}_{\text{NG}}, 0)} \quad (\text{Im } P_{\text{NG}} = 0),$$

$$Q_{\text{NG}} = -iM_{\text{NG}}^{-1}(A - B)^{-1}P_{\text{NG}} = i\frac{X(0) - Y(0)}{2S(\hat{\mathcal{P}}_{\text{NG}}, 0)} \quad (\text{Re } P_{\text{NG}} = 0).$$

Center of mass mode

trivial case

$$\hat{Q}_{\text{CM}} = \frac{1}{A} \sum_{i=1}^A \hat{r}_i, \quad \hat{P}_{\text{CM}} = -i \sum_{i=1}^A \hat{\nabla}_i, \quad M_{\text{CM}} = mA$$

finite HO basis:

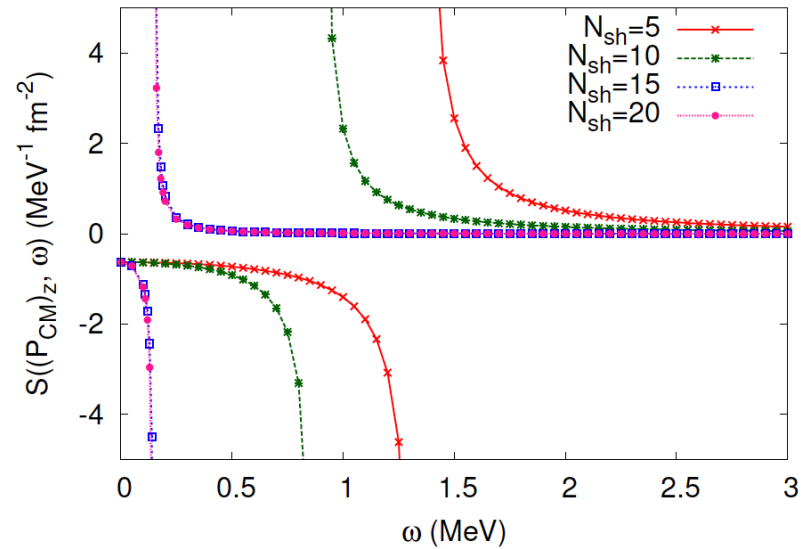
translational mode is not at zero energy

$$S(\hat{\mathcal{P}}_{\text{NG}}, \omega) \sim \frac{M_{\text{NG}} \Omega_{\text{NG}}^2}{\omega^2 - \Omega_{\text{NG}}^2}$$

$$\Omega_{\text{NG}}^2 = \frac{1}{S(\hat{\mathcal{P}}_{\text{NG}}, 0) S(\hat{Q}_{\text{NG}}, 0)}$$

HFBTHO, SLy4+volume pairing, 26Mg (oblate)

Response to momentum operator



N_{sh}	$1/2m$ from $(\hat{Q}_{\text{CM}})_z$	$1/2m$ from $(\hat{P}_{\text{CM}})_z$	Inglis-Belyaev	Ω_{CM} MeV	$\langle [(\hat{Q}_{\text{CM}})_z, (\hat{P}_{\text{CM}})_z] \rangle / i$
5	20.69748	20.74676	26.04977	1.346	0.998836
10	20.78073	20.82140	25.87571	0.889	0.999310
15	20.73573	20.73232	25.73650	0.151	1.000026
20	20.73946	20.73666	25.74138	0.146	1.000041
exact	20.73553	20.73553	-	0	1

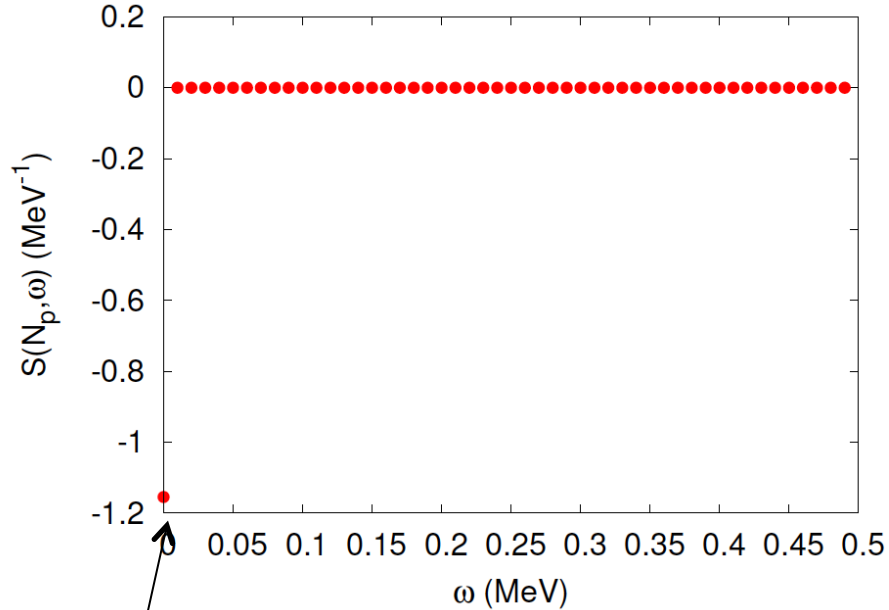
$$1/2m = A/2M_{\text{CM}}$$

Inglis-Belyaev cranking inertia does not reproduce the mass

Pairing rotational mode

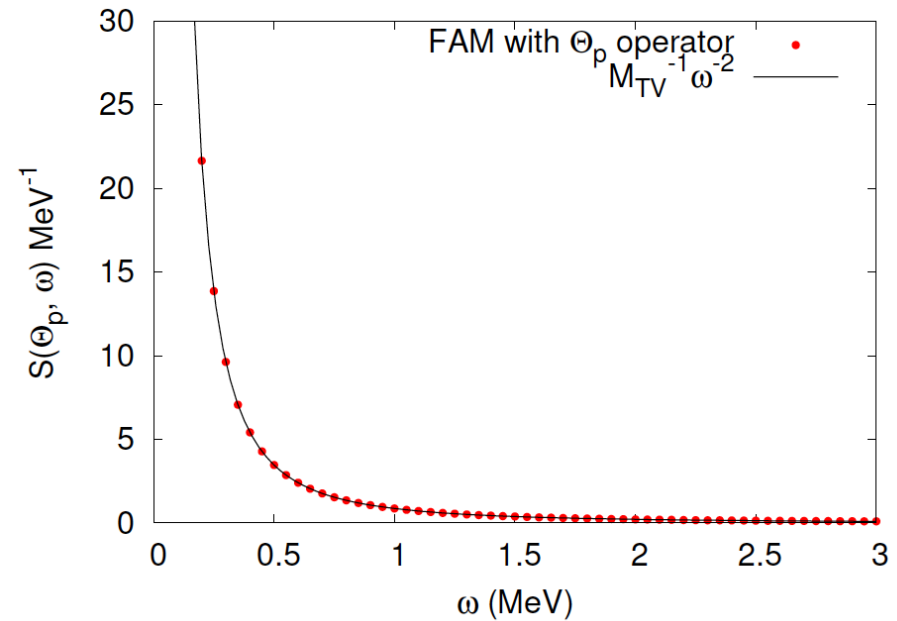
SLy4+volume pairing, ^{26}Mg (oblate, proton number broken) Nsh=5

FAM with proton number external field



Thouless-Valatin inertia

$$S(\hat{P}_{\text{NG}}, \omega) = \begin{cases} 0 & (\omega \neq \pm\Omega_i) \\ -M_{\text{NG}} & (\omega = 0) \end{cases}$$

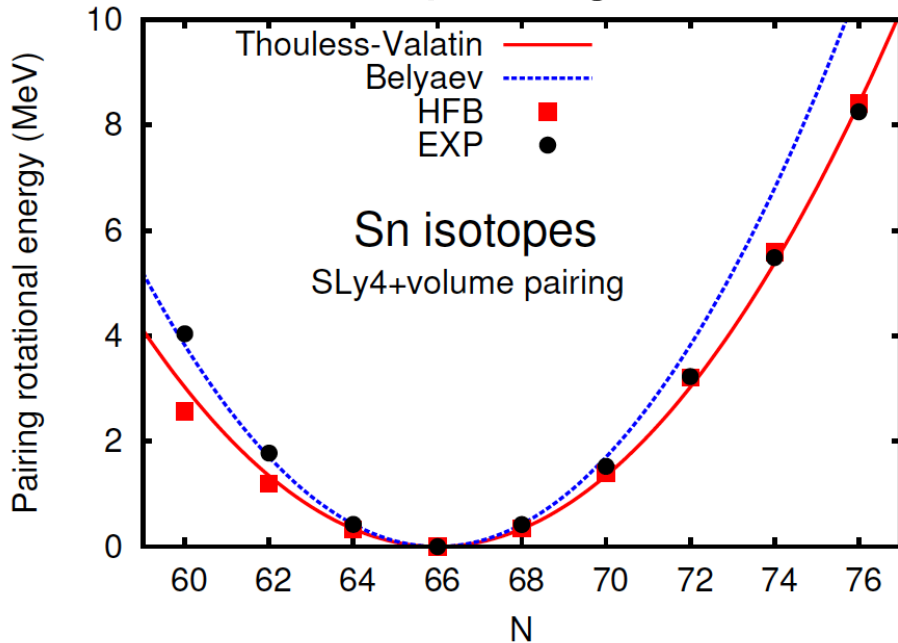


$$S(\hat{F}, \omega)_{\text{NG}} = \sum_{i, \Omega_i=0} \frac{1}{\omega^2} \left\{ \frac{1}{M_i} |\langle P_i | \hat{F} | 0 \rangle|^2 + \omega [QP]_i(\hat{F}) \right\}$$

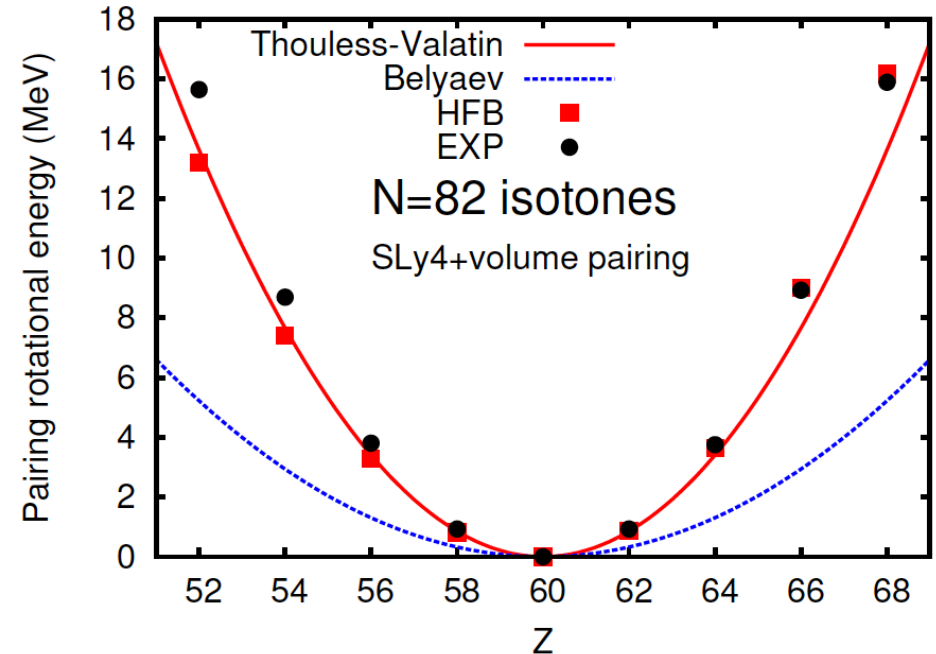
Pairing rotations : realistic cases

HFBTHO, SLy4 + volume pairing, pairing strength adjusted at 116Sn, Nsh=20

neutron pairing rotation



proton pairing rotation



$$B(N, Z_0) = B(N_0, Z_0) + \lambda_n(N_0, Z_0)\Delta N + \boxed{\frac{(\Delta N)^2}{2\mathcal{J}_n(N_0, Z_0)}}$$

$$\Delta N = N - N_0$$

pairing rotational energy

ground states form “pairing rotational bands”

proton pairing: effect of residual Coulomb significant

Mixing of neutron and proton pairing rotations

when neutron and proton are in a superconducting phase

broken symmetries: neutron number and proton number

NG modes (QRPA eigenmodes): two, but mixing of two

TV inertias from two NG modes \rightarrow three moments of inertia

QRPA eigenmodes

$$\hat{N}_1 = \hat{N}_n \cos \theta + \alpha \hat{N}_p \sin \theta,$$

$$\hat{\Theta}_1 = \hat{\Theta}_n \cos \theta + \frac{1}{\alpha} \hat{\Theta}_p \sin \theta$$

$$\hat{N}_2 = -\hat{N}_n \sin \theta + \alpha \hat{N}_p \cos \theta,$$

$$\hat{\Theta}_2 = -\hat{\Theta}_n \sin \theta + \frac{1}{\alpha} \hat{\Theta}_p \cos \theta$$

Thouless-Valatin mass of eigenmodes

$$M_1 = -S(\hat{N}_n, \hat{N}_n) \cos^2 \theta - \alpha^2 S(\hat{N}_p, \hat{N}_p) \sin^2 \theta \\ - 2\alpha S(\hat{N}_n, \hat{N}_p) \sin \theta \cos \theta,$$

$$S(\hat{N}_n, \hat{N}_n) = -2N_n(A+B)^{-1}N_n$$

$$S(\hat{N}_n, \hat{N}_p) = -2N_n(A+B)^{-1}N_p$$

$$M_2 = -S(\hat{N}_n, \hat{N}_n) \sin^2 \theta - \alpha^2 S(\hat{N}_p, \hat{N}_p) \cos^2 \theta \\ + 2\alpha S(\hat{N}_n, \hat{N}_p) \sin \theta \cos \theta,$$

$$S(\hat{N}_p, \hat{N}_p) = -2N_p(A+B)^{-1}N_p$$

constraint from orthogonality of two modes:

$$\tan 2\theta = \frac{2\alpha S(\hat{N}_n, \hat{N}_p)}{S(\hat{N}_n, \hat{N}_n) - \alpha^2 S(\hat{N}_p, \hat{N}_p)}$$

Mixing of neutron and proton pairing rotations

TV inertias from two NG modes \rightarrow three moments of inertia

$$E_{\text{rot}}(N, Z) = \frac{(\Delta N_1)^2}{2M_1} + \frac{(\Delta N_2)^2}{2M_2},$$

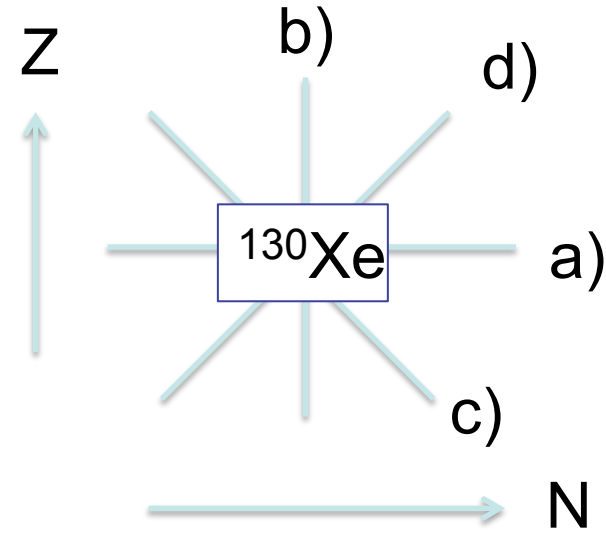
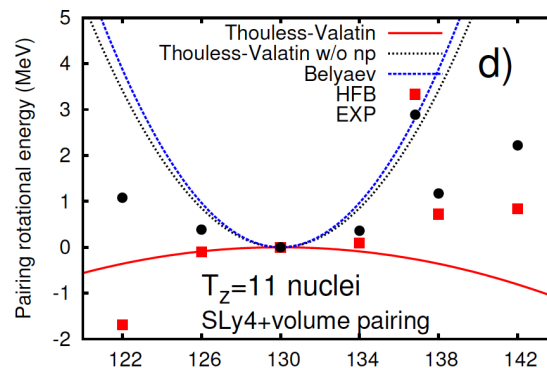
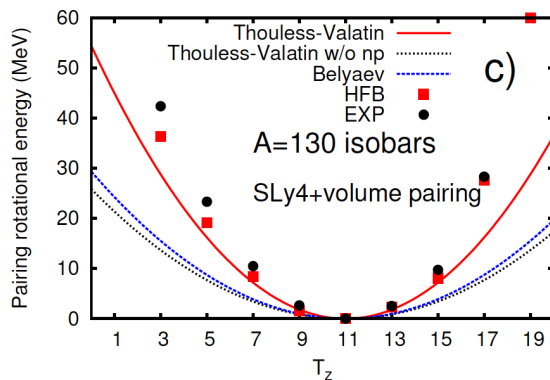
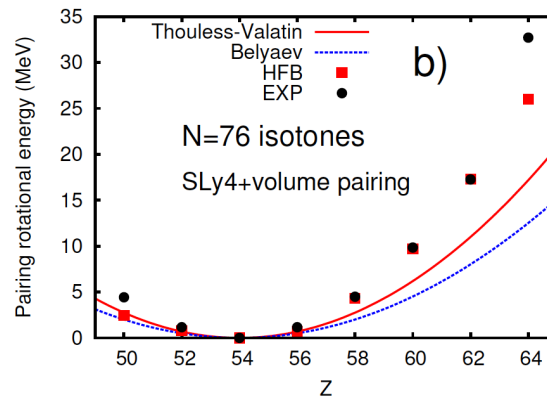
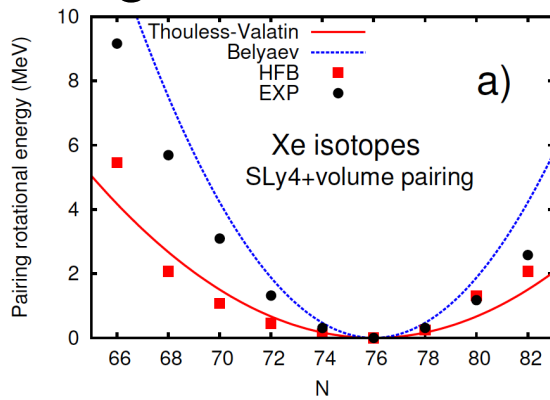
$$= \frac{1}{2} (\Delta N \quad \Delta Z) \mathbb{J}^{-1} \begin{pmatrix} \Delta N \\ \Delta Z \end{pmatrix}$$

$$= \frac{(\Delta N)^2}{2\mathcal{J}_{nn}} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{np}} + \frac{(\Delta Z)^2}{2\mathcal{J}_{pp}}$$

inertia tensor

$$\mathbb{J}^{-1} = \begin{pmatrix} S(\hat{N}_n, \hat{N}_n) & S(\hat{N}_n, \hat{N}_p) \\ S(\hat{N}_p, \hat{N}_n) & S(\hat{N}_p, \hat{N}_p) \end{pmatrix}^{-1} = \begin{pmatrix} 1/\mathcal{J}_{nn} & 1/\mathcal{J}_{np} \\ 1/\mathcal{J}_{pn} & 1/\mathcal{J}_{pp} \end{pmatrix}$$

pairing rotation around ^{130}Xe nucleus



d): global gauge symmetry breaking, not associated with isovector pairing

Summary

Recent development of finite-amplitude method for NG modes

- ❑ Thouless-Valatin inertia
- ❑ Coordinate operator → useful for decoupling the spurious modes

Outlook

- ❑ pairing collective Hamiltonian
- ❑ rotational moment of inertia, $E(2+)$ systematics
- ❑ isorotation

Collaborators

- ❑ Double-beta decay
 - ❑ Jon Engel (UNC-Chapel Hill, USA)
 - ❑ Javier Menendez (Tokyo, Japan)
- ❑ DFT
 - ❑ Javid Sheikh (Kashmir, India)
 - ❑ Koichi Sato (RIKEN, Japan)
 - ❑ Takashi Nakatsukasa (Tsukuba/RIKEN, Japan)
 - ❑ Jacek Dobaczewski (York/Warsaw/Jyvaskyla)
 - ❑ Witold Nazarewicz (NSCL/FRIB, MSU, USA)
- ❑ FAM
 - ❑ Markus Kortelainen (Jyvaskyla, Finland)
 - ❑ Erik Olsen (MSU, USA)
 - ❑ Witold Nazarewicz

Calculation

Killdevil, UNC-Chapel Hill



KRAKEN XT5 (NICS, UT)



COMA(PACS-IX), Tsukuba



iCER, MSU

