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Constraining Finite-Range Momentum Dependent Effective Interactions

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Skyrme Interaction

Contact interaction

$$v_{\mathrm{Skyrme}}(r) = t_0(1 + x_0 P^{\sigma})\delta(r)$$

$$+t_1(1 + x_1 P^{\sigma})(\delta(r)k^2 + k'^2\delta(r))/2$$

$$+ t_2(1 + x_2 P^{\sigma})\boldsymbol{k}'^{*}\delta(r) \cdot \boldsymbol{k}$$

Momentum Dependent

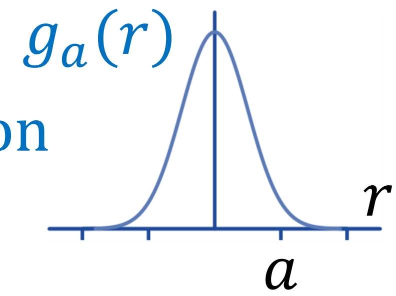
$$+ t_3/6(1 + x_3 P_{\sigma})\rho^{\alpha}\delta(r)$$

Density dependent $1/6 < \alpha < 2/3$

$$+iW_0\hat{\sigma} \cdot [\boldsymbol{k}'^{*}\delta(r) \times \boldsymbol{k}]$$

Spin-orbit

Regularized Interaction



Gaussian interaction

$$v_{\mathrm{Reg}} = t_0(1 + x_0 P^\sigma - y_0 P^\tau - z_0 P^{\sigma\tau}) g_a(r)$$

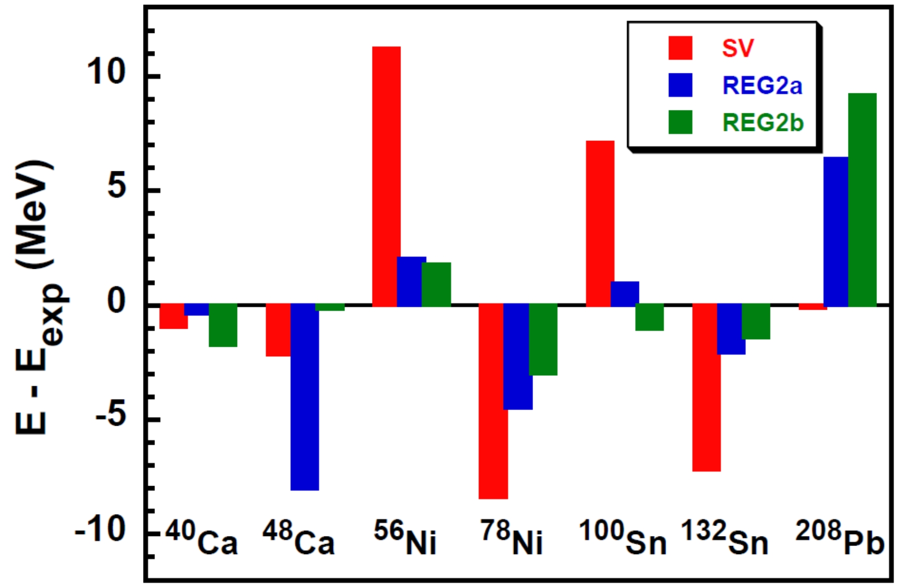
$$+t_1(1 + x_1 P^\sigma - y_1 P^\tau - z_1 P^{\sigma\tau})(g_a(r) k^2 + k'^2 g_a(r))/2 + \qquad \hat{T}_1$$

$$+t_2(1 + x_2 P^\sigma - \underbrace{y_2 P^\tau - z_2 P^{\sigma\tau}}) \boldsymbol{k}'^* g_a(r) \cdot \boldsymbol{k} \qquad \hat{T}_2$$

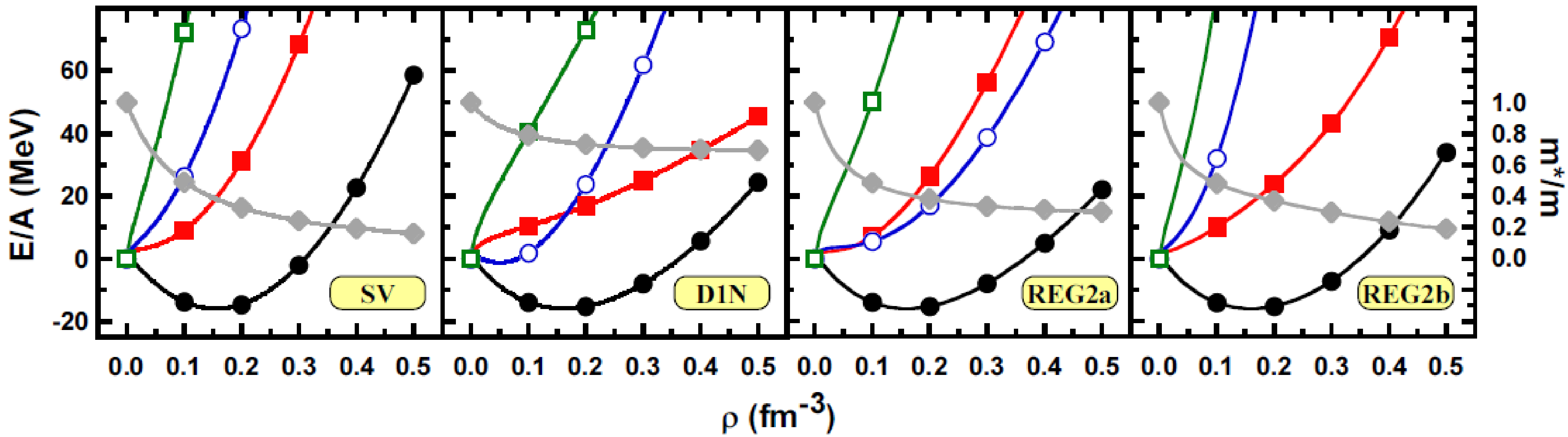
- $P^\tau \neq \pm 1$
- coefficients y_i, z_i not reabsorbed
- more parameters at a given order

	Ref.	REG2a
K_∞ (MeV)	230	230.00
L (MeV)	75	100.2
E/A (sat.) (MeV)	-16	-16
ρ_{sat} (fm)	0.16	0.160
J (MeV)	32	32

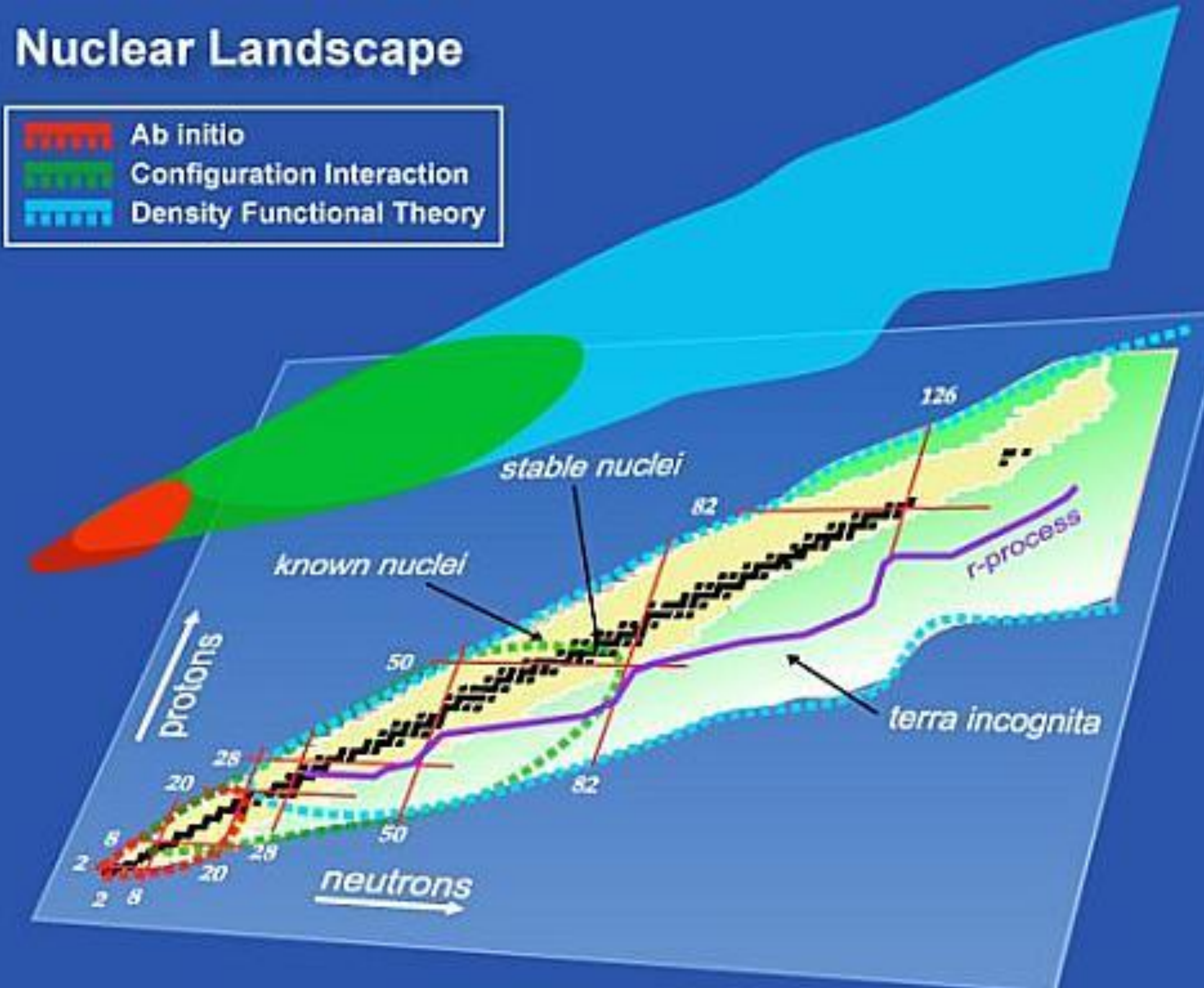
Close Shell masses



Infinite Matter Properties



Bennaceur et al., EPJ Web of Conferences 66 (2014) 02031



from UNEDF website

Pairing Matrix Elements to constrain properties of open shell finite nuclei

Gomez, Prieto, Navarro, NPA 549 (1992) 125

$$V_{nlj}^{ST} = \left\langle (nlj, nlj)_{J=0} \middle| v_{Reg}^{ST} \middle| (nlj, nlj)_{J=0} \right\rangle$$

one can gauge the matrix element on the pairing energy of a close shell + 2ν nucleus. In $V_{f_{7/2}^2}^{ST}$ case, ^{42}Ca

Pairing Matrix Elements to constrain properties of open shell finite nuclei

Gomez, Prieto, Navarro, NPA 549 (1992) 125

$$V_{nlj}^{ST} = \left\langle (nlj, nlj)_{J=0} \middle| v_{Reg}^{ST} \middle| (nlj, nlj)_{J=0} \right\rangle$$

$V_{f_{7/2}^2}^{ST}$

Wavefunction

$$\psi_{nljq}^{S=0}(\vec{r}_1\sigma_1, \vec{r}_2\sigma_2) = \sum_{m_{l_1}} \frac{\sqrt{2j+1}}{2(2l+1)} (-)^{l-m_{l_1}} \delta_{\sigma_1 - \sigma_2}$$

$$Y_{m_{l_1}}^{l}(\hat{r}_1) Y_{-m_{l_1}}^{l}(\hat{r}_2) R_{nljq}(r_1) R_{nljq}(r_2)$$

Multipole expansion

$$g_a(\vec{r}_1 - \vec{r}_2) = \frac{e^{-|\vec{r}_1 - \vec{r}_2|^2 / a^2}}{(a\sqrt{\pi})^3}$$

$$= \frac{4\pi}{(a\sqrt{\pi})^3} e^{-(r_1^2 + r_2^2)/a^2} \sum_{LM} i_L\left(2\frac{r_1 r_2}{a^2}\right) Y_M^{L\,*}(\hat{r}_1) Y_M^L(\hat{r}_2)$$

The M.E. is an integral over a
total of 6 Spherical Harmonics
with gradients!

Local NLO contribution

Difference between Momenentum dependent operators

$$\overbrace{\hat{T}_1 - \hat{T}_2} = (k'^2_{12} + k^2_{12})/2 - \vec{k}'_{12} \cdot \vec{k}_{12} = \left(\vec{k}'^*_{12} - \vec{k}_{12}\right)^2 /2$$

Local part of the interaction

$$\left[\hat{T}_1 - \hat{T}_2, \delta(r'_1 - r_1)\delta(r'_2 - r_2)\right] = 0$$

Commutes with locality deltas

$$\left(\hat{T}_1 - \hat{T}_2\right) \delta(r'_1 - r_1)\delta(r'_2 - r_2) g_a(r_1 - r_2) \rightarrow -\frac{\left(\vec{\nabla}_1 - \vec{\nabla}_2\right)^2}{2} g_a(r_1 - r_2)$$

thus is the laplacian of the gaussian

That is conveniently related to the

Derivative respect to the range

$$\frac{\left(\vec{\nabla}_1 - \vec{\nabla}_2\right)^2}{2} g_a(r_1 - r_2) = \frac{1}{a}\frac{\partial}{\partial a} g_a(r_1 - r_2)$$

Local N^{n}LO contribution to matrix elements for regularized interaction

$$\left(\frac{\left(\vec{\nabla}_1 - \vec{\nabla}_2\right)^2}{2}\right)^n g_a(r_1 - r_2) = \left(\frac{1}{a}\frac{\partial}{\partial a}\right)^n g_a(r_1 - r_2)$$

The matrix element for a local regularized interaction ($t_2 = -t_1$),

$$V_{N^n LO}[t^{(n)}] = \left(-\frac{1}{a}\frac{\partial}{\partial a}\right)^n V_{LO}[t^{(n)}]$$

	Ref.	REG2a	New
K_∞ (MeV)	230	230.00	230.51
L (MeV)	75	100.20	84.87
E/A (sat.) (MeV)	-16	-16.00	-16.17
ρ_{sat} (fm)	0.16	0.160	0.160
J (MeV)	32	32.00	33.31
$\langle\psi_{f^2_{7/2}}\|V^{S=0}_{Loc}\|\psi_{f^2_{7/2}}\rangle$ (MeV)	-3	***-0.54***	**-1.76**
$\langle\psi_{f^2_{7/2}}\|V^{S=1}_{Loc}\|\psi_{f^2_{7/2}}\rangle$ (MeV)	$\gtrsim 0$	0.12	**0.13**

Pairing Energy ^{44}Ca
-9.153 MeV.

One Body
Density Funct.

$$E^{ST}_{p-p}[\rho] = \frac{1}{2} \sum_{\substack{nlj \\ n'l'j'}} (-)^{l'+l} \sqrt{(2j+1)(2j'+1)} (u \cdot v)_{nlj} (u \cdot v)_{n'l'j'}$$

$$\left\langle (n'l'j', n'l'j')_{00} \middle| v^{ST}_{Reg} \middle| (nlj, nlj)_{00} \right\rangle$$

Two Body
Matrix Element

J. Sadoudi, M. Bender and T. Duguet, unp. (2012)

In the case of pairing for close shell + 2ν

$$n'l'j' = nlj$$

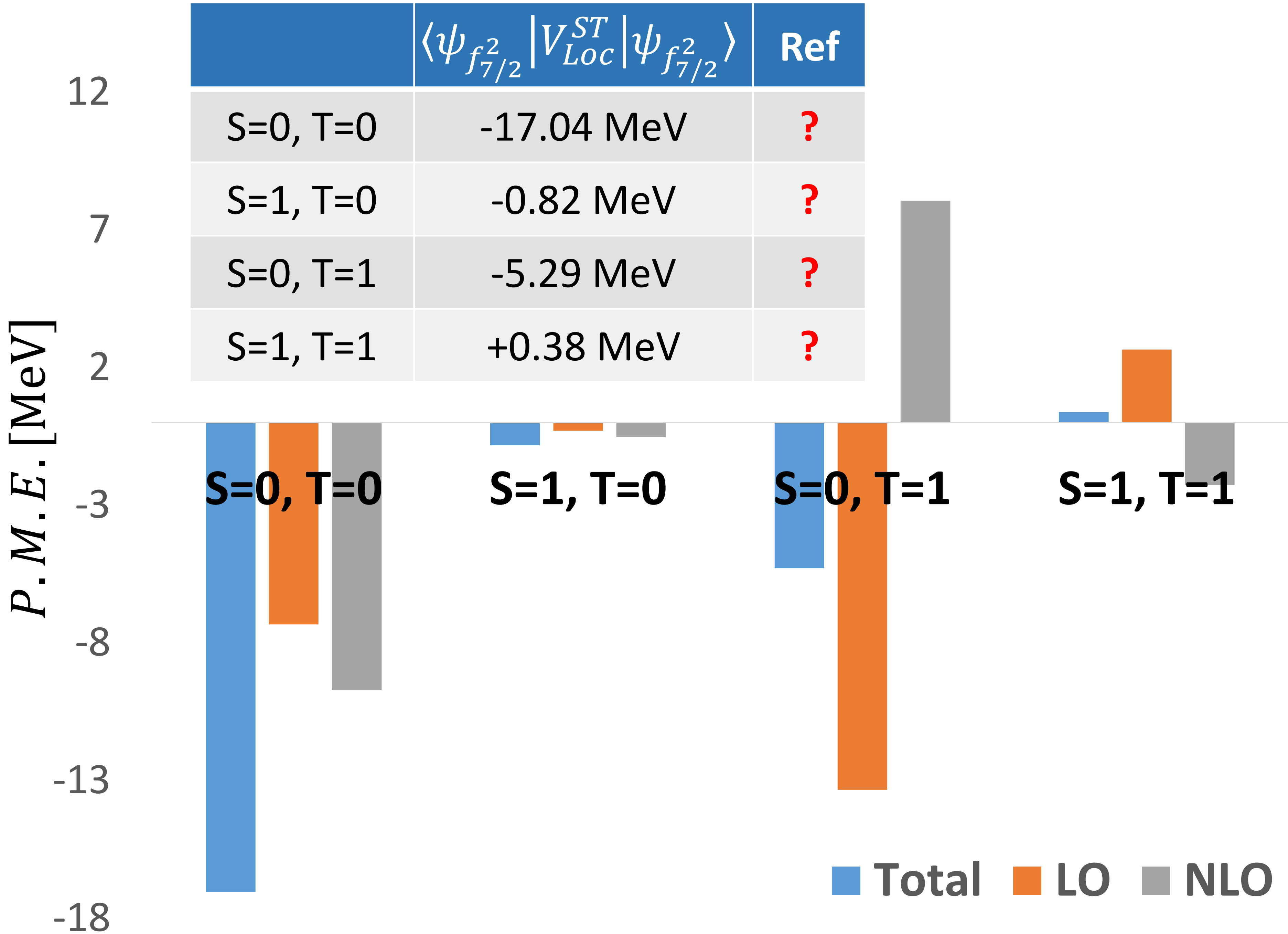
One Body
Density Funct.

$$E_{p-p}^{ST}[\rho] = \frac{1}{2}(-)^{l'+l}\sqrt{(2j+1)(2j'+1)}\,(u\cdot v)_{nlj}^{2}$$

$$\langle (nlj,nlj)_{00} | v_{Reg}^{ST} | (nlj,nlj)_{00} \rangle$$

Two Body
Matrix Element

J. Sadoudi, M. Bender and T. Duguet, unp. (2012)



	$\langle \psi_{f_{7/2}^2} \| V_{Loc}^{ST} \| \psi_{f_{7/2}^2} \rangle$	**Ref**
S=0, T=0	-17.04 MeV	?
S=1, T=0	-0.82 MeV	?
S=0, T=1	-5.29 MeV	?
S=1, T=1	+0.38 MeV	?

Landau Parameters are the coefficient of the p-h interaction expanded in the legendre polynomial basis for the different spin-isospin channels, calculated at the Fermi surface.

channel

$$\sum_l f_l^{(\alpha)} P_l\left(\vec{k}\cdot\vec{k}'\right) = v_{p-h}(\vec{k},\vec{k}')$$

Fourier Transform

Particle-hole interaction

$= v(1 - P^x P^\sigma P^\tau)$

$$\sum_l f_l^{(\alpha)} P_l\left(\vec{k}\cdot\vec{k}'\right) = v_{p-h}(\vec{k},\vec{k}')$$

$$v_{\text{Reg}}^{LO}(r_{12},r'_{12}) = t_0(1 + x_0 P^\sigma - y_0 P^\tau - z_0 P^{\sigma\tau}) g_a(r_1 - r_2)$$
$$(\delta(r_1 - r'_1)\delta(r_2 - r'_2) - \delta(r_1 - r'_2)\delta(r_2 - r'_1) P^\sigma P^\tau)$$

Direct term

Exchange term

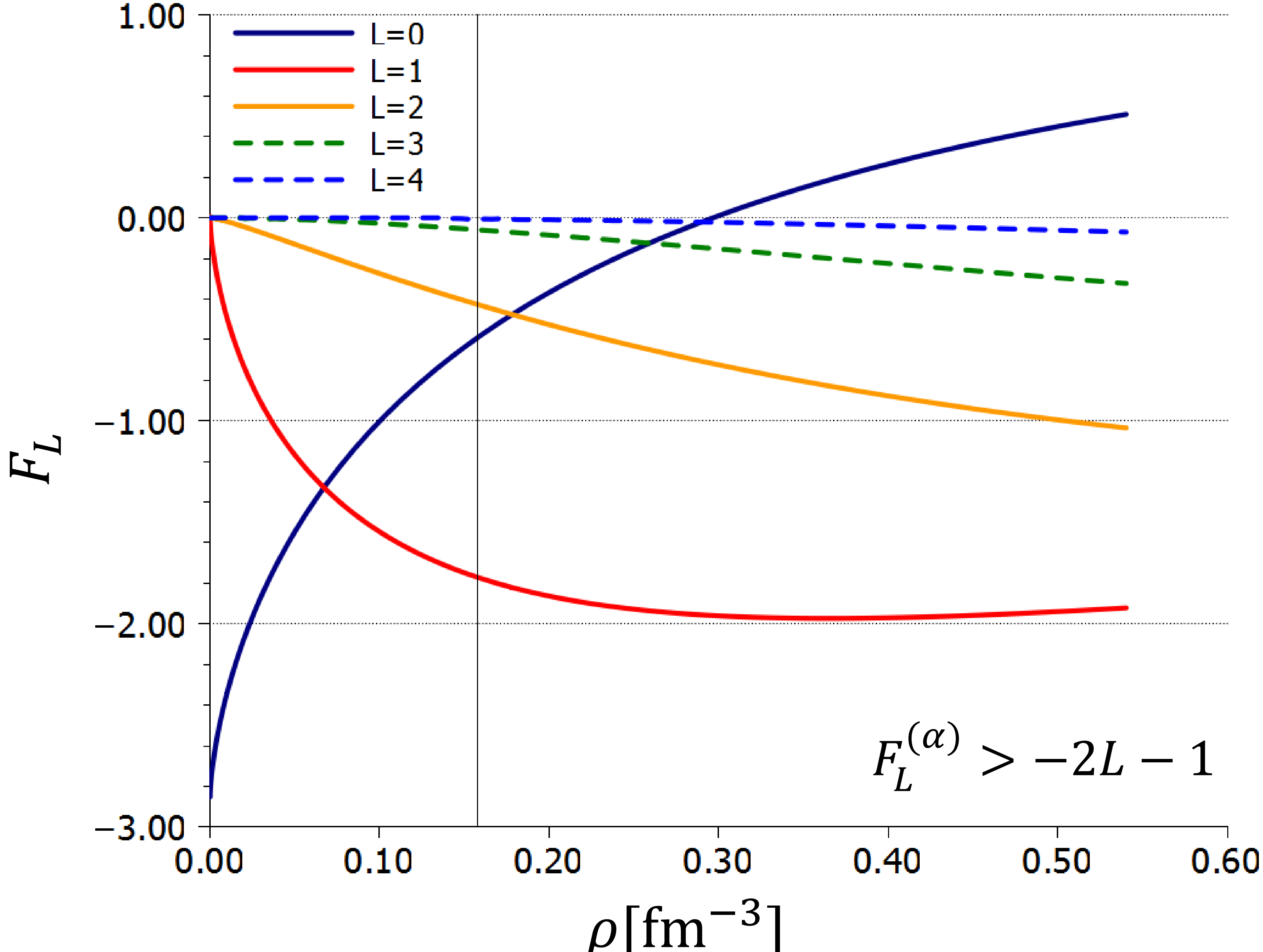
$$\mathcal{F}\left[v_{\text{Reg}}^{LO}\right] = D^{(\alpha)}(q) - E^{(\alpha)}(\vec{k} - \vec{k}')$$

$$f_l = D^{(0,0)}\delta_{l,0} - E^{(0,0)} \mathrm{f}_l(a,k_F)$$

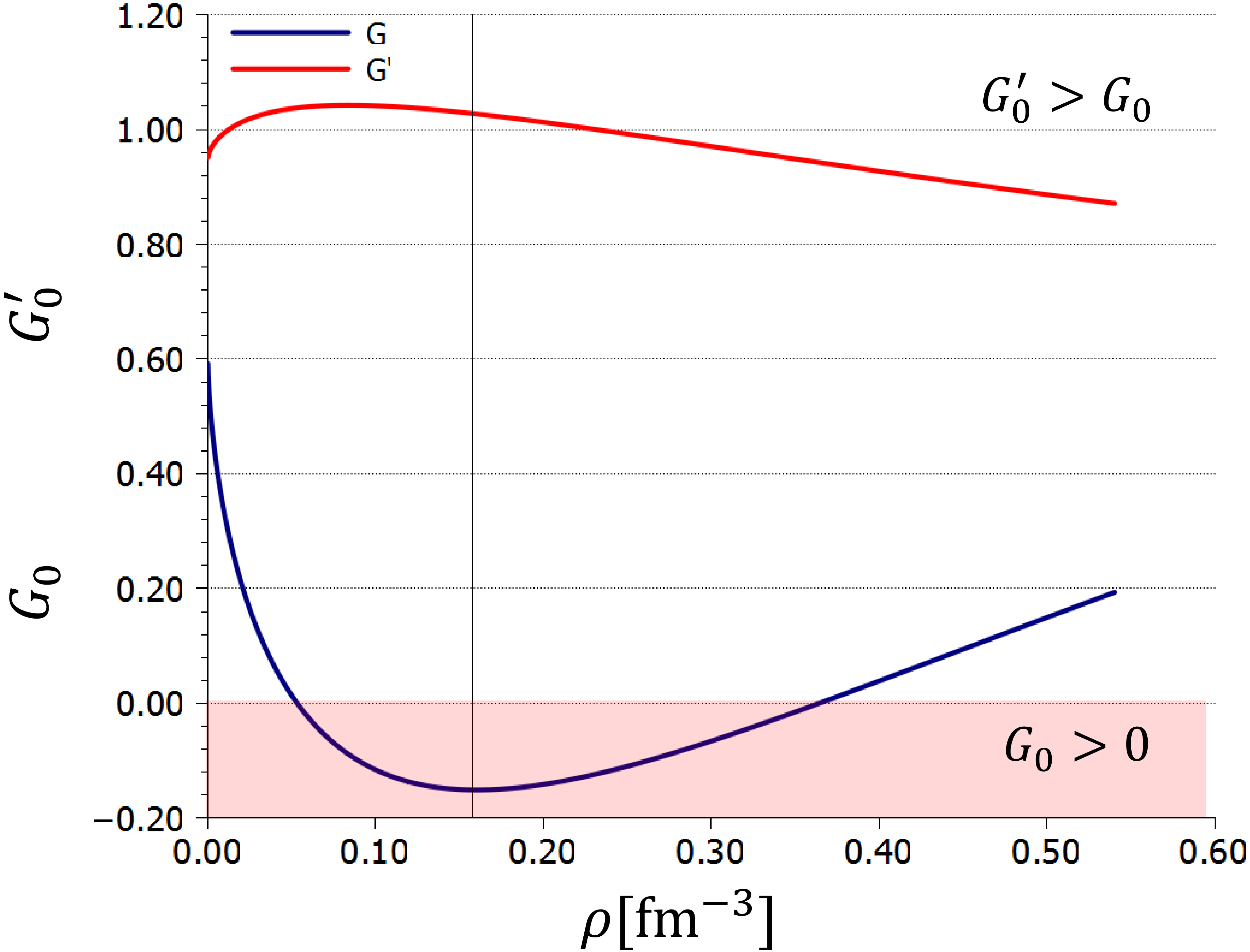
$$g_l * \vec{\sigma}\cdot\vec{\sigma} = \left(D^{(1,0)}\delta_{l,0} - E^{(1,0)} \mathrm{f}_l(a,k_F)\right)$$

$$f'_l * \vec{\tau}\circ\vec{\tau} = \left(D^{(0,1)}\delta_{l,0} - E^{(0,1)} \mathrm{f}_l(a,k_F)\right)$$

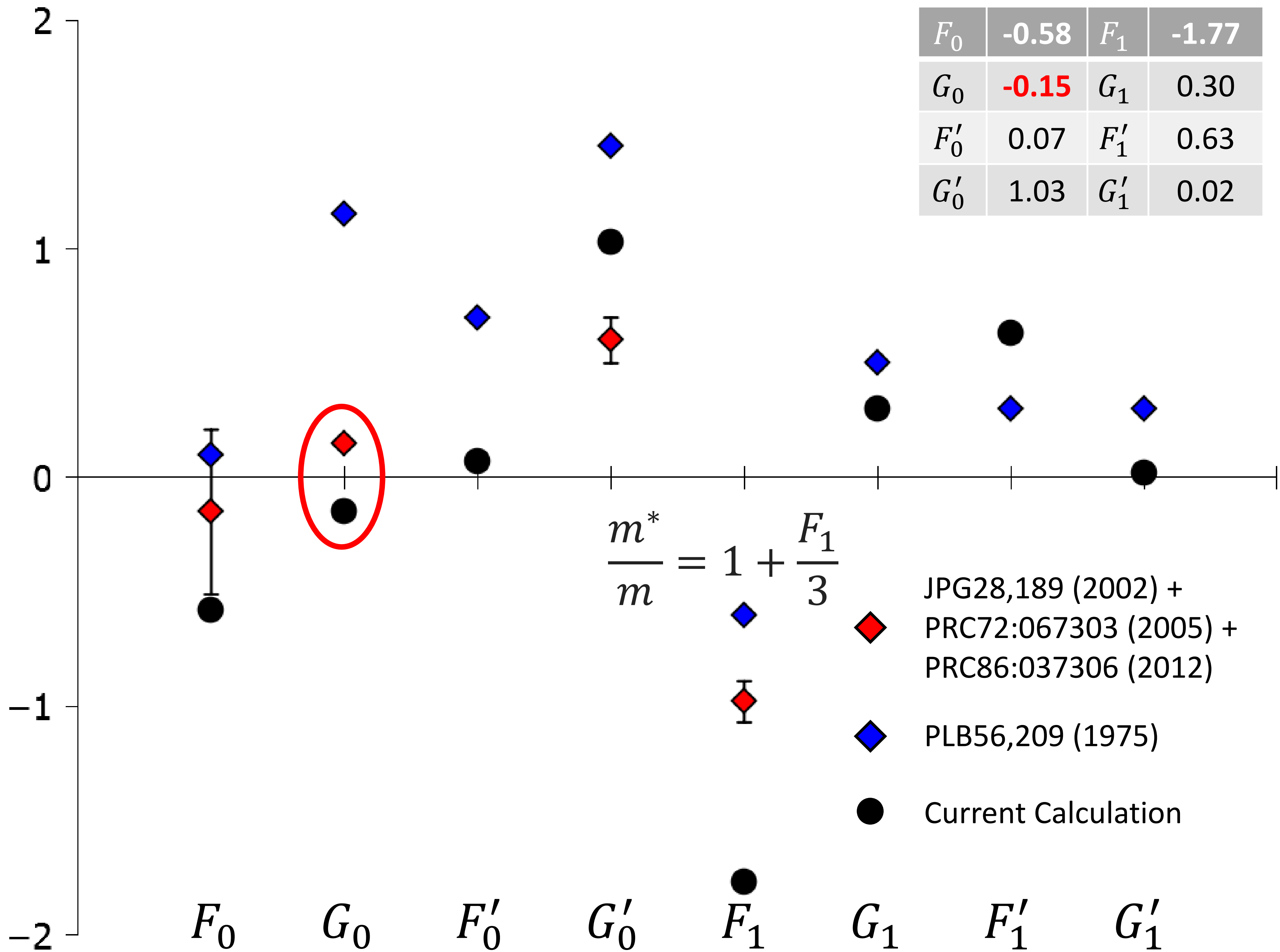
$$g'_l * \vec{\sigma}\cdot\vec{\sigma}\,\vec{\tau}\circ\vec{\tau} = \left(D^{(1,1)}\delta_{l,0} - E^{(1,1)} \mathrm{f}_l(a,k_F)\right)$$



L=0
L=1
L=2
L=3
L=4
1.00
0.00
−1.00
−2.00
−3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
F_L
$\rho[\mathrm{fm}^{-3}]$
$F_L^{(\alpha)} > -2L - 1$



G
G'
$G_0' > G_0$
$G_0 > 0$
G_0'
G_0
1.20
1.00
0.80
0.60
0.40
0.20
0.00
−0.20
0.00
0.10
0.20
0.30
0.40
0.50
0.60
$\rho[\mathrm{fm}^{-3}]$


2
1
0
−1
−2
F_0 -0.58 F_1 -1.77
G_0 -0.15 G_1 0.30
F'_0 0.07 F'_1 0.63
G'_0 1.03 G'_1 0.02
$\frac{m^*}{m} = 1 + \frac{F_1}{3}$
JPG28,189 (2002) +
PRC72:067303 (2005) +
PRC86:037306 (2012)
PLB56,209 (1975)
Current Calculation
F_0
G_0
F'_0
G'_0
F_1
G_1
F'_1
G'_1

dziękuję

Multipole Expansion of the Gaussian

$$v_{Reg} \propto g_a\left(\vec{k}_1 - \vec{k}_2\right) = \frac{e^{-\left|\vec{k}_1-\vec{k}_2\right|^2/a^2}}{\left(a\sqrt{\pi}\right)^3}$$

$$= \frac{4\pi}{\left(a\sqrt{\pi}\right)^3} e^{-(k_1^2+k_2^2)/a^2} \sum_{LM} i_L\left(2\frac{k_1 k_2}{a^2}\right) Y_M^{L^*}(\hat{k}_1) Y_M^L(\hat{k}_2)$$

$$\downarrow$$

$$(2L+1)P_L(\hat{k}_1 \cdot \hat{k}_2)/4\pi$$

NLO contribution to the Matrix Element

Gradient on wavefunction

$$\vec{\nabla}\left[\mathcal{R}(r)Y^{l}_{m_l}(\hat{r})\right]=\sqrt{\frac{l+1}{2l+1}}\left(\frac{\partial\mathcal{R}(r)}{\partial r}-\frac{l}{r}\mathcal{R}(r)\right)\vec{Y}^{l+1}_{l\,m_l}(\hat{r})+$$
$$+\sqrt{\frac{l}{2l+1}}\left(\frac{\partial\mathcal{R}(r)}{\partial r}-\frac{l+1}{r}\mathcal{R}(r)\right)\vec{Y}^{l-1}_{l\,m_l}(\hat{r})$$

Vector Spherical Harmonics

- Term $\propto \hat{T}_1 = (k'^2 + k^2)/2$

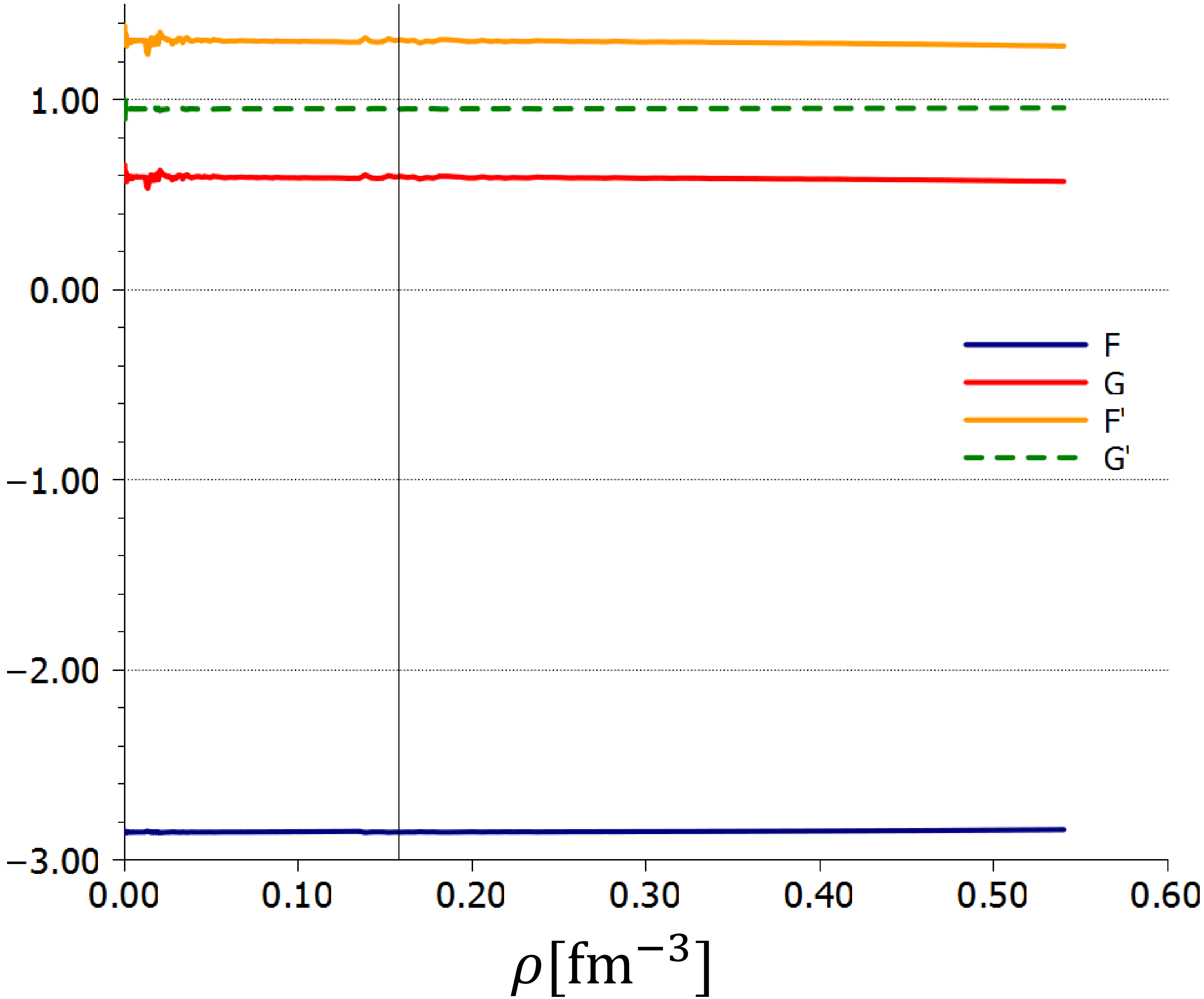
$$\propto g_a(\vec{r}_1-\vec{r}_2)\vec{\nabla}'_1\cdot\vec{\nabla}_1 \propto \int \vec{Y}^{l\pm1\,*}_{l,m_l}(\hat{r}_1)\cdot\vec{Y}^{l\pm1}_{l,m_{l'}}(\hat{r}_1)Y^{L\,*}_{M}(\hat{r}_1)dr_1 \int Y^{l}_{-m_l}{}^{*}(\hat{r}_2)Y^{l}_{-m_{l'}}(\hat{r}_2)Y^{L}_{M}(\hat{r}_2)dr_2$$

Solved

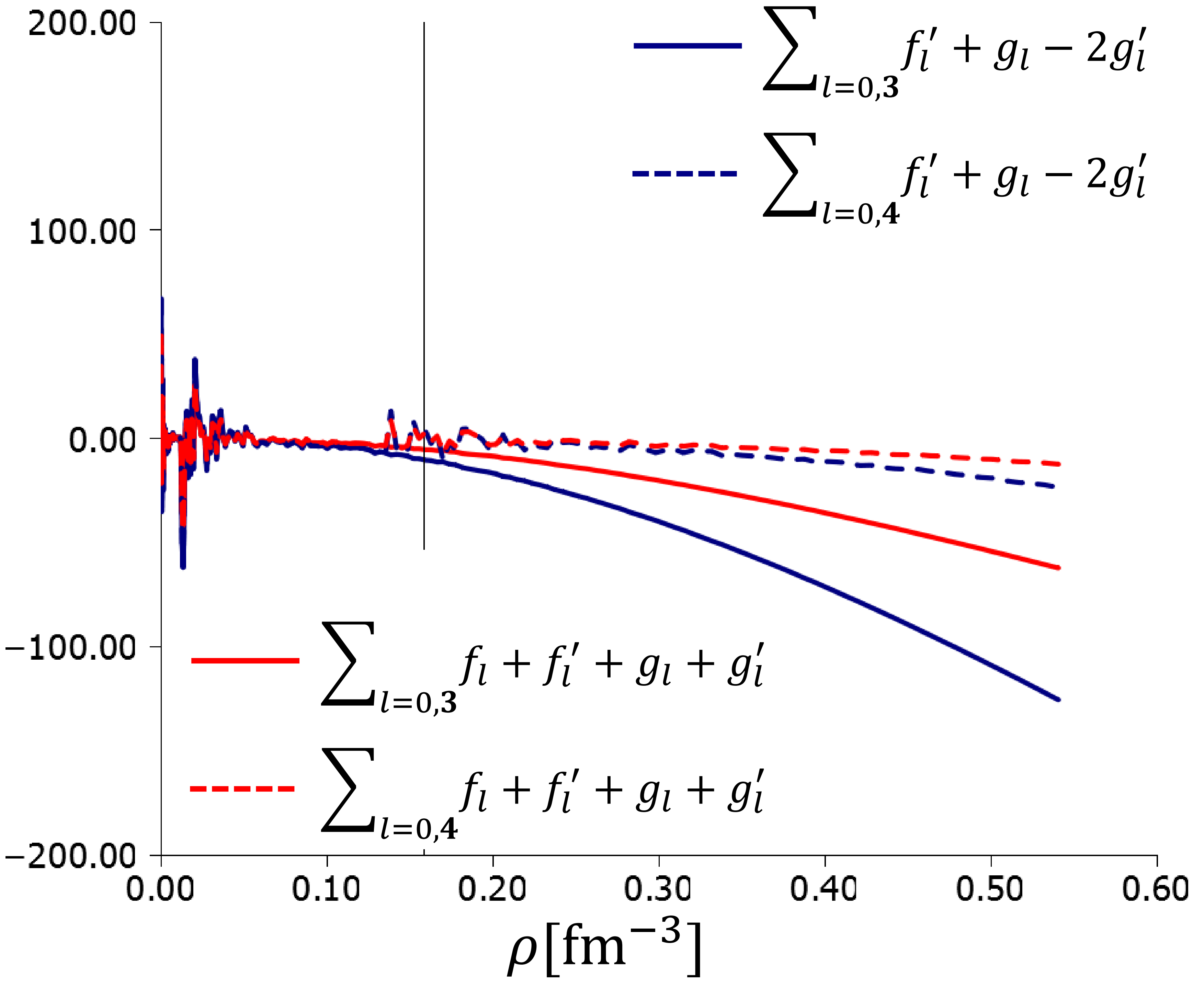
- Term $\propto \hat{T}_2 \propto \vec{k}\cdot\vec{k}'$

$$\propto g_a(\vec{r}_1-\vec{r}_2)\vec{\nabla}'_1\cdot\vec{\nabla}_2 \propto \int \vec{Y}^{l\pm1\,*}_{l,m_{l_1}}(\hat{r}_1)Y^{l}_{-m'_{l_1}}(\hat{r}_1)Y^{L\,*}_{M}(\hat{r}_1)dr_1 \cdot \int \vec{Y}^{l\pm1\,*}_{l,m_{l_2}}(\hat{r}_1)Y^{l}_{-m'_{l_2}}(\hat{r}_2)Y^{L}_{M}(\hat{r}_2)dr_2$$

???



1.00
0.00
−1.00
−2.00
−3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
$\rho\,[\mathrm{fm}^{-3}]$
F
G
F'
G'



200.00
100.00
0.00
−100.00
−200.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
$\rho[\mathrm{fm}^{-3}]$
$\sum_{l=0,3} f_l' + g_l - 2g_l'$
$\sum_{l=0,4} f_l' + g_l - 2g_l'$
$\sum_{l=0,3} f_l + f_l' + g_l + g_l'$
$\sum_{l=0,4} f_l + f_l' + g_l + g_l'$