Collaboration Workshop "The future of MR-DFT" Thursday, 25 June, 2015

MR-TDDFT for low-energy heavy ion reactions: Ideas

<u>Kazuyuki Sekizawa</u>

(Warsaw Univ. of Technology, Poland)

2015. 4: I started working at WUT in collaboration with P. Magierski2015. 3: I have finished my PhD at Univ. of Tsukuba (Supervisor: K. Yabana)

What I'd like to talk about

K. Sekizawa

I'll discuss a use of a "multi-Slater-determinant" to describe:

- multi-nucleon transfer (MNT) reaction
- sub-barrier fusion
- superheavy element (SHE) synthesis

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" Let's imagine the future of MR-DFT!! "

Any comments/questions/criticisms are welcome!!

K. Sekizawa

- 1. Introduction: Drawbacks in "SR-TDEDF"
- 2. Method: "MR-TDEDF" with a multi-Slater-determinant
- 3. "Ideas" for MNT / subbarrier fusion / SHE synthesis
- 4. Summary

1. Introduction: Drawbacks in "SR-TDEDF"

- 2. Method: "MR-TDEDF" with a multi-Slater-determinant
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About the "SR-TDEDF" (or TDHF)

- Action to be minimized:

$$S = \int_{t_1}^{t_2} dt \, \left\langle \Phi(t) \left| i\hbar \partial_t - \hat{H} \right| \Phi(t) \right\rangle \qquad \left(S = \int_{t_1}^{t_2} dt \, \left[i\hbar \sum_{i=1}^N \left\langle \phi_i(t) \left| \partial_t \right| \phi_i(t) \right\rangle - E[\rho(t)] \right] \right)$$

Trial w.f.: a *single* Slater determinant $\Phi(x_1, \cdots, x_N, t) = \frac{1}{\sqrt{N!}} \det \left\{ \phi_i(x_j, t) \right\}$ Single-particle wavefunctions

$$\phi_i(x,t) \qquad (i=1,\cdots,N; x \equiv (\mathbf{r},\sigma))$$

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Variation $\frac{\delta S}{\delta \phi_i^*(t)} = 0$ leads to: $i\hbar \partial_t \phi_i(x,t) = \hat{h}[\rho(t)]\phi_i(x,t)$: TDEDF (or TDHF, TDKS) equations One-body density Single-particle Hamiltonian $\rho(\mathbf{r},t) = \sum_{i,\sigma} |\phi_i(x,t)|^2$ $\hat{h}[\rho(t)] = \frac{\delta E}{\delta \rho(t)}$

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Variation
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 leads to:
 $i\hbar \partial_t \phi_i(x,t) = \hat{h}[\rho(t)]\phi_i(x,t)$: TDEDF (or TDHF, TDKS) equations
One-body density
 $\rho(\mathbf{r},t) = \sum_{i,\sigma} |\phi_i(x,t)|^2$ Single-particle Hamiltonian
 $\hat{h}[\rho(t)] = \frac{\delta E}{\delta \rho(t)}$

✓ No empilical parameters:

: Input is an EDF only

✓ Sucessfully applied:

Giant resonances (linear response, RPA)

Heavy ion reactions (fusion, transfer, quasi-fission, fission, ...)

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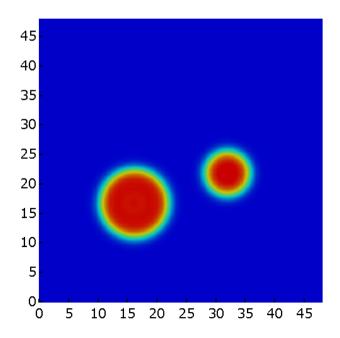
- MNT:Subbarrier fusion:SHE synthesis:

(••) : Many-body dynamics is described within a *single* mean-field potential

- MNT: Insufficient description of transfer channels far from average
- Subbarrier fusion:
- SHE synthesis:

Ex. 1: Multi-nucleon transfer (MNT) reaction

 ${}^{40}\text{Ca}+{}^{124}\text{Sn}$ at $E_{\text{lab}}=170$ MeV, b=3.96 fm

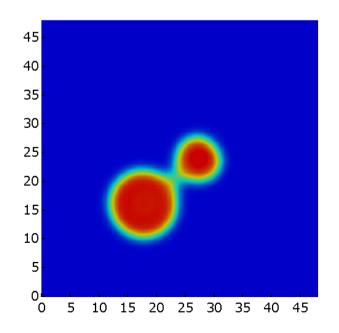


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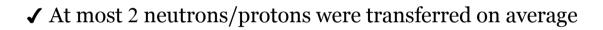


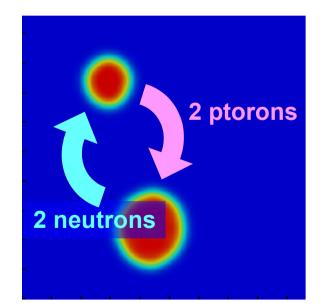
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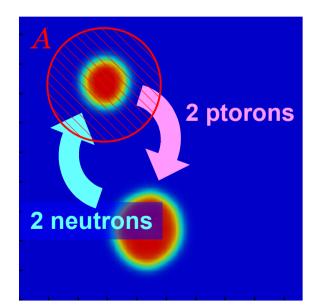
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PNP: C. Simenel, PRL105(2010)192701

 ${}^{40}\text{Ca}+{}^{124}\text{Sn} \text{ at } E_{\text{lab}}=170 \text{ MeV}, b=3.96 \text{ fm}$



- ✓ At most 2 neutrons/protons were transferred on average
- ✓ Particle-number projection technique can be applied: - Probability: *n* nucleons are inside the volume *A* $P_n = \langle \Phi | \hat{P}_n | \Phi \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{in\theta} \det \{ \langle \phi_i | \phi_j \rangle_B + e^{-i\theta} \langle \phi_i | \phi_j \rangle_A \}$ $\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_A)\theta} \text{ : Particle-number projection op.}$ - Cross section for a nucleus containing *N*, *Z* nucleons

$$\sigma_{N,Z} = 2\pi \int db \ b \ P_N(b) P_Z(b)$$

(2) : Many-body dynamics is described within a *single* mean-field potential

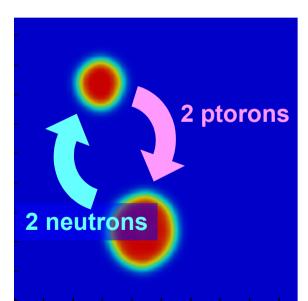
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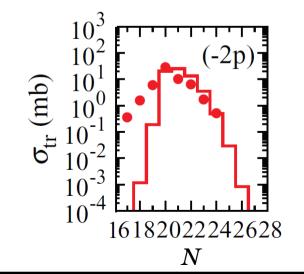
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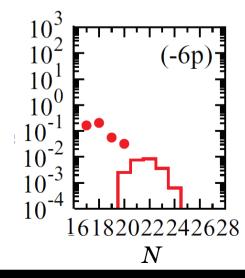
K.S., K.Yabana, PRC88(2013)064614

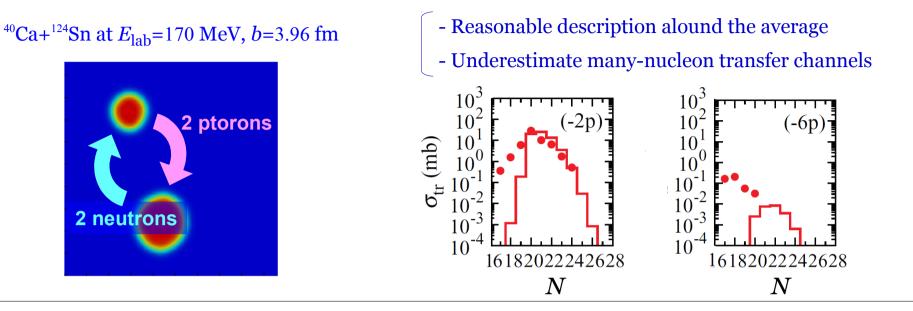
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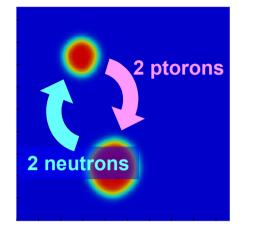
- ✓ At most 2 neutrons/protons were transferred on average
- ✓ Result:
- Reasonable description alound the average
- Underestimate many-nucleon transfer channels



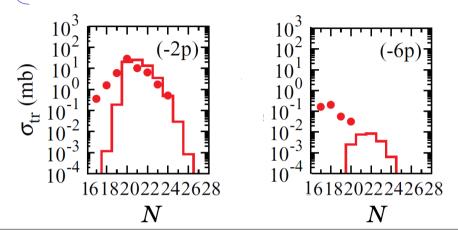


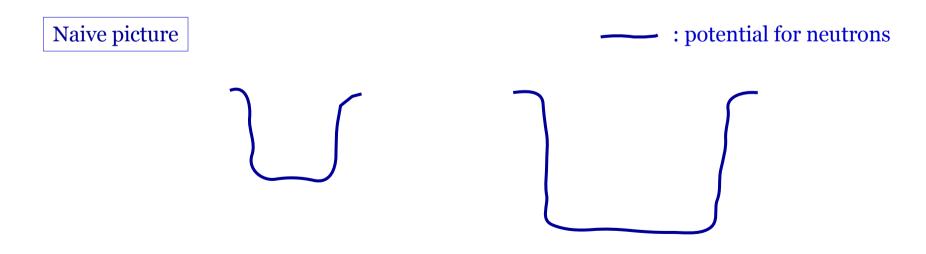


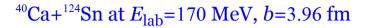


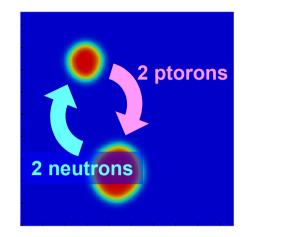


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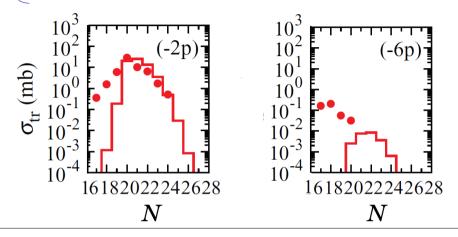


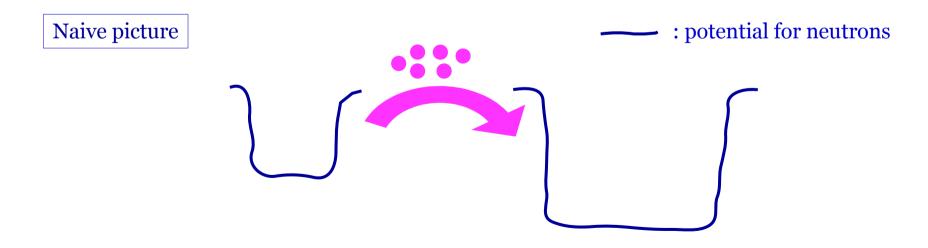


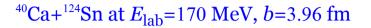


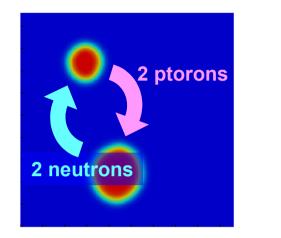


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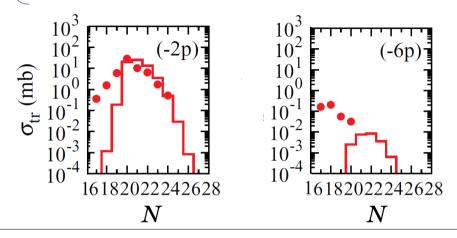


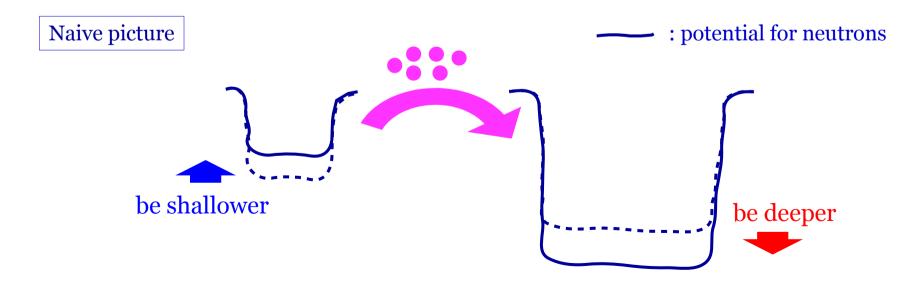




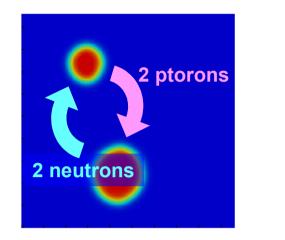


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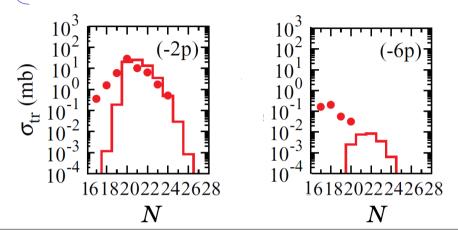


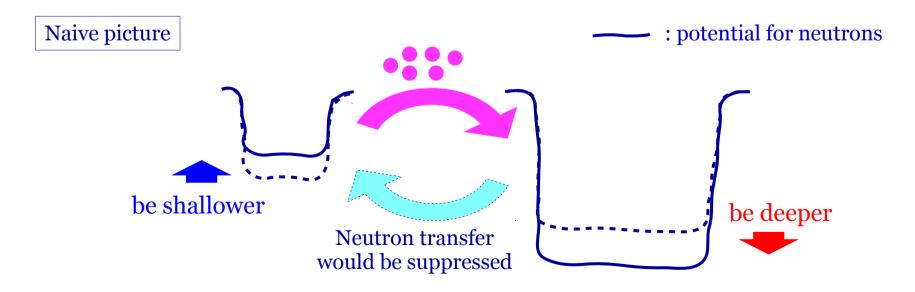


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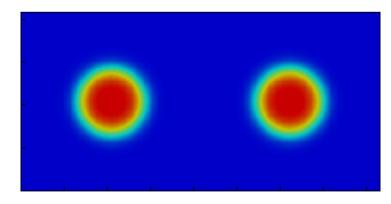
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- Subbarrier fusion: Absence of quantum tunneling of the many-body wavefunction
- SHE synthesis:

Ex. 2: Subbarrier fusion

 40 Ca+ 40 Ca at $E_{c.m.}$ =64 MeV < V_{B}



 $\checkmark P_{\text{fusion}} = 0 \text{ when } E < V_{\text{R}}$



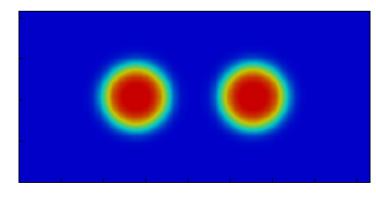
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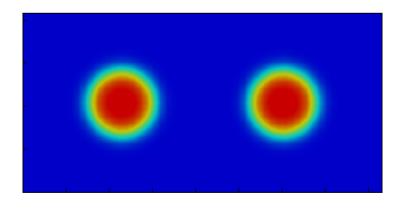


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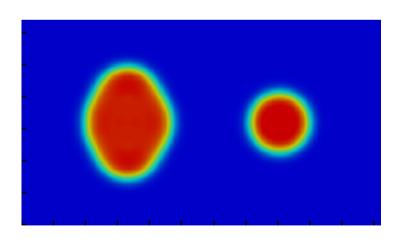
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✓ $P_{\text{fusion}} = 0$ when $E < V_{\text{B}}$

Ex. 3: SHE synthesis

 ${}^{64}\text{Ni}+{}^{238}\text{U} \rightarrow {}^{302}\text{120}$? (E_{lab} =428 MeV, b=0 fm)



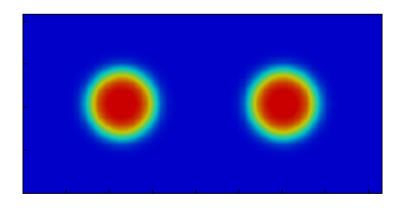
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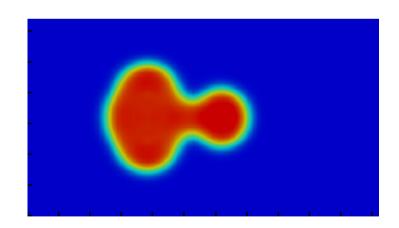
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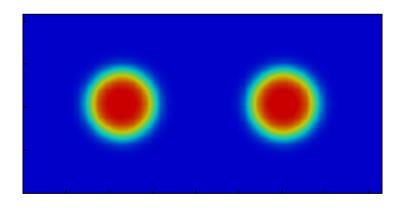
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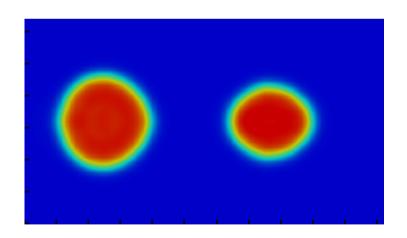
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✓ In this case P_{fusion}=0, but we would also like to have a tiny, finite fusion probability

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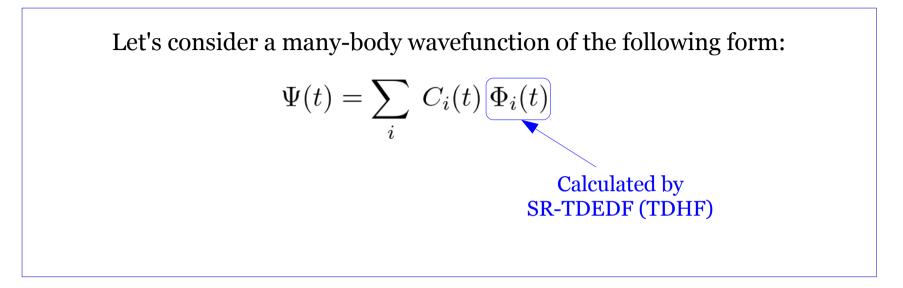
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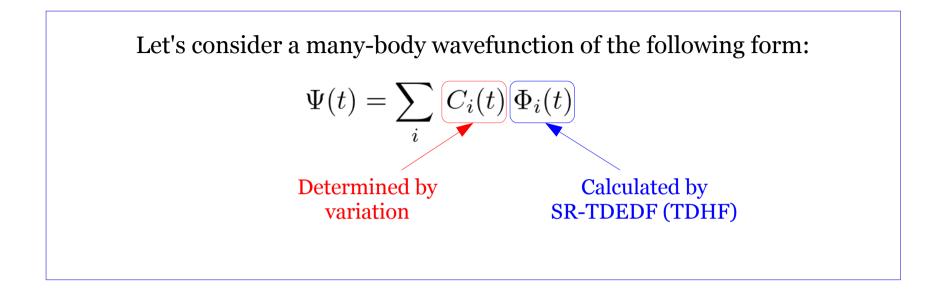
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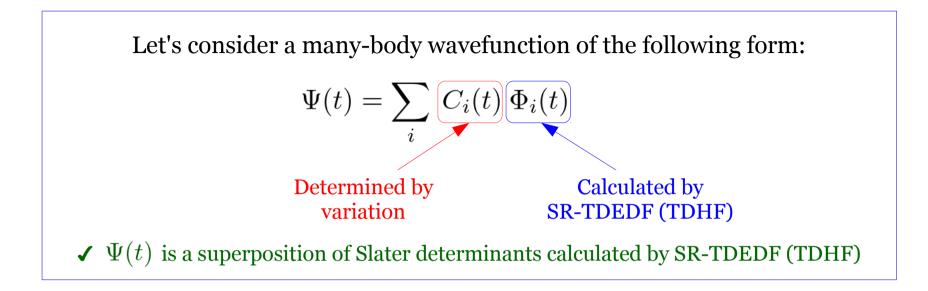
Let's consider a many-body wavefunction of the following form:

$$\Psi(t) = \sum_{i} C_i(t) \Phi_i(t)$$

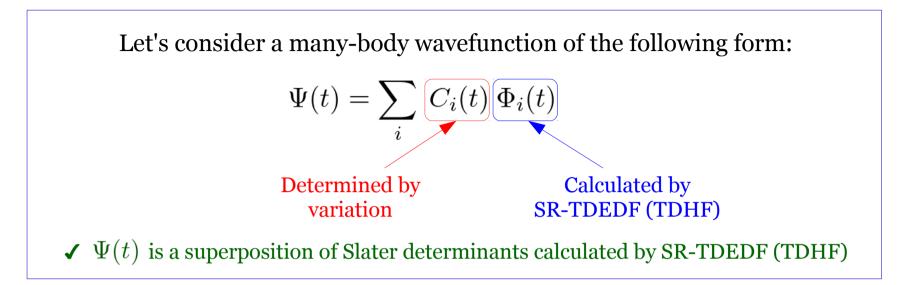




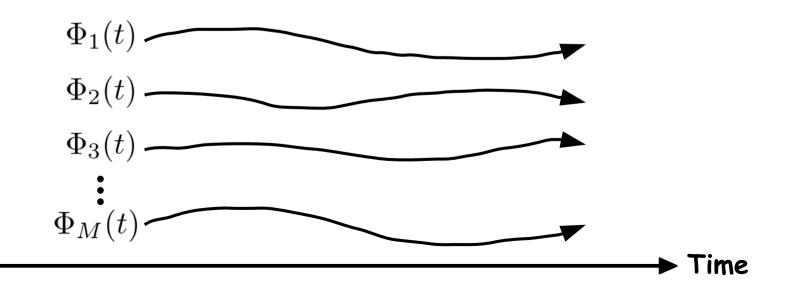
"MR-TDEDF": Structure of the many-body wavefunction



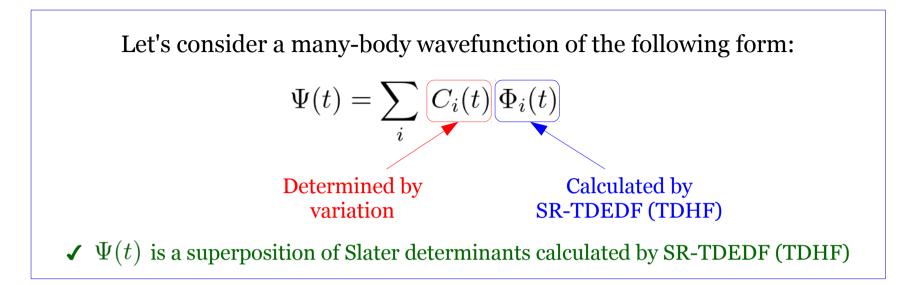
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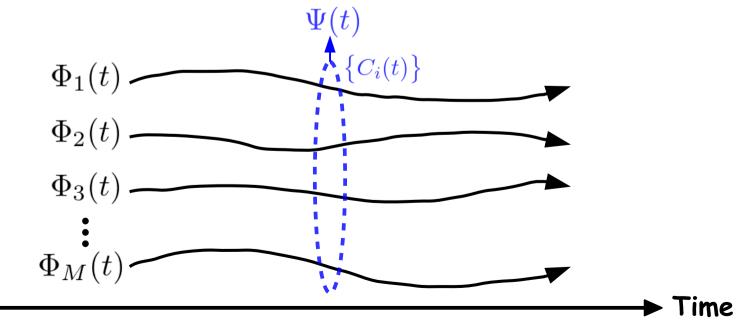
* $\Phi_i(t)$ is already fixed, and only the coefficients $C_i(t)$ are the variables of the variation



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"MR-TDEDF": Equation for the coefficients $\overline{C_i(t)}$

Action to be minimized:

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 $\Psi(t) = \sum_{i} C_{i}(t) \Phi_{i}(t)$: Multi-Slater-determinant

┌ Action to be minimized: -

$$S = \int_{t_1}^{t_2} dt \, \langle \Psi(t) | i\hbar \partial_t - \hat{H} | \Psi(t) \rangle \qquad \Psi(t) = \sum_i C_i(t) \, \Phi_i(t) : \text{Multi-Slater-determinant}$$
$$= \int_{t_1}^{t_2} dt \, \sum_{ij} C_i^*(t) \langle \Phi_i(t) | \left(i\hbar \, \dot{C}_j(t) | \Phi_j(t) \rangle + i\hbar \, C_j(t) \partial_t | \Phi_j(t) \rangle - C_j(t) \hat{H} | \Phi_j(t) \rangle \right)$$

┌ Action to be minimized: -

$$\begin{split} S &= \int_{t_1}^{t_2} dt \ \left\langle \Psi(t) \middle| i\hbar \partial_t - \hat{H} \middle| \Psi(t) \right\rangle & \Psi(t) = \sum_i C_i(t) \ \Phi_i(t) : \text{Multi-Slater-determinant} \\ &= \int_{t_1}^{t_2} dt \ \sum_{ij} C_i^*(t) \left\langle \Phi_i(t) \middle| \left(i\hbar \dot{C}_j(t) \middle| \Phi_j(t) \right\rangle + i\hbar C_j(t) \partial_t \middle| \Phi_j(t) \right\rangle - C_j(t) \hat{H} \middle| \Phi_j(t) \right\rangle \right) \\ &= \int_{t_1}^{t_2} dt \ \sum_{ij} C_i^*(t) \left(i\hbar \dot{C}_j(t) \mathcal{N}_{ij}(t) + C_j(t) \mathcal{A}_{ij}(t) - C_j(t) \mathcal{H}_{ij}(t) \right) \\ &\text{Norm kernel} & \text{A transition matrix} & \text{Interaction kernel} \\ \mathcal{N}_{ij}(t) &= \left\langle \Phi_i(t) \middle| \Phi_j(t) \right\rangle & \mathcal{A}_{ij}(t) = \left\langle \Phi_i(t) \middle| i\hbar \partial_t \middle| \Phi_j(t) \right\rangle & \mathcal{H}_{ij}(t) = \left\langle \Phi_i(t) \middle| \hat{H} \middle| \Phi_j(t) \right\rangle \\ &= \det \left\{ \left\langle \phi_k^{(i)}(t) \middle| \hat{h}^{(j)}(t) \middle| \phi_l^{(j)}(t) \right\rangle \right\} \end{split}$$

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$$\begin{split} S &= \int_{t_1}^{t_2} dt \ \left\langle \Psi(t) \middle| i\hbar\partial_t - \hat{H} \middle| \Psi(t) \right\rangle & \Psi(t) = \sum_i C_i(t) \ \Phi_i(t) : \text{Multi-Slater-determinant} \\ &= \int_{t_1}^{t_2} dt \ \sum_{ij} C_i^*(t) \left\langle \Phi_i(t) \middle| \left(i\hbar \dot{C}_j(t) \middle| \Phi_j(t) \right\rangle + i\hbar C_j(t) \partial_t \middle| \Phi_j(t) \right\rangle - C_j(t) \dot{H} \middle| \Phi_j(t) \right\rangle \right) \\ &= \int_{t_1}^{t_2} dt \ \sum_{ij} C_i^*(t) \left(i\hbar \dot{C}_j(t) \mathcal{N}_{ij}(t) + C_j(t) \mathcal{A}_{ij}(t) - C_j(t) \mathcal{H}_{ij}(t) \right) \\ &\text{Norm kernel} & \text{A transition matrix} & \text{Interaction kernel} \\ \mathcal{N}_{ij}(t) &= \left\langle \Phi_i(t) \middle| \Phi_j(t) \right\rangle & \mathcal{A}_{ij}(t) = \left\langle \Phi_i(t) \middle| i\hbar\partial_t \middle| \Phi_j(t) \right\rangle & \mathcal{H}_{ij}(t) = \left\langle \Phi_i(t) \middle| \hat{H} \middle| \Phi_j(t) \right\rangle \\ &= \det \left\{ \left\langle \phi_k^{(i)}(t) \middle| \hat{h}^{(j)}(t) \middle| \phi_l^{(j)}(t) \right\rangle \right\} \end{split}$$

Variation
$$\frac{\delta S}{\delta C_i^*(t)} = 0$$
 leads to:
Equation for the coefficients $\{C_i(t)\}$
 $i\hbar \mathcal{N}(t) \dot{\mathcal{C}}(t) = (\mathcal{H}(t) - \mathcal{A}(t))\mathcal{C}(t)$ where $\mathcal{C}(t) \equiv \begin{pmatrix} C_1(t) \\ C_2(t) \\ \vdots \\ C_M(t) \end{pmatrix}$

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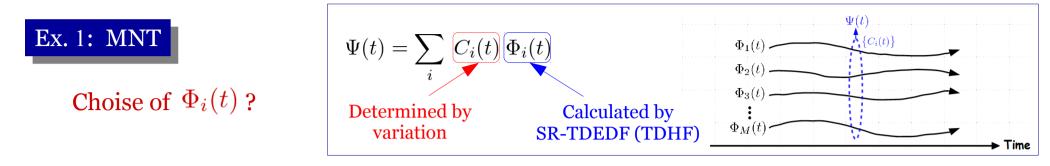
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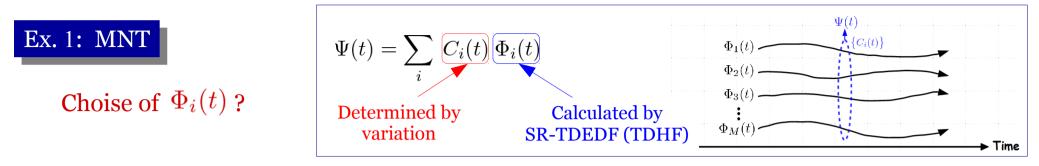
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4. Summary

Ideas: MR-TDEDF for MNT reaction

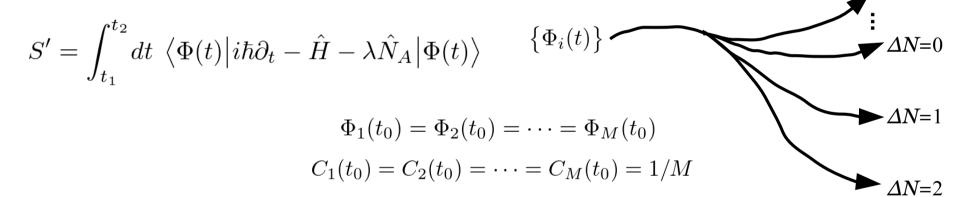


Ideas: MR-TDEDF for MNT reaction

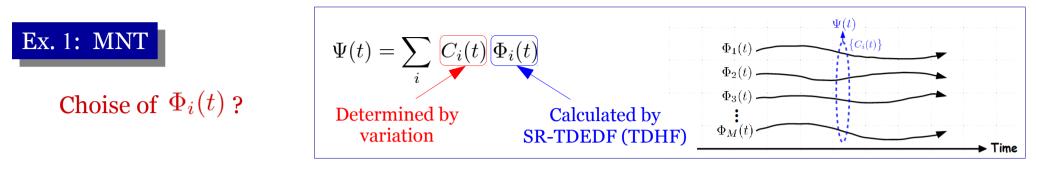


1 Transfer-constrained SR-TDEDF (TDHF)

C. Simenel, J. Phys. G **41**(2014)094007

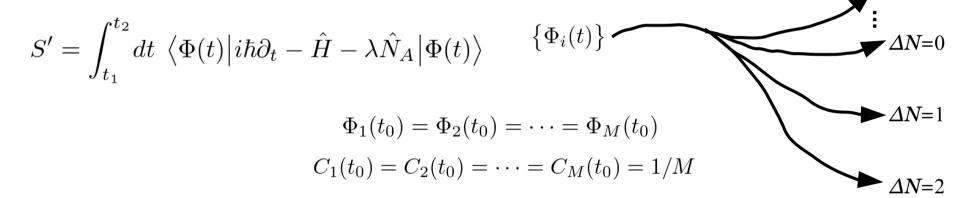


Ideas: MR-TDEDF for MNT reaction



1 Transfer-constrained SR-TDEDF (TDHF)

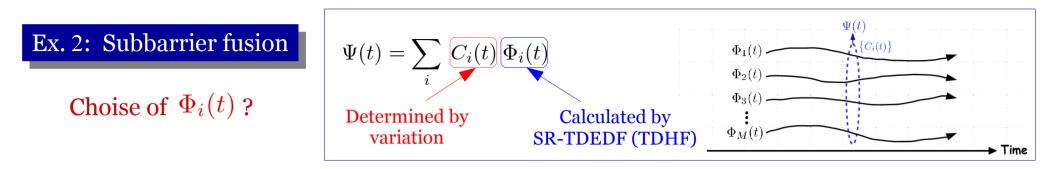




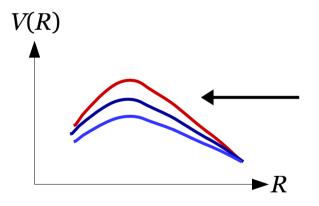
2 Use different projectile / target combinations

$$\Phi_1(t) = {}^{40}\text{Ca} + {}^{124}\text{Sn}$$
, $\Phi_2(t) = {}^{42}\text{Ca} + {}^{122}\text{Sn}$, $\Phi_3(t) = {}^{18}\text{Ar} + {}^{126}\text{Te}$, ...
 $C_1(t_0) = 1$ $C_{i \neq 1}(t_0) = 0$

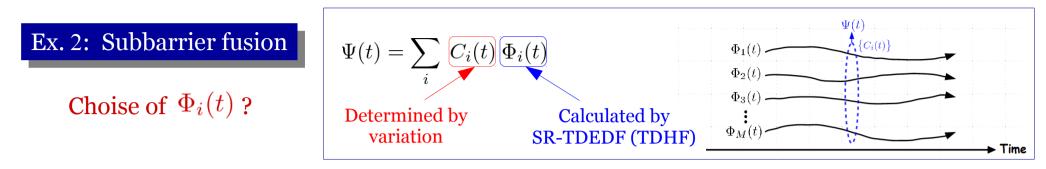
Ideas: MR-TDEDF for subbarrier fusion / SHE synthesis



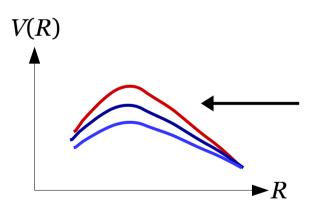




Ideas: MR-TDEDF for subbarrier fusion / SHE synthesis



1) Modify the nucleus-nucleus potential



2 Put static solutions in addition to SR-TDEDF

$$\Phi_1(t)$$
: SR-TDEDF

1. Introduction: Drawbacks in "SR-TDEDF"

2. Method: "MR-TDEDF" with a multi-Slater-determinant

3. "Ideas" for MNT / subbarrier fusion / SHE synthesis

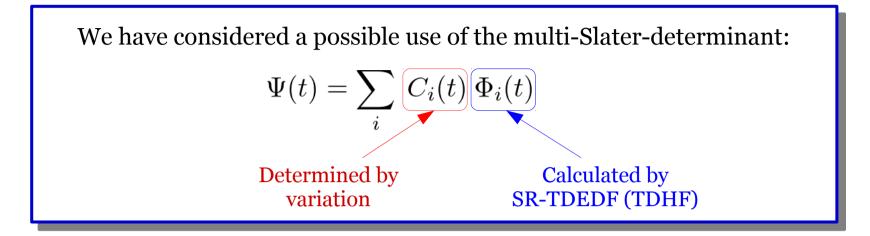
4. Summary

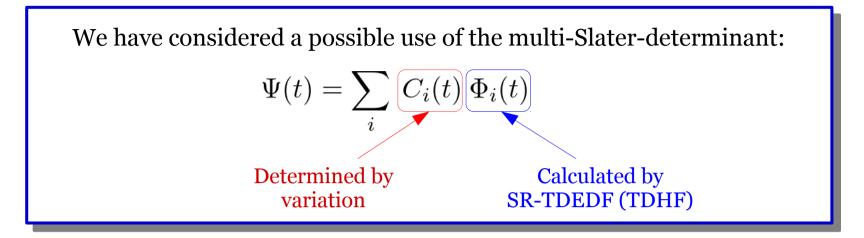
1. Introduction: Drawbacks in "SR-TDEDF"

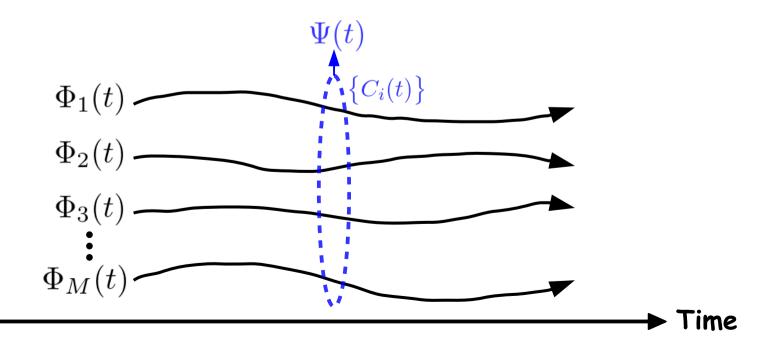
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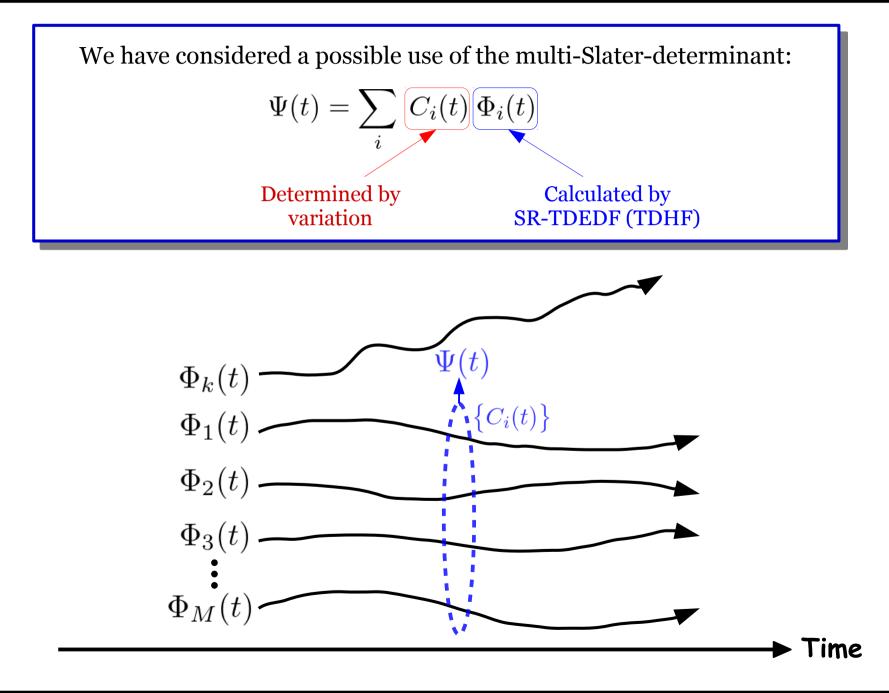
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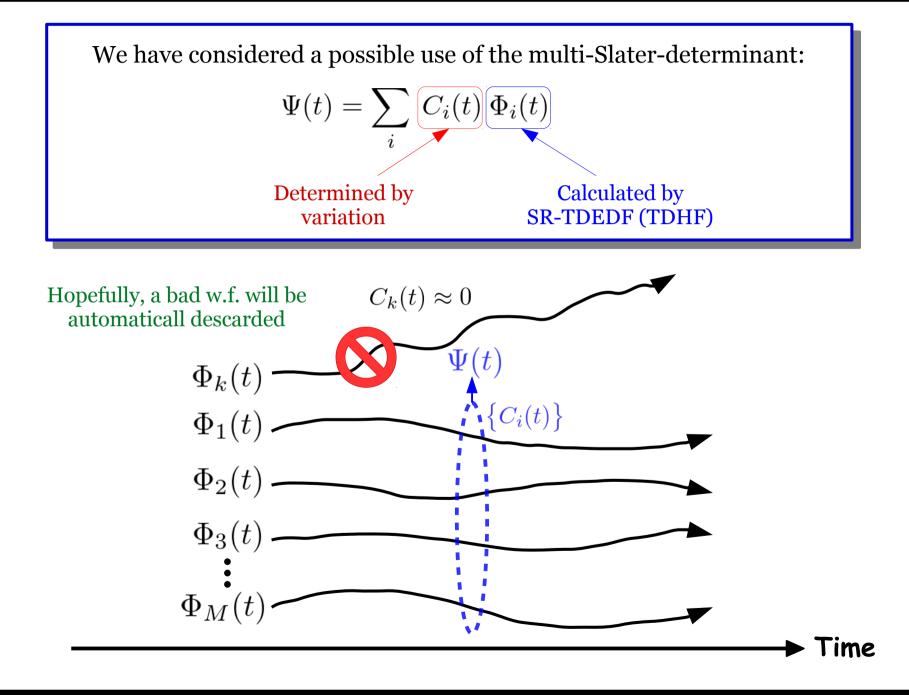
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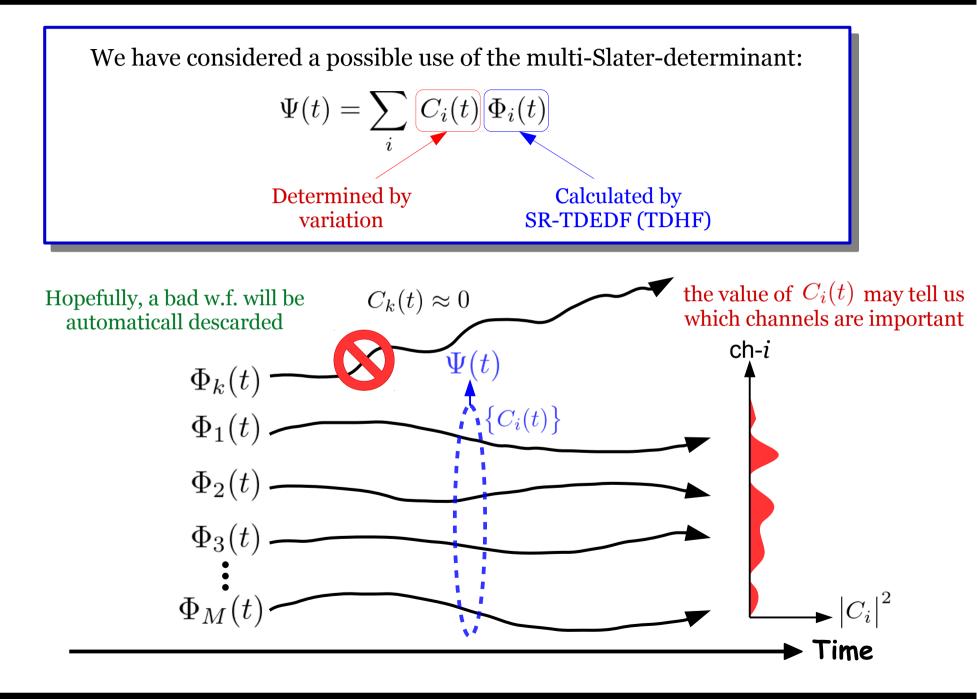




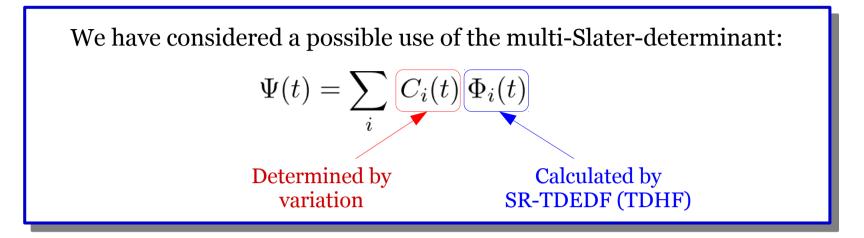




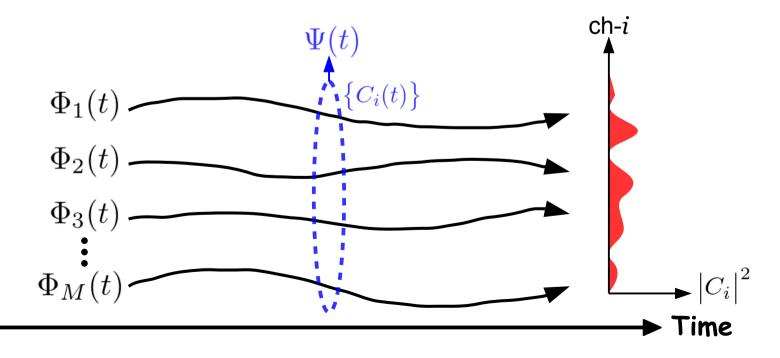




K. Sekizawa



Although there are many practical problems, we can try it in the near future!



About me:

Kazuyuki SEKIZAWA Nuclear Theory Group Faculty of Physics, Warsaw University of Technology, Poland Research Assistant Professor (adiunkt naukowy) E-mail: sekizawa @ if.pw.edu.pl URL: http://sekizawa.fizyka.pw.edu.pl/english/